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## Sound field computations in the Bay of Bengal using parabolic equation method

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Effect of the cold core eddy in the Bay of Bengal on acoustic propagation was analysed by parabolic equation (PE) method. Source depth, frequency and propagation range considered respectively for the two numerical experiments are 150 m, 400 Hz, 650 km (case i) and 1000 m, 70 Hz, 5000 km (case ii). Acoustic intensity levels varied from -20 to -35 dB in the upper 500 m (case i). In the presence of the eddy, isolines exhibited deviations between 250 and 500 km range extending from 100 m to the abyssal region. Maximuminsonified region is present in the vicinity of the channel axis depth which showed variations from 800 to 1500 m in case ii.

Parabolic equation (PE) method has proven to be a powerful and effective means for the estimation of sound field for specific oceanic environments such as - a) range dependent sound speed profiles, b) variable bottom topography, c) specific input sound field (source directivity) pattern and d) source frequency spectra<sup>1</sup>. In the present study, the solution of the PE based on implicit finite difference method (Crank Nicholson's scheme) is used for determining the acoustic propagation loss along the vertical domain (slice) from the source to the receiver. Two numerical experiments are performed to study I) the effect of the cold core eddy in the Bay of Bengal on acoustic propagation and ii) the acoustic field in the SOFAR channel over ranges of thousands of kilometers.

### Materials and Methods

In the PE method introduced by Tappert<sup>2,3</sup>, the acoustic wave equation (Helmholtz eqn.) which is an elliptic partial differential equation is solved by transforming into a parabolic partial differential equation as detailed below. In a 2-D cylindrical co-ordinate system (r, z) for a harmonic source at r=0, z=z<sub>s</sub>, the acoustic pressure 'p' is given by the Helmholtz equation (HE)<sup>2</sup>.

$$\frac{\delta^2 p}{\delta r^2} + \frac{1}{r} \frac{\delta p}{\delta r} + \frac{\delta^2 p}{\delta z^2} + k_0^2 n^2 p = 0 \quad \dots (1)$$

where  $n(r, z) = C_0 / C(r, z)$  is acoustic index of refraction and  $k_0 = 2\pi f / C_0$  is the reference wave number, f is the frequency of the source and  $C_0$  is the reference sound speed.

The approximations involved in the conversion of HE to PE<sup>4</sup> are i) far field approximation i.e.  $k_0 r \gg 1$ , ii) zero back scattering and iii) paraxial approximation, i.e. angular spectrum of propagating energy is  $< 40^\circ$ .

The standard PE of Lee & McDaniel<sup>4</sup> is given by

$$\frac{\delta^2 u}{\delta r^2} + 2ik_0 \frac{\delta u}{\delta r} + k_0^2 (n^2 - 1)u = 0 \quad \dots (2)$$

and

$$p(r, z) = u(r, z) * s(r) \text{ and } s(r) = \sqrt{(2 / \pi k_0 r)} e^{i(kr - (\pi/4))}$$

The above equation governs the outgoing field and can be solved using the split step fourier algorithm<sup>2</sup> by imposing an artificial zero absorption bottom boundary and pressure release surface conditions. In cases where bottom interaction is strong, a more general implicit finite difference (IFD) scheme as well as an ordinary differential equation<sup>4</sup> (ODE) method are useful. The IFD model<sup>4,7</sup> takes into consideration, the range

