

PARAMETRIC MODELLING OF NONSTATIONARY PLATFORM DECK MOTIONS

S.Mandal
Ocean Engineering Division
National Institute of Oceanography
Dona Paula, Goa - 403 004, India
email: smandal@csnio.ren.nic.in

ABSTRACT

Univariate system identification models are, in general, developed based on the assumption that offshore dynamic systems are stationary random processes. For nonstationary processes, a method is adopted to transform the time series into wide-sense-stationary processes. Then the time series are modelled by the maximum entropy method which is formulated here for spectral estimation of platform deck displacements. The lower order maximum entropy spectra of nonstationary platform deck displacements are compared with fast Fourier transform spectra and show good agreement. However, the higher order maximum entropy model can be used for better representation of nonstationary motions. This method also reduces long time series of nonstationary offshore data into a few values which are the coefficients of the maximum entropy model.

INTRODUCTION

This paper is concerned with the application of maximum entropy (ME) method to nonstationary time series data collected from offshore structural monitoring systems. This technique has been applied to ocean engineering problems which are wide-sense stationary processes [Burg (1975), Briggs and Vandiver (1982), Samii and Vandiver (1984), Mandal et al (1995)]. The advantages of ME method over the commonly known fast Fourier transform (FFT) method are: (a) the ME model with few parameters describes the dynamic processes and (b) significantly minimizes data storage problems for long term measuring systems. Proper ME modelling not only could solve mass data storage and handling problems, but also be used to detect structural failures and other unknown phenomena (Worden et al, 1992) in the presence of noise in measured offshore dynamic systems.

The offshore dynamic systems are, strictly speaking, nonstationary. For application purposes, when a random process follows a normal (Gaussian) distribution, a offshore dynamic system can be broadly used as a wide-sense stationary process. However, this is not always the case. Offshore platform deck displacements measured at North Sea are strictly nonstationary. Nonstationary time series data can not be directly modelled using autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) algorithms which are valid for univariate stationary offshore dynamic processes [Spanos (1983), Witz and Mandal (1991),

Mandal et al (1992)]. However, nonstationary processes can be more precisely modelled using transformed stationary processes and the ME model.

This paper presents the use of a ME model for a nonstationary time series motions of offshore platform's deck. The method of establishing dynamic models of nonstationary full scale data described here is simple and easy to formulate provided one is familiar with ME modelling of stationary processes which is described by Burg (1975). This is possible because of the differencing and integrating techniques for transforming the nonstationary time series to the stationary time series, and vice versa. These techniques along with the ME spectral algorithm are described here.

FORMATION OF STATIONARY PROCESSES

It is very difficult to directly analyze the nonstationary processes using any conventional method. There are some simple steps (described below) to be undertaken and thereafter we use the ME technique.

Transformation Technique

A nonstationary process, y_t is subjected to some form of data transformation and differencing, d times to yield stationary process, z_t . If the number of differencing is zero, i.e., $d = 0$, an ME of z_t enables a wide class of stationary processes. If the number of differencing is greater than zero ($d = 1, 2, 3, \dots$), the appropriately transformed and differenced time series can be modelled by the ME technique.

The differencing d is a simple operation which involves determining successive changes in the values of a time series. Using the first differences of y_t , define a new variable, z_t as

$$z_t = y_t - y_{t-1} \quad \text{for } t = 2, 3, 4, \dots, n \quad (1)$$

Here the time series, z_t is called the first differences of the time series, y_t . If the first differencing does not exhibit a stationary process, z_t can be redefined by using the second differences ($d = 2$).

Differences of the first differencing can be expressed as

$$\begin{aligned} z_t &= (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) \\ &= y_t - 2y_{t-1} + y_{t-2} \quad \text{for } t = 3, 4, \dots, n \end{aligned} \quad (2)$$

Similarly we can continue the differencing until the new time series shows a stationary process.

