

## WAVE DIRECTIONAL SPREADING AT SHALLOW AND INTERMEDIATE DEPTH

V Sanil Kumar<sup>1</sup>, M.C. Deo<sup>2</sup> and N.M. Anand<sup>1</sup>

<sup>1</sup>Ocean Engineering Division, National Institute of Oceanography  
Dona Paula, Goa - 403 004, India.

Fax : (0832)221360; e-mail: oed@csnio.ren.nic.in

<sup>2</sup>Civil Engineering Department, Indian Institute of Technology  
Powai, Mumbai - 400 076, India.

Fax : (022)578 3480; e-mail: mcdeo@gemini.civil.iitb.ernet.in

### ABSTRACT

Based on wave measurements carried out at shallow and intermediate water depths variation of directional spread parameter 's' is studied. A comparison of the mean wave direction and principal wave direction shows that the two are the same for frequencies close to peak frequency. It is found that at both shallow and intermediate water depths, directional width is marginally more than the mean spreading angle. The estimated Fourier coefficient related parameters 's<sub>1</sub>' and 's<sub>2</sub>' are compared with each other and found that 's<sub>1</sub>' is smaller than 's<sub>2</sub>'. The spreading parameter 's' estimated from significant wave height and wave period corresponding to spectral peak is found to be more than 's<sub>1</sub>' and difference between 's<sub>4</sub>' and 's<sub>1</sub>' is not large. The spectrum computed from measured data shows that Scott spectrum approximates the observations in a fairly satisfactory way. A comparative study was carried out based on the directional spectrum estimated from Fourier coefficients and the model directional spectrum obtained by multiplying the Scott spectrum by the spreading function estimated from significant wave height and wave period. It shows that the model estimates directional distribution reasonably well.

### INTRODUCTION

Although there are several models available to represent the wave directional spreading, the following cosine power model, originally proposed by Longuet-Higgins et al., (1963) is most widely used due to simplicity and general effectiveness in representing the wave directional spreading.

$$D(f, \theta) = G(S) \cos^{2s} [(\theta - \theta_p)/2] \quad (1)$$

where  $G(S)$  is the normalising parameter and 's' is spreading parameter which controls directional spreading around the mean wave direction,  $\theta_p$ . Cartwright (1963) showed that 's' can be related to the Fourier coefficients  $a_0, a_1, b_1, a_2$  and  $b_2$  in two ways,  $s_1$  and  $s_2$ , where,

$$s_1 = \frac{r_1}{1-r_1} \quad (2)$$

$$s_2 = \frac{1+3r_2+(1+14r_2+r_2^2)^{1/2}}{2(1-r_2)} \quad (3)$$

where  $r_1$  and  $r_2$  are estimated from the Fourier coefficients.

For fairly steady wind condition and mild sea states parameter 's' can be expressed as a function of relative frequency ( $f/f_p$ ) and wave age ( $c_p/U$ ) (Mitsuyasu et al., 1975), as under:

$$s_3 = 11.5 (c_p/U)^{2.5} (f/f_p)^b \quad (4)$$

where  $b = 5$  for  $(f/f_p) \leq 1$   
 $= -2.5$  otherwise.

In the above equations  $c_p$  is wave speed associated with peak frequency ( $f_p$ ) and  $U$  is measured wind speed at 10 m height above mean sea level.

Wang (1992) showed that 's' can be related to design wave height and wave period as under:

$$s_4 = 0.2 (H_s/L_p)^{-1.28} (f/f_p)^b \quad (5)$$

where  $L_p$  is wave length associated with peak frequency of the spectrum ( $f_p$ ) estimated based on linear dispersion relation and  $H_s$  is significant wave height.

In the present study the Fourier coefficients and the parameter 's' were estimated from the measured time series data at shallow and intermediate water depth and the parameter 's' was represented by empirical equations. The variation of other directional wave parameters are also discussed.