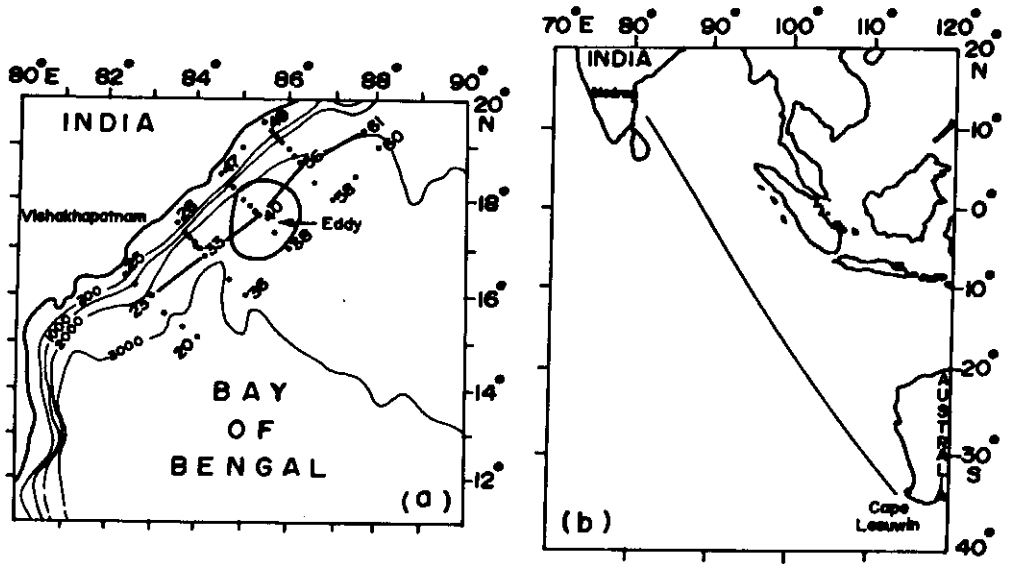


Fig. 1 - Location of sections considered for this study, a) across a cold core eddy in the western Bay of Bengal, b) Cape Leeuwin (Australia) to Madras

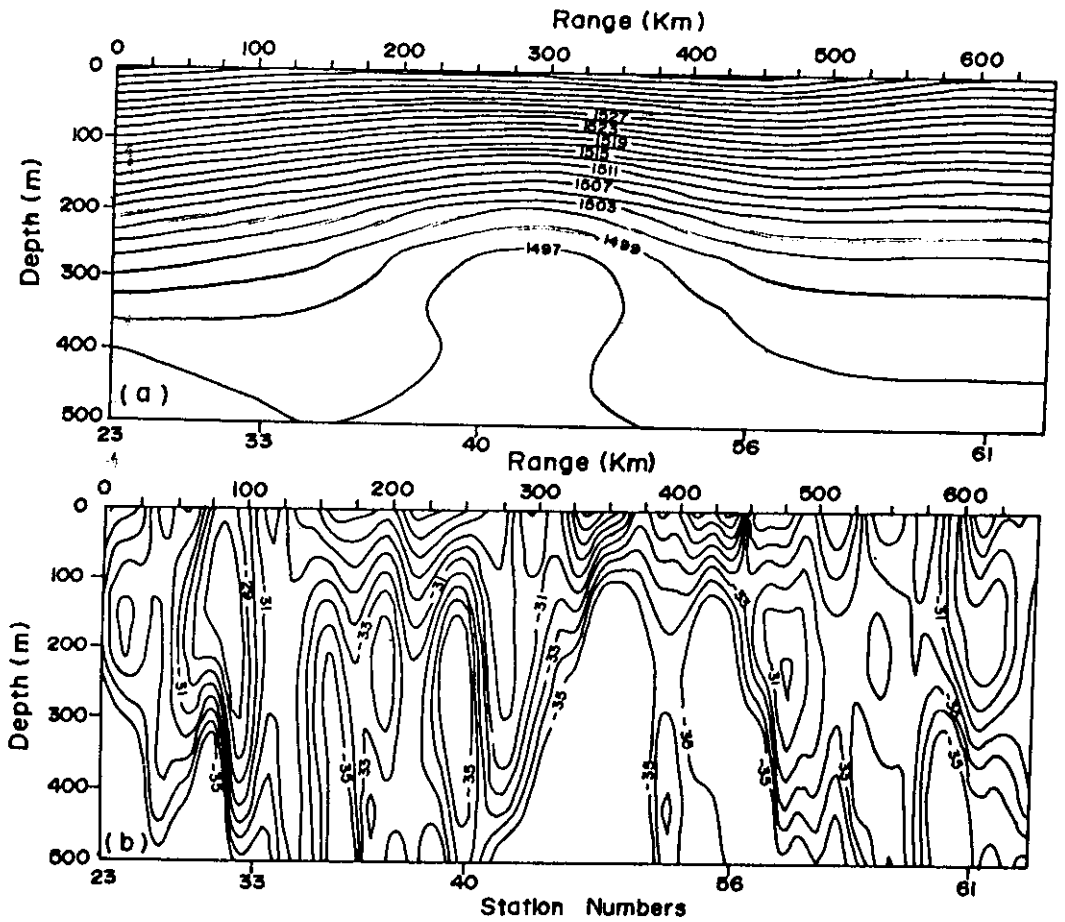


Fig. 2 - a) Vertical section of sound speed across the subsurface cold core eddy in the western Bay of Bengal, b) contours of acoustic intensity levels computed using IFD program

