### Study of the directional spectrum of ocean waves using array, buoy and radar measurements

### A. A. Fernandes

#### Dissertation

submitted to Goa University

for the Degree of

#### **Doctor of Philosophy**

in

Marine Science

### **Goa University**

June 1999

To my near and dear ones

## Statement

As required under the University ordinance 0.19.8.(vi), I state that this thesis entitled *Study of the directional spectrum of ocean waves using array, buoy and radar measurements* is my original contribution and it has not been submitted on any previous occasion.

The literature related to the problem investigated has been cited. Due acknowledgements have been made wherever facilities and suggestions have been availed of.

A. A. FERNANDES

National Institute of Oceanography, Goa June 1999

## Certificate

This is to certify that the thesis entitled *Study of the directional spectrum of ocean waves using array, buoy and radar measurements*, submitted by A. A. Fernandes to Goa University for the degree of Doctor of Philosophy, is based on his original studies carried out under my supervision. The thesis or any part thereof has not been previously submitted for any other degree or diploma in any university or institution.

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- Ocean waves
- Tides
- Surges
- Currents
- Palaeoclimate
- Computer programming in FORTRAN

#### Software developed

- Short-term wave statistics from wave by wave analysis and spectral analysis
- Directional spectrum of ocean waves from arrays using the method of Esteva and Borgman
- Directional spectrum of ocean waves from an orbital following buoy using the digital band pass filter method
- Directional spectrum of ocean waves from SAR imagery
- Spectral, cross spectral and rotary spectral analysis
- Digital low pass, high pass and band pass filtering in time domain as well as frequency domain
- Factor analysis or empirical orthogonal function analysis
- Multiple regression analysis
- Statistical prediction of palaeoclimate

#### Select List of Publications other than ocean waves

- Nautical and physical oceanographic observations off the drilling rig Chancellors Ville in the Mahanadi Basin. Project Leaders: Ehrlich Desa, A. A. Fernandes and P. V. Sathe. NIO Technical Report No. 05/80, December 1980.
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## **Synopsis**

A brief review of the available methods of measurement and analysis of the wave (1-30 s period) directional spectrum is given in Chapter 1. The review also includes the directional spectrum of surf beat (30 s - 5 minutes period).

Chapter 2, contents of which are summarized below, discusses my original contribution in the determination of the directional spectrum of ocean waves from array measurements.

Wave direction has for the first time been consistently, accurately and unambiguously evaluated from array measurements using phase/time/path difference (PTPD) methods of Esteva [1977] in case of polygonal arrays and Borgman [1974] in case of linear arrays. A description of these methods is given. Cross spectrum analysis, which is employed in these methods, is briefly described. Time series measurements of water surface elevation at a 15-gauge polygonal array, in  $\approx 8m$  water depth, operational at the Coastal Engineering Research Center's (CERC) Field Research Facility at Duck, North Carolina, USA, have been used. Two modifications have been made in the methodology. One modification is that instead of the *apparent* phase in the range  $(-\pi, \pi)$  the *true* phase in the range  $(-\infty, \infty)$ , has been used; the other modification being that estimates of wave direction are registered only if the relevant gauges are coherent at 0.01 significance level.

It is shown that PTPD methods, which were not successfully used heretofore, are adept in describing the propagation of waves generated by distant storms (swell), locally generated waves (sea), infragravity waves (surf beat) locally generated by energetic incident swell, and surf beat of remote origin occurring when low swell energies are present. PTPD methods fail in case of multimodal directional spreads, which can be easily spotted from the large value of the standard deviation of redundant estimates of wave direction. Specifically it was found that at Duck: (i) the directions of swell and surf beat, when energetic swell is present, conform to the schematic diagram of surf beat generation given by Herbers et al. [1995] (ii) surf beat of remote origin occurs when the significant wave height,  $H_{mo}$  falls below 0.41 m (iii) the surf beat of remote origin is not normally incident at the shore contrary to Herbers et al. [1995]. In fact it was found that the surf beat of remote origin is incident at angles in excess of  $45^{\circ}$  with respect to the shore normal, and

(iv) the surf beat of remote origin is largely trans-oceanic in origin.