## DATA MEASUREMENT AND ANALYSIS

Wave measurements were made by deploying waverider buoy at 15 m water depth on east coast of India in Bay of Bengal (Location A) and at 23 m water depth off Goa, on the west coast of India (Location B). The wave data were collected during November and December 1995 at location A and during June 1996 at location B. The sampling interval was 0.78125 s and data were recorded for 20 minutes duration at every 3 hr. interval. The directional

parameters obtained are the mean wave direction,  $\theta_m$ , principal wave direction,  $\theta_p$ , directional width,  $\sigma$  and mean spreading angle,  $\theta_k$  (Sanil Kumar et al., 1996).

During the period of wave data collection at both locations an Aanderaa self recording weather station was installed on the open coast at 10m height above the mean sea level and the wind speed and direction were recorded at every 3 hr. interval.

## RESULTS AND DISCUSSION

During the study period at location A, significant wave height ( $H_s$ ) varied from 0.5 m to 2 m while the wind speed varied from 0.5 m/s to 7 m/s. The regression analysis carried out shows that the regression coefficient between wind speed and significant wave height is 0.64. The zero crossing wave period varied from 3 s to 9 s. The maximum wave height ( $H_{max}$ ) determined from each 20 minutes record based on the zero crossing method shows that it is 1.65 times the significant wave height (Fig. 1a). The regression analysis shows that the regression coefficient between the observed significant wave height and the maximum wave height is 0.95. Following the Rayleigh distribution for an average wave period of 4 to 5 s, the maximum wave height has to be 1.6 to 1.7 times the significant wave height. At location B, during the study period,  $H_s$  varied from 1 m to 6 m while the wind speed varied from 0.5 m/s to 13 m/s. The regression analysis shows that the regression coefficient between these two parameters is 0.97.  $T_z$  varied from 5 s to 9 s and there was a correlation between  $H_s$  and  $T_z$  with a regression coefficient of 0.76.  $H_{max}$  shows that it is 1.65 times the  $H_s$  (Fig. 1b) with a regression coefficient of 0.98. So at both locations it is found that the relation between significant wave height and maximum wave height follows the Rayleigh distribution.

A comparison of mean wave direction and principal wave direction estimated at location A shows that mean wave direction and principal wave direction for frequency ranging from 0.07 to 0.25 Hz (close to peak frequency) is same. But mean wave direction and principal wave direction corresponding to higher (>0.25 Hz) and lower (<0.07 Hz) frequencies are not same. At intermediate water depth also this was found to be the same. The variation of the mean and principal wave direction corresponding to the peak frequency for both locations are shown in Figs. 2a and 2b.

The mean wave direction corresponding to peak frequency at location A varied from 30° to 120° with respect to north (Fig. 3a). The orientation of the coastline at this region is north-south and is a straight and long open coast. The wind direction recorded at the coast shows that it varied predominately between 0° to 90° and between 270° to 360°. This shows that waves are generated in deep water and the transformed wave characteristics are only recorded by the buoy, since the waverider buoy was only 5 km away from the coast in 15 m water depth. At location B, the wave direction with respect to north is from the sector between 210° and 270° (Fig. 3b). The predominant wind direction is from the sector between 210° and 360°. The sudden change in wind direction and speed are due to gusty winds associated with the south west monsoon. It indicates that the presence of swells, which contributes a significant amount of energy to the spectrum, are consistently from the same direction even