Selection of Number of Differencing

Here the transformation of a nonstationary process to a stationary process is depended on the number of differencing operations, d which can be selected based on the following procedures:

- a) Checking the time series data: This generally gives a clue to the approximate degree of differencing required. Using nonstationary time series data, initially we use the first difference ($d=1$) and then examine the first differenced time series. If it still shows nonstationarity, we use $d=2$ and then check the second differenced time series. This is the way of checking the trend of differenced time series and its mean value which tends towards zero.
- b) In order to check the estimated autocorrelations of the original time series and the differenced time series, variation of the autocorrelation values with time lag can also sometimes give the proper selection of d values for which differenced time series form a stationary process. For wide sense stationary processes, autocorrelations do not vary with time.

Backshift Notation

Using above simple differencing techniques we are transforming a nonstationary time series into a stationary time series and then estimating the ME parameters. However, we are interested in predicting the original time series. This is not a problem as we can regain the original time series using a backshift operator, \mathfrak{R} .

The backshift operator, \mathfrak{R} , is such that the time series data can be expressed as

$$\mathfrak{R}^d y_t = y_{t-d} \quad \text{for } d = 1, 2, \dots \quad (3)$$

To make meaningful sense of the backshift operator, equation (3) states that \mathfrak{R} shifts time subscripts. When \mathfrak{R} is used in an algebraic expression it must be multiplied by some other variable such as y_t . Therefore, the backshift operator is meaningful because it changes the time subscript on the variable by which it is multiplied as shown in equation (3). Multiplying a constant, C by any number of backshift operations \mathfrak{R} does not alter the constant, i.e.,

$$\mathfrak{R}^d C = C \quad \text{for } d = 1, 2, \dots \quad (4)$$

The first differenced time series, z_t of the original time series, y_t can be expressed by using the differencing operator $(1-\mathfrak{R})$ as

$$\begin{aligned} z_t &= y_t - y_{t-1} \\ &= (1-\mathfrak{R})y_t \end{aligned} \quad (5)$$

As \mathfrak{R} is not a number, $(1-\mathfrak{R})$ has no numerical value; it is denoted as an operator. In above equation, $(1-\mathfrak{R})$ is multiplied by an original time series variable to express the first differences of that variable.

The second differenced time series can be expressed as

$$\begin{aligned} z_t &= y_t - 2y_{t-1} + y_{t-2} \\ &= y_t - 2\mathfrak{R}y_t + \mathfrak{R}^2y_t \\ &= (1-\mathfrak{R})^2y_t \end{aligned} \quad (6)$$

Similarly d differenced time series can be expressed as

$$z_t = (1-\mathfrak{R})^d y_t \quad (7)$$

A relation between the differenced stationary time series, z_t with respect to the original nonstationary time series, y_t by the operator $(1-\mathfrak{R})^d$ is shown in above equation.

Regaining Nonstationary Processes

Equation (7) shows that the z values are the differences of the y values and the y values are the sums of the z values. Therefore, y values can be obtained by integrating (summing) the z values. Even though, we can build up the ME model by transforming the nonstationary time series to the stationary time series, our main aim is to regain the original nonstationary time series as described below.

Let the stationary time series z_t for a differencing of $d = 1$, then equation (5) can be rewritten as

$$\begin{aligned} y_t &= (1-\mathfrak{R})^{-1} z_t \\ &= [1 + \mathfrak{R} + \mathfrak{R}^2 + \mathfrak{R}^3 + \dots] z_t \\ &= z_t + \mathfrak{R}z_t + \mathfrak{R}^2z_t + \mathfrak{R}^3z_t + \dots \\ &= z_t + z_{t-1} + z_{t-2} + z_{t-3} + \dots \\ &= \sum z_t \end{aligned} \quad (8)$$

Now, suppose we have a differencing of $d = 2$, equation (6) can be rewritten as

$$\begin{aligned} y_t &= (1-\mathfrak{R})^{-2} z_t \\ &= (1-\mathfrak{R})^{-1} [(1-\mathfrak{R})^{-1} z_t] \end{aligned} \quad (9)$$

Let the second term (in square bracket) in equation (9) be defined as x_t . Equation (9) can then be rewritten as

$$y_t = (1-\mathfrak{R})^{-1} x_t \quad (10)$$

where

$$x_t = (1 - \mathfrak{R})^{-1} z_t \quad (11)$$

Therefore, once x_t is obtained from equation (11) in the same way as shown in equation (8), substituting x_t in equation (10) determines the original nonstationary time series, y_t . In this way we can continue to integrate d number of times to get back the original time series.