PTPD methods assume that in a spectral frequency band, the waves can approach from a single direction, the wave direction may be different for different spectral bands and are simple, expedient and provide redundant estimates of wave direction. Two gauges are sufficient for determining wave direction using the method of Borgman [1974] while 3 non-collinear gauges are required in case of the method of Esteva [1977].

In the wind wave regime (0.04 - 0.32 Hz), estimates of wave direction as a function of frequency using Esteva's method with the two modifications described above, are consistent with the results of frequency-direction spectra obtained by the CERC using the sophisticated Iterative Maximum Likelihood Estimation (IMLE) method described by Pawka [1983], which assumes that at a given frequency band waves can approach simultaneously from all directions.

Wave direction estimates using the method of Borgman [1974] suffer from an ambiguity within a mirror symmetry as the method is applied to linear arrays. In contrast wave direction estimates using the method of Esteva [1977] are unambiguous as the method is applied to polygonal arrays.

There are two reasons why PTPD methods for determining wave direction from array measurements have not become popular; the first being the insufficient documentation of Borgman [1974] in case of linear arrays; and the second being the failure of Esteva [1977] in determining wave direction correctly over the design range 25–7 s of her 5-gauge polygonal array at Pt. Mugu, California. Esteva was able to determine correctly the direction of the observed swell of 16 s but failed to do so in case of the observed swell of 8 s. Fernandes et al. [1988] provided the necessary documentation in case of linear arrays, and repeating the computer simulations of Esteva for her polygonal array were successful in consistently, accurately and unambiguously determining the direction of both 16 s as well as 8 s swell. Their success was due to their adroit use of the criterion of Barber and Doyle [1956], which stipulates that for ensuring correct directions, the distances between the gauges 12 and 13 in the semi-ordered gauge triad 123, should both be less than half a wave length for the particular frequency band. Thus Fernandes et al. [1988] established that the method of Esteva for determing wave direction unambigously from polygonal arrays works in case of computer simulated data. **The principal object of the present study is to establish that the method Esteva works in case of measured data also.** 

Chapter 3, contents of which are discussed below discusses my work in the determination of the wave directional spectrum using an orbital following buoy.

Software has been developed for the determination of wave direction from time series measurements of heave, pitch and roll of an orbital following buoy. The method of digital band pass filtering described by Edward C. Brainard II of Endeco Inc., USA was used. A derivation of the basic formula used is presented. The software developed by me, viz. BUOY-D-P.FOR, and the software provided by ENDECO, viz. 1156DBP.EXE, were both run for observed as well as computer simulated data. In the case of observed data, wave directions obtained with BUOY-D-P agreed within  $\pm 5^{\circ}$  with the single available visual estimate of swell direction and within  $\pm 5^{\circ}$  with directions obtained with 1156DBP. In the case of computer simulated data, BUOY-D-P was able to recover wave directions accurately for both mono-chromatic and multi-chromatic wave trains. 1156DBP was equally successful, except that if the simulated wave direction was  $\theta$ , the computed direction was found to be  $\pi + \theta$ . This offset has been explained.

Chapter 4, contents of which are described below discusses my work in the determination of the wave directional spectrum using Synthetic Aperture Radar (SAR), which is the modern method for measuring waves.

Gaussian smoothed SAR image spectra have been evaluated from 512X512 pixel sub-scenes of image mode ERS-1 SAR scenes off Goa, Visakhapatnam, Paradeep and Portugal. The two recently acquired scenes off Portugal showed the signature of swell of wavelength  $\approx 200m$  and internal waves of wavelength > 400m. Only internal waves of wavelength > 400m were seen in the scene off Goa observed on 11 March 1992. The scenes off Visakhapatnam and Paradeep did not show any wave like features, the latter appearing to be of "white noise" nature.