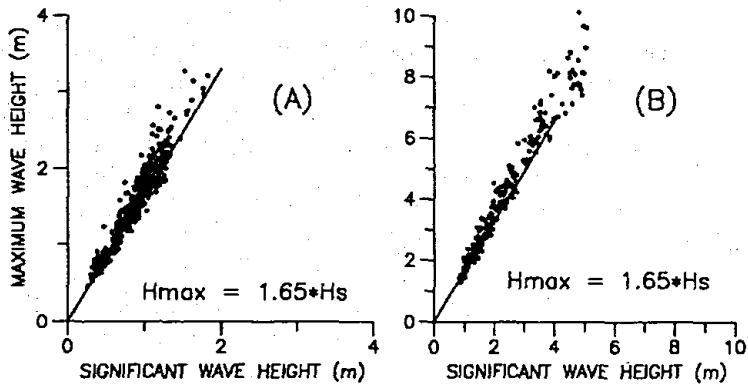


Fig.1 Variation of maximum and significant wave height

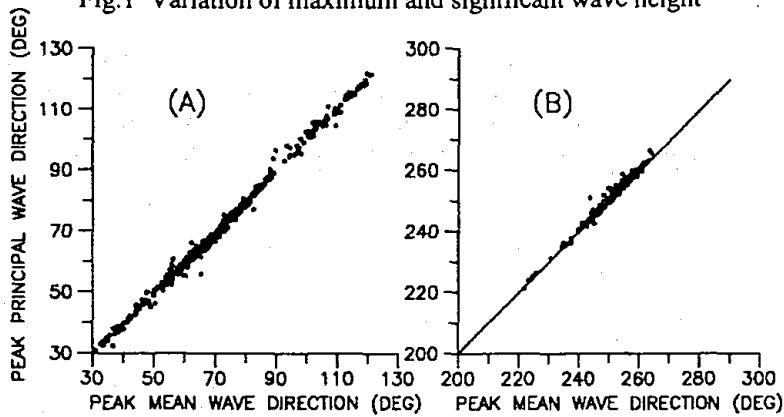


Fig.2 Mean and principal wave direction at peak frequency

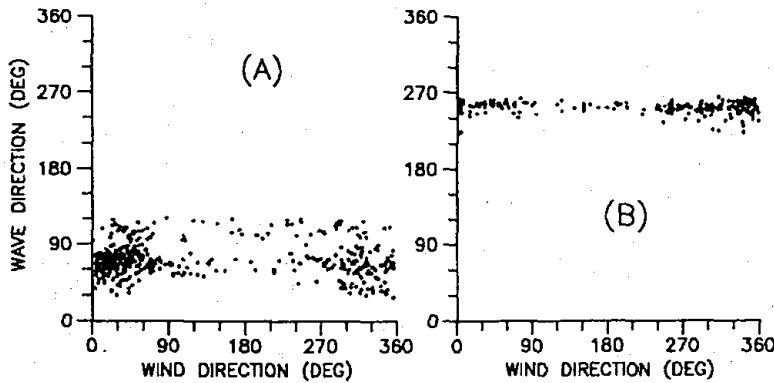


Fig.3 Variation of wind and mean wave direction

when the wind direction changed. From Figs. 3a and 3b it is clear that care has to be taken while assuming mean wave direction to be same as the wind direction in modelling of shallow and intermediate water waves.

A two dimensional spectral model can easily be represented by considering product of a unidirectional spectrum and a directional spreading model. For the west coast of India, Scott spectrum (Scott, 1965) was recommended for use by Dattatri et al. (1977), Narasimhan and Deo (1979) and Sanil Kumar et al. (1994) by analysing the data collected off Mangalore, Bombay High and Karwar respectively. The probable reason might be the fact that validation of Scott's spectrum was carried out using considerable swell dominated data. Similar situation usually prevails along many sites along the Indian coast.

The variation of wave spectrum, recorded on 25 November 1995 at 0900 hrs along with Scott spectrum with frequency for location A is shown in Fig. 4a and the spectrum recorded on 15 June 1996 at 1200 hrs along with the Scott spectra for location B is shown in Fig. 4b. It shows that at both locations Scott spectrum represents the wave spectrum.

Fig. 4c shows variation of mean wave direction, principal wave direction and wind direction with frequency for location A and Fig. 4d shows it for location B. It is found that the mean wave direction and principal wave direction differ at higher and lower frequencies. The local wind direction was consistent with the wave direction at higher frequencies. This indicates that the waves at higher frequencies are generated by local wind.