MAXIMUM ENTROPY SPECTRUM

The time series z_r can be expressed by the maximum entropy method, if it is generated from the relationship

$$z_r = - \sum_{k=1}^p a_k z_{r-k} + \sqrt{\rho_w} w_r \quad (12)$$

Here z_r is the r th sample of the discrete random process, a_k are the ME parameters, and w_r is the white noise with variance, ρ_w .

The estimated ME spectrum, $S_{zz}(\omega)$ of the time series z_r can be expressed as

$$S_{zz}(\omega) = \frac{\rho_w}{|1 + \sum_{k=1}^p a_k e^{-ik\omega T}|^2} \quad (13)$$

Here T is the sampling period of the time series and ω is the frequency.

The ME parameters are estimated using the forward-backward method (Marple, 1987).

RESULTS

Lateral displacements of a large fixed offshore platform's deck (M/S British Petroleum, UK) located in the North Sea are investigated here to illustrate the application of the ME model to an offshore engineering problem. The displacements of 1024 data points at 5Hz sampling rate are used. The displacements were nonstationary which can be observed in Figure 1. Before ME modelling, the nonstationary displacement time series data are transformed to stationary time series data by the number of differencings. Figure 2 shows the first differenced ($d=1$) time series data. It is observed that the nonstationarity still remains. Further differencing yields the second differenced ($d=2$) time series data which is shown in Figure 3. This appears to form a stationary process which is consistent. By integrating (summing) twice (using