There exists a 180° ambiguity in wave direction observed from radar imageries. Based on the method of Atanassov et al. [1985], a computer program has been developed for removal of the 180° ambiguity by using two images of the same area separated by a time interval which is small compared to the period of the dominant waves. The computer program has been successfully tested with computer simulated images. A brief review of the method of Atanassov is given.

The major conclusions of the thesis, summarized in Chapter 5, are:

- 1. PTPDM of Esteva [1977] and Borgman [1974] have for the first time been successfully used to determine wave directional spectrum from *measured* data. The software for the same has been indigenously developed.
- 2. Software has indigenously been developed for the determination of the wave directional spectrum from time series measurements of heave, pitch and roll of an orbital following buoy using the digital band pass filter method. The software has been validated with computer simulated data. Some modifications are needed to deal with measured (field) data.
- 3. Software developed for removal of the 180° ambiguity in wave direction from computer simulated images should be useful to remove the said ambiguity from actual satellite imagery if at least two looks of the multi-look data is available.

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### Chapter 1

## INTRODUCTION

Ocean waves (1–30 s period) are generated by wind, the restoring force being gravity. Growth of waves depends on the fetch and duration of the wind. Waves smaller than 1 s period are known as ripples for which the restoring force is surface tension. Kinsman [1965] is an excellent book on ocean waves.

Generally waves are measured for a period of 20 minutes, every three hours. The analysis of the 20 minute record yields short term statistics, viz., significant wave height, zero crosssing period, etc.. An analysis of significant wave height, zero crosssing period, etc., over a period of 1–5 years yields long term wave statistics, viz., highest significant wave height that will occur once in 50 or 100 years known as return period.

The directional spectrum means the distribution of wave energy density as a function of frequency and direction. A cross-section of the directional spectrum surface for a given frequency, which is a curve depicting the wave energy density as a function of direction is also referred to as the directional spectrum. Essentially the directional spectrum is a representation of the 2dimensional wave number spectrum obtained using the dispersion relation for surface waves. The directional spectrum gives a more complete description of the waves occuring at a particular place and time than other representations such as significant wave height and the predominant wave height and direction.

The earliest and the most enduring application of the directional spectrum was in the field of wave forecasting and hindcasting by Pierson et al. [1955], the WAM model of Hasselmann et al. [1988] being its present day analogue. In another case Komar and Inman [1970] have used directional spectra for the study of littoral transport. Directional spectra have been used by for studying the diffraction of wind waves. Directional spectra are also useful for the design and analysis of moorings, offshore towers and piles. For the analysis of vibration and three dimensional analysis of structures with torsional loads, directional spectrum is an input function. Directional spectra are necessary for the study of growth and decay of waves and to acertain the frequency and di-

rectional spread of energy density by processes like resonant interactions. Occurrence of "freak" waves which have been responsible for the destruction of breakwaters in Iceland, Norway and France is thought to be linked with the wave directional spectrum of ocean waves [Bruun, 1985].

Panicker [1974] listing as many as 91 references gives an extensive review of the methods of measurement and analysis of the wave directional spectrum.

The three main methods for measuring the wave directional spectrum are through (i) arrays (ii) buoys and (iii) radar.

Arrays and buoys directly measure the wave directional spectrum, while radar measures it through remote sensing using radio or microwaves. In array type of measurement just one parameter, viz., wave elevation is measured at several points in a linear or polygonal array. In buoy type of measurements several different parameters are measured at essentially the same point. As we shall see later two types of buoys are available, viz., slope following buoys and orbital following buoys. In radar type of measurements, the backscatter from Bragg resonant surface waves is measured. It was Crombie [1955] who identified Bragg scattering as the mechanism for imaging "sea-clutter" (ocean waves) from radar echo from sea at High Frequency (dekameter waves).