It is found that at both locations, directional width was marginally more than mean spreading angle (Fig. 4e & 4f). The difference between these parameters are not much, with a regression coefficient of 0.985. Benoit (1992) and Besnard and Benoit (1994) observed similar values for these parameters.

The variation of ' $s_1$ ', ' $s_2$ ' and ' $s_4$ ' with frequencies for location A is presented in Fig. 4g and for location B in Fig. 4h. Here in both cases, the values of ' $s_1$ ' is found to be smaller than ' $s_2$ ' which is consistent with the observations of Cartwright (1963), Mitsuyasu et al. (1975) and Wang (1992). In the present study difference in ' $s_1$ ' and ' $s_2$ ' is not found as large. A better estimate of spreading parameter ' $s$ ' in such a situation would be average of ' $s_1$ ' and ' $s_2$ ' (Cartwright, 1963). The spreading parameter ' $s_3$ ' is found to be not reliable for location A, since the waves are not generated locally and the difference in wave direction and wind direction is large. Hence this parameter is not considered for location A. For location B the spreading parameter ' $s_3$ ' is estimated based on Eq. 4 with a modification to the empirical relation. The constant 11.5 proposed in the original equation is changed to 5 to represent it suitably for the present data. It is found that ' $s_3$ ' and ' $s_4$ ' showed similar results for location B. The spreading parameter ' $s_4$ ' estimated based on significant wave height and wave period was also presented in Figure. The empirical Eq. 5 for estimating ' $s_4$ ' is modified by changing the coefficient from 0.2 to 0.14. It is found that ' $s_4$ ' values are marginally higher than ' $s_1$ ' values. Since the expression for ' $s_4$ ' involves only significant wave height and period corresponds to maximum energy, it can be used in situations when only the design parameters are specified.