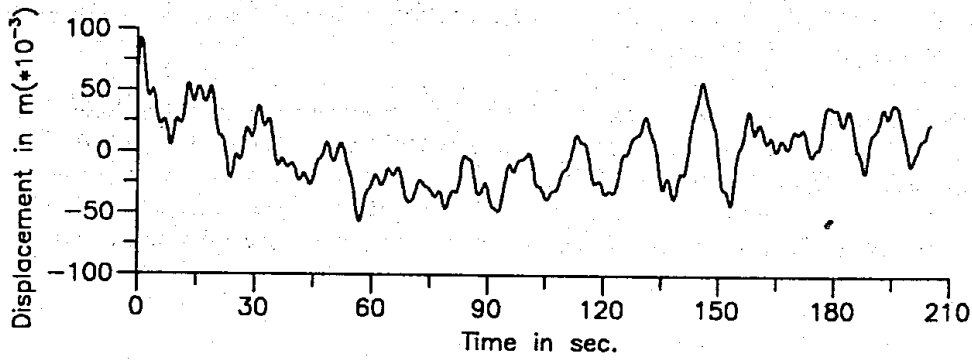


Fig.1 Time series offshore platform deck displacement

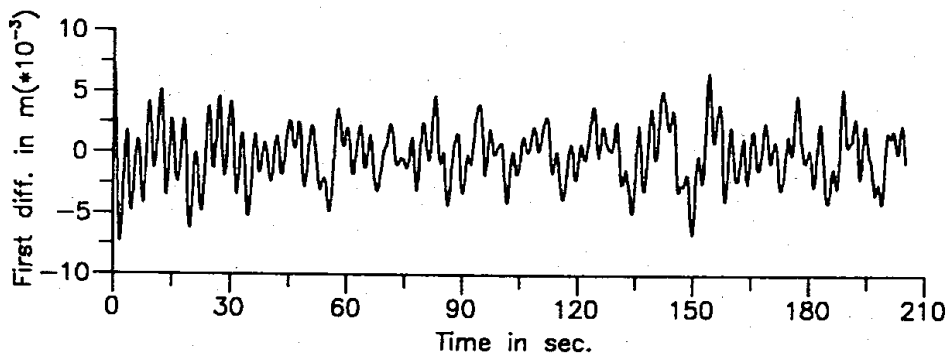


Fig.2 First differenced time series estimated from Fig.1

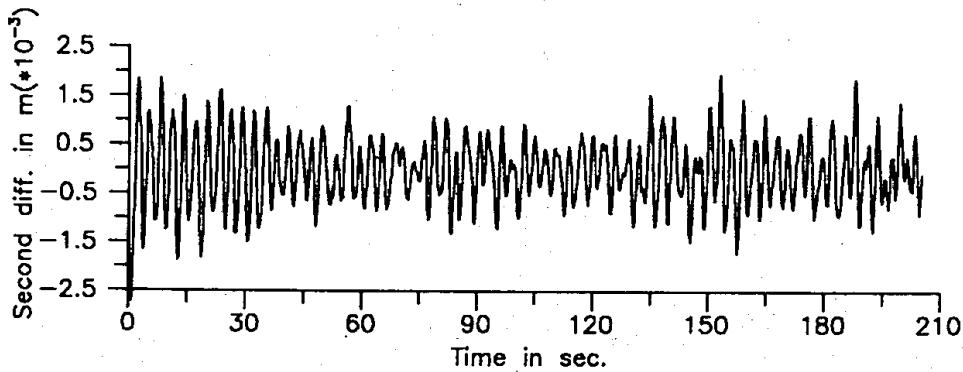


Fig.3 Second differenced time series estimated from Fig.1

equations 11-14), the time series data of Figure 3 can be converted back exactly to the original nonstationary data of Figure 1.

Variation of the least squares error (LSE) with ME model order is shown in Figure 4. It indicates that the LSE does not decrease appreciably for ME model orders greater than 18. The estimated power spectral energy (PSE) of the ME model with model order 18 is compared with the PSE determined by the fast Fourier transform (FFT) method as shown in Figure 5. This shows that the lower order ME model can not only well represent the first highest PSEs