The wave directional spectrum can also be measured using optical methods including stereophotogrammetry which has been done in the Stereo Wave Observation Project reported by Cote et al. [1960] and Chase et al. [1957], who used two simultaneously taken photographs of the sea surface.

Uberoi [1964] used an optical analogue method to determine the wave directional spectra of 2-dimensional functions from single photographs. Stilwell and Pilon [1974] using the concept of continuous skylight luminance function obtained wave directional spectra from single photographs of the sea surface using optical 2-dimensional Fourier Transform technique, utilizing auxiliary data of altitude and azimuth, the method being a generalization of the sun glitter method of Cox and Munk [1954] and Cox [1958], which provided statistics of surface slopes but could not determine the wave directional spectrum.

Panicker [1974] briefly describes the following mathematical techniques available for determining the wave directional spectra from cross-spectra between the measured quantities: (i) Direct Fourier Transform method, which is useful when large number of cross-spectra are available as in the case of the Stereo Wave Observation Project (ii) Parametrized Estimation method, which assumes a parametrized representation of the wave directional spectrum and uses an inverse relationship to evaluate the defining parameters from the cross-spectra. This technique has been used to estimate the directional spectrum when only a limited number of cross-spectra are available as in the case of arrays and buoys (iii) Amplitude and phase detection techniques (iv) Discrete wave train analysis which may be used when energy is concentrated in a finite number of directions, and (v) Data adaptive Techniques, which include Maximum Likelihood Estimation method (MLE) and Maximum Entropy Method. These data adaptive methods specially the MLE method and its refinement the Iterative Maximum Likelihood Estimation (IMLE) method yield high resolution wave directional spectra and are being extensively being used in case of buoys and arrays.

It has been found, as elaborated in Chapter 2, from array, buoy as well as radar measurements that the wave directional spectrum has a unimodal distribution, so that the computation of the "mean" wave direction as a function of frequency is both meaningful as well as useful and is routinely performed in case of commercially available wave directional buoys marketed by ENDECO, WAVEC and NEREIDES.

Chapter 2 discusses about arrays giving the design of some important arrays reported in literature alongwith a review of the techniques of analysis used to determine the wave directional spectrum from array measurements. The highlight of this thesis is that for the first time, wave direction has been successfully evaluated as a function of frequency from actual array measurements over a wide range of frequencies including locally generated waves (sea), waves generated by distant storms (swell), and infra gravity waves (surf beat) – using what we have called as Phase/time/path difference methods, which are essentially amplitude and phase detection techniques, a description of which is given in case of linear as well as polygonal arrays. A description of cross-spectrum analysis, which forms the basis of all techniques for determining the wave directional spectrum from arrays and buoys, is also given.

Chapter 3 gives a brief review of some important buoys reported in literature and the techniques of analysis of the wave directional spectrum for both slope following buoys as well as orbital following buoys. A salient part of this thesis is that I have indegeneously developed the software for determining the wave direction as a function of frequency from an ENDECO orbital following buoy using the ENDECO Digital Band Pass Filter method and applied it to actual field measurements.

Chapter 4 essentially discusses the problem of determining the wave directional spectrum from Synthetic Aperture Radar imagery. Wave directional spectrum has been determined from actual SAR scenes observed off Goa, Visakhapatnam, Paradeep and Portugal. A method for removal of the 180<sup>o</sup> ambiguity in determination of wave direction from radar imagery which I have successfully implemented is described. Some information about radar, radiowaves, microwaves etc., is also given.

Chapter 5 lists the conclusions of the work carried out and reported in this thesis on the determination of wave directional spectrum using arrays, buoys and radar.

Tables in Appendix A and Appendix B give the wave directional estimates of measured data at Duck, North Carolina, USA, using phase/time/path difference methods in case of linear and polygonal arrays respectively – these tables constitute the principal results of this thesis.