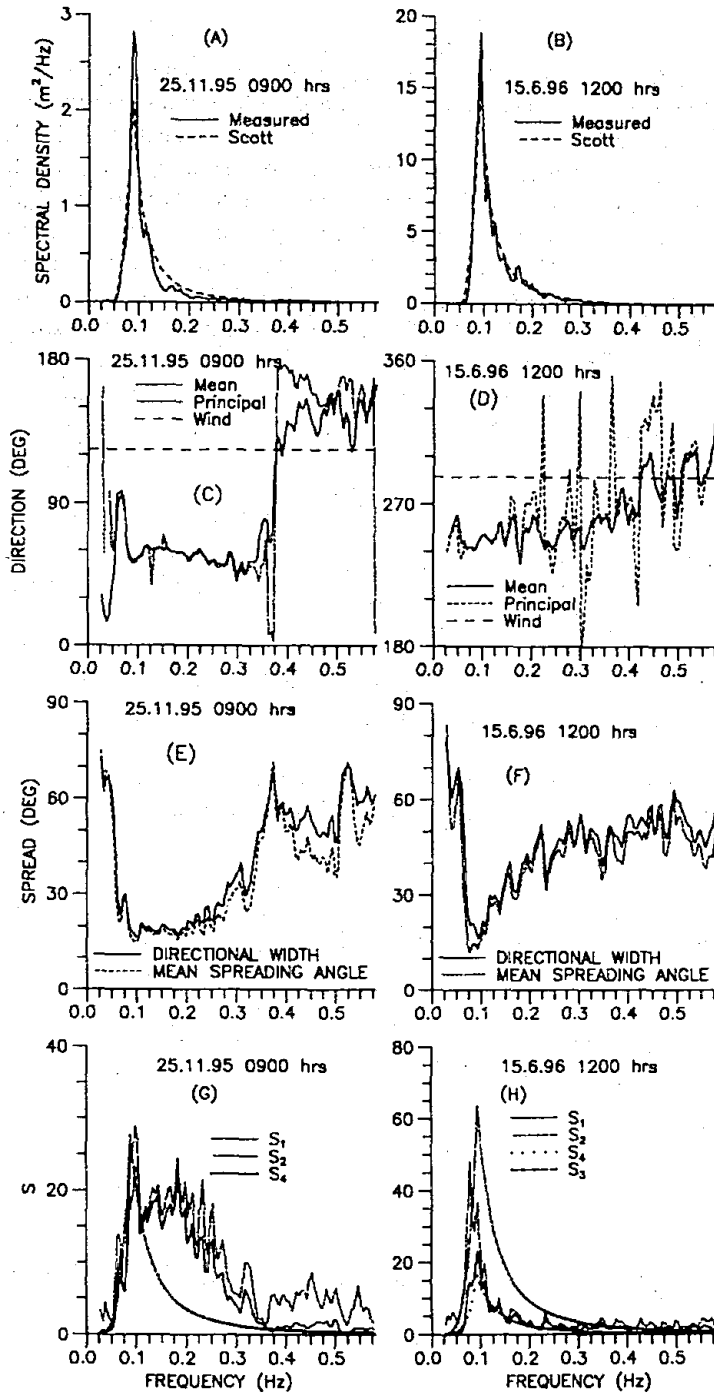


Fig.4 Variation of spectral energy, wave direction, spread and S

Based on the above studies it is found that the directional wave spectrum can be modelled fairly well by multiplying the Scott spectrum by the spreading function given in Eq. 1 and the spreading parameter 's' can be estimated based on the Eq. 5. A comparison study was carried out based on the directional spectrum estimated from the Fourier coefficients and the model proposed above. The contour plots of directional spectrum are shown in Fig. 5a. The continuous line shows the contour plots of directional spectrum computed based on the Fourier coefficients and the dotted line shows the same estimated from the model for location A. The same for location B is shown in Fig. 5b. It shows that the model follows the directional distribution reasonably well. But in the model the mean wave direction is an input and this has to be obtained for each location. In the present case the mean wave direction obtained from the measurement is taken as input since there was not much correlation between the mean wave and the wind direction in both the locations.

## CONCLUSIONS

- a) It is found that the wind direction deviates from the wave direction recorded during the same period this is due to the fact that waves were recorded in nearshore, where waves were almost parallel to the coast.
- b) The mean spreading angle and the directional width showed similar variation with directional spread marginally more than the mean spreading angle.
- c) The observed relative large values of spreading parameter near the peak frequency to lower and higher frequencies agrees fairly well with suggestions in the literature for shallow and intermediate water waves also.
- d) It is found that the values of ' $s_2$ ' is always more than ' $s_1$ '. For design purposes when only the design wave parameters are specified, spreading parameter can be estimated from Eq. 5.
- e) The directional spectrum can be modelled by the product of Scott spectrum and the directional spreading function given by Eq. 1 using the spreading parameter estimated from Eq. 5.

## Acknowledgments

The authors thank the Director and staff of Ocean Engineering Division, National Institute of Oceanography, Goa for encouragement and support.