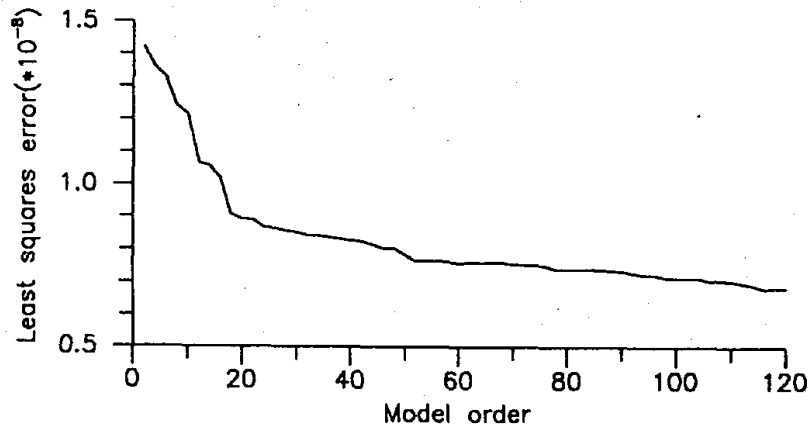


Fig.4 Least squares error variation

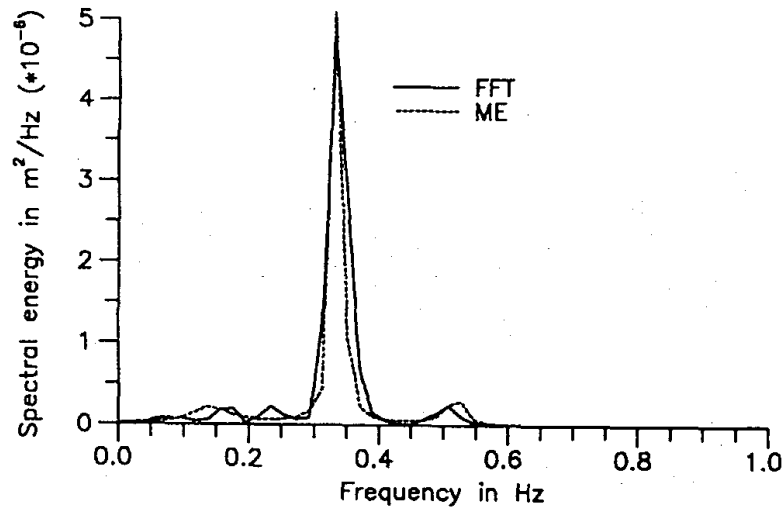


Fig.5 Spectra calculated by FFT and ME (18) model

near frequency 0.332 Hz, it could also pick up the second higher PSEs near 0.508 Hz. In addition the agreement is reasonable at frequencies below 0.3 Hz which are associated with wave excitation frequencies. Therefore, the second differenced time series data of platform deck displacements can be represented by the ME model with model order 18. However, for better representation of the nonstationary process, one needs to use higher order ME model. The ME model with model order 114 gives a better representation of the nonstationary time series (Figure 6). Here the spectral energies calculated by the FFT and ME models are in good

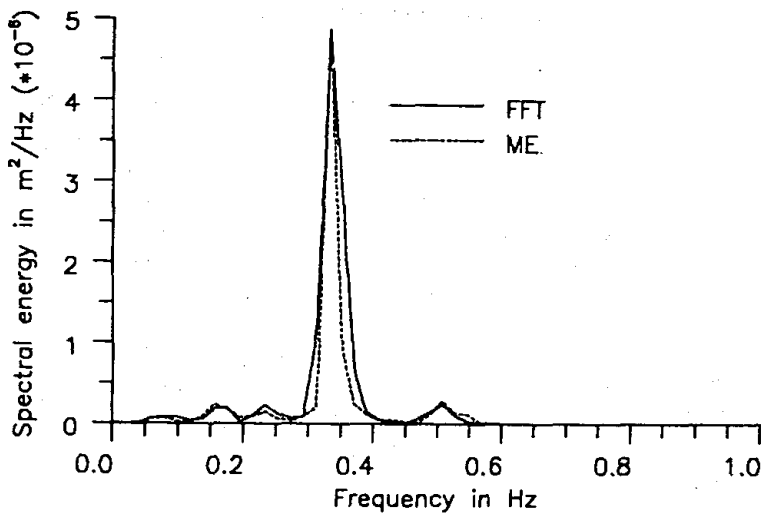


Fig.6 Spectra calculated by FFT and ME(114) model

agreement at the lower frequencies (0.098, 0.156 and 0.234 Hz). Therefore, effect of wave forces measured by the deck acceleration time histories can be well described by the ME model.

Often the power spectrum is estimated from the discrete time series record using the FFT method. Here the number of spectral values representing the time series of n data points is $n/2$. Whereas from the above result it is observed that the relatively lower order ME spectrum provides reasonable agreement with the FFT spectrum of nonstationary platform deck motion.

DISCUSSION

The ME model can be directly applied only to stationary offshore dynamic processes, that is, mean, variance and autocorrelations of any random process are constant through time. The time series can be nonstationary in many ways. A time series can have nonstationary mean, variance, autocorrelations, or combination of these statistical parameters. The most common nonstationary processes can be those with some steady trend in the mean of the time series. Such types of time series can simply be transformed to a stationary time series by using the

differencing technique. Other types of time series may be such that their trend in variability is more complex in nature, and transformation of such time series to a stationary process may not be achieved so easily. In these cases suitable algebraic transformation techniques need to be developed.

In offshore dynamic systems nonstationary processes are sometimes measured. ME modelling of this type of time series data can be possible through transformation of a nonstationary process to a stationary process by the differencing technique. Complete removal of nonstationarity may not always be easy. From the practical application point of view an approximate transformed stationary time series is likely to be adequate for estimation of the ME parameters.

The application of ME modelling to nonstationary time series offshore data is examined in previous section. The nonstationary time series of platform displacements, here are homogeneous, i.e., different segments of each time series seem to behave similar to the rest of the time series if we allow for changes in each segment's trend which may be level and/ or sloped. In other words, if we can eliminate the slope and level of different segments of a nonstationary process, it forms an apparently stationary process. Such type of time series can be simply transformed to stationary process by one or two time differencings.

After differencings to form a stationary process, the ME spectral model is constructed. The lower order ME model can provide good fit to describe the presented nonstationary offshore platform deck displacements with two differencings. However, the present study shows that the higher order model seems to yield a much improved representation of the deck displacements as compared to a lower order model.

We should avoid unnecessary differencing which may create artificial patterns in the time series data and reduces the estimation accuracy. The number of differencings can be chosen based on formation of wide-sense-stationary processes and checking autocorrelations of original and differenced time series data.

To recover the original nonstationary time series the differenced time series needs to be integrated, which involves summing successive values in a differenced time series.

CONCLUSIONS

The nonstationary offshore structural motions can not be directly modelled using ME algorithms which are generally used for wide sense stationary processes. One has to observe the time series in respect to its nature of nonstationarity, i.e., levels and trends of nonstationarity. The study presented herein considered the cases where the observed nonstationary time series having nonstationary mean are modelled using differencing techniques. To regain nonstationary process, one has to integrate by a number of times equal to the number of differencings already undertaken. The main purpose of forcing the nonstationary time series to a stationary time series is to model the dynamic process by formulating ME algorithms.

The measured nonstationary platform deck displacements have been successfully modelled using ME algorithms. The FFT spectral analysis requires relatively large amount of information to be retained in order to ensure proper description of the dynamic system. The present study has shown that, for example, 1024 digital platform deck motion data can be reduced to only few values which are the ME parameters. This has significant advantages in terms of not only analyzing nonstationary offshore dynamic processes but also the data storage requirements for long-term structural monitoring system.

Acknowledgements

The author would like to gratefully acknowledge the support of the Director, National Institute of Oceanography, Goa.

References

- Briggs, J. and Vandiver, J.K., 1982. Multichannel maximum entropy method of spectral analysis applied to offshore platforms. *Proc. Offshore Technology Conference*, Houston, Texas, OTC 4286, pp 647-659.
- Burg, J.P., 1975. Maximum entropy spectral analysis. PhD thesis, Dept of Geophysics, Stanford University, CA.
- Ljung, L., 1991. Signal Processing Toolbox, MATLAB for VAX/VMS Computers, Math Works Inc., MA 01760, USA.
- Mandal, S., Witz, J.A. and Lyons, G.J., 1992. Reduced order ARMA spectral estimation of ocean waves. *Applied Ocean Research*, 14(5), pp 303-312.
- Mandal, S., Sanil Kumar, V. and Nayak, B.U., 1995. Autoregressive modelling of measured sea waves off west coast of India, Proc. 4th Intl. Conf. on Coastal and Port Engineering in developing countries, Rio-de-Janeiro, Brazil, pp 1752-1763.
- Marple, S.L., 1987. *Digital Spectral Analysis with Applications*. Prentice-Hall, Englewood Cliffs, N.J.
- Samii, K. and Vandiver, J.K., 1984. A numerically efficient technique for the simulation of random wave forces on offshore structures. *Proc. Offshore Technology Conference*, Houston, Texas, OTC 4811, pp 301-305.
- Spanos, P.D., 1983. ARMA algorithms for ocean wave modelling. *J of Energy Resources Technology*, ASME, 105(9), pp 300-309.
- Witz, J.A. and Mandal, S., 1991. Application of system identification to the reduction of structural monitoring data. *Integration of Design Techniques project report MON5 (1989-'91)*, Dept. of Mechanical Engineering, University College London, October'91.
- Worden, K. Stansby, P.K. Tomlinson, G.R. and Billings, S.A., 1992. On wave force analysis using system identification. Proc. 6th International Conference on *Behaviour of Offshore Structures*, BOSS-92, London, Vol-I, pp 415-430.