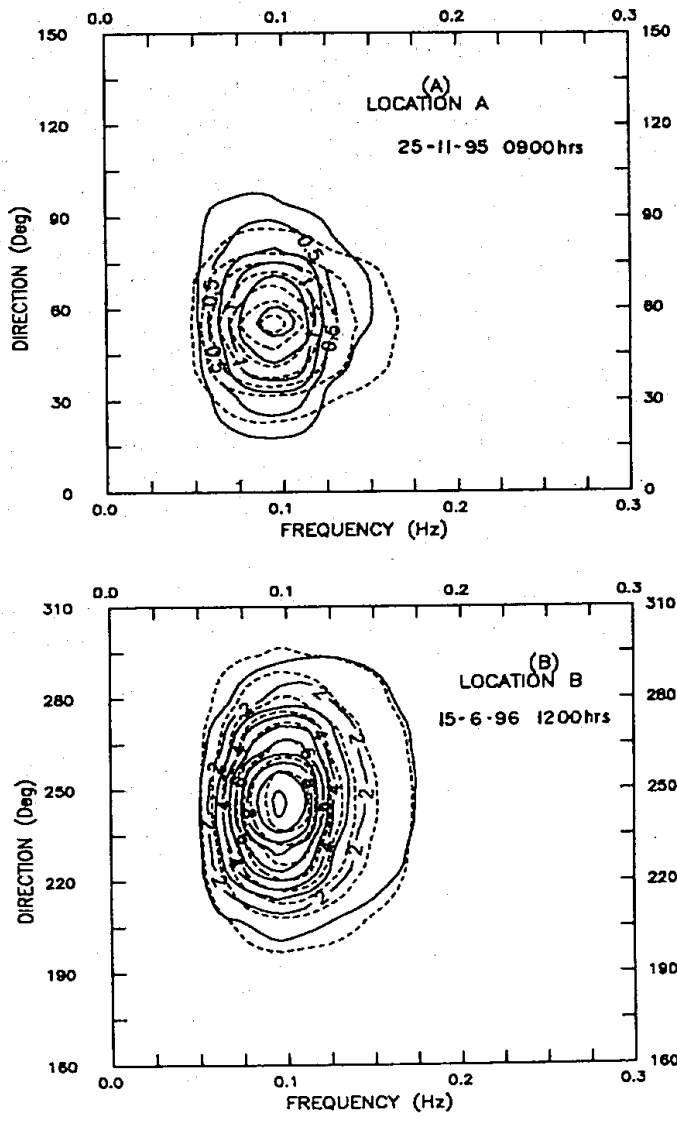


Fig.5 Directional wave spectra



## References

- Benoit, M., 1992. Practical comparative performance survey of methods used for estimating directional wave spectra from Heave-Pitch-Roll data, *Proceedings of the Conference Coastal Engineering*, pp. 62-75.
- Besnard, J.C. and Benoit, M., 1994. Representative directional wave parameters - Review and comparison on numerical simulation, *International Symposium: Waves - Physical and Numerical Modeling*, IAHR, pp. 901-910.
- Cartwright, D.E., 1963. The Use of directional spectra in studying the output of a wave recorder on a moving ship, *In Ocean Wave Spectra*, Prentice Hall, pp. 203-219.
- Dattatri, J. Jothishanker, N. and Raman, H., 1977. Comparison of Scott spectra with ocean wave spectra, *Journal of Waterways Port Coastal and Ocean Engineering*, ASCE, 103, pp. 375-378.
- Longuet-Higgins, M.S. Cartwright, D.E. and Smith, N.D., 1963. Observations of the Directional spectrum of sea waves using motions of a floating buoy, *In Ocean Wave Spectra*, Prentice Hall, pp. 111-136.
- Mitsuyasu, H. Tasai, F. Suhara, T. Mizuno, S. Ohkusu, M. Honda, T. and Rikiishi, K., 1975. Observations of the directional spectrum of ocean waves using a Cloverleaf buoy, *Journal of Physical Oceanography*, 5, pp. 750-760.
- Narasimhan, S. and Deo, M.C., 1979. Spectral analysis of waves - A Study, *Proceedings Civil Engineering in the Oceans IV Conference*, San Francisco, pp. 877-892.
- Sanil Kumar, V. Mandal, S. Anand, N.M. and Nayak, B.U., 1994. Spectral representation of measured shallow water waves, *Proceedings India National Conference On Harbor and Ocean Engineering*, Vol. I, pp. 23-32.
- Sanil Kumar, V. Deo, M.C, Anand, N.M. and Gowthaman, R., 1996. Directional wave spectra off southeast coast of Tamilnadu, , *Proceedings International Conference in Ocean Engineering*, Madras, pp. 254-258.
- Scott, J.R., 1965. A Sea Spectrum For Model Test and Long-Term Ship Prediction, *Journal of ship research*, 9, pp. 145-152.
- Wang, D.W., 1992. Estimation Of Wave Directional Spreading In Severe Seas, *Proceedings of the Second International Offshore and Polar Engineering Conference*, Vol. III, pp. 146-153.