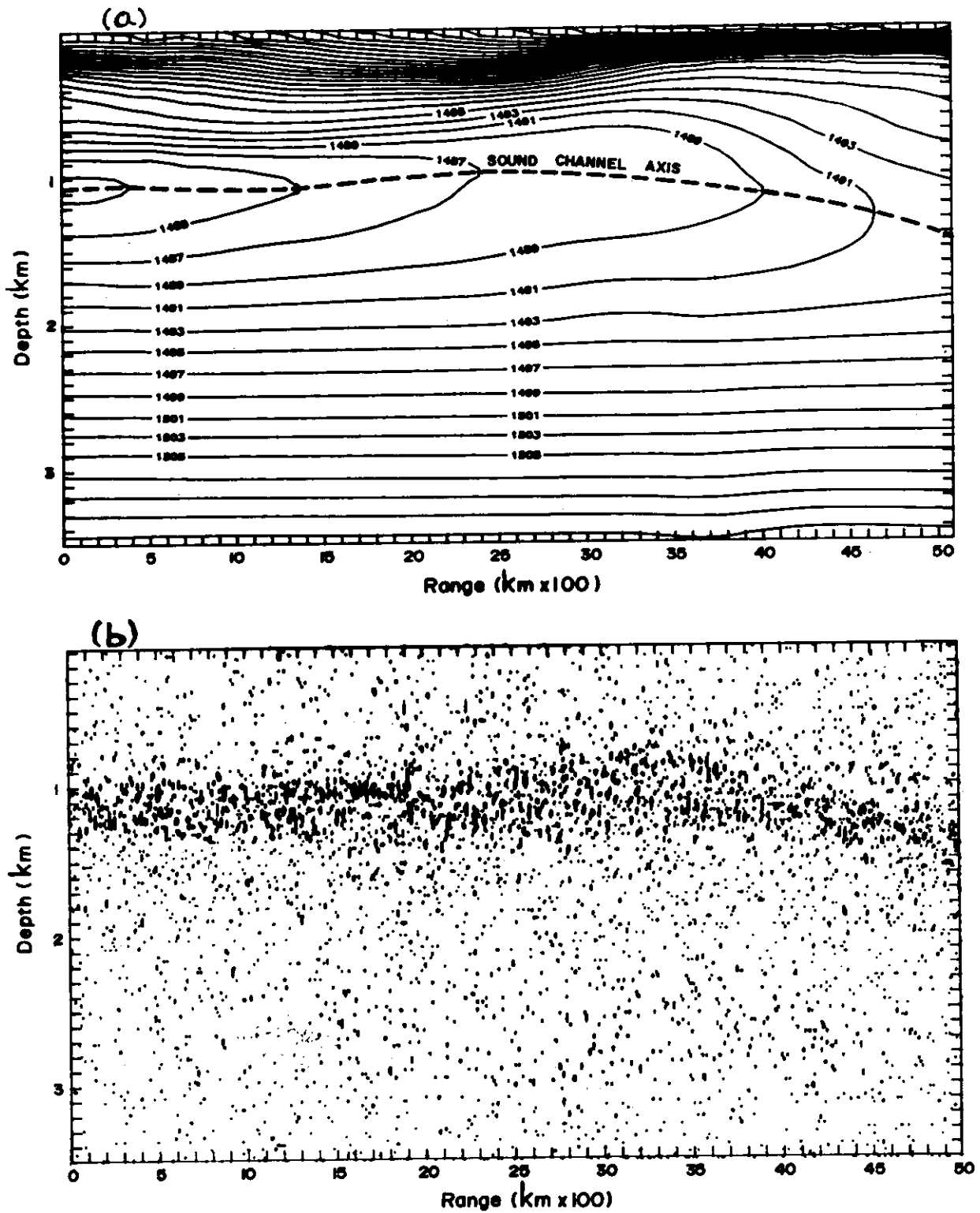


Fig 3 – a) Vertical Structure of sound speed from Cape Leeuwin (Australia) to Madras along the Geodesic path, b) contours of acoustic intensity level in the SOFAR propagation

dependent nature of the sound speed profile and the variable bottom for computing the acoustic field in the frequency domain. This can be Fourier transformed into the time domain to obtain the ray arrivals or time response of the acoustic channel. The IFD program has been documented and published<sup>4</sup> for general use. The usefulness of this program to model acoustic propagation under different conditions has also been test validated in some of the recent studies<sup>8-10</sup>. This study is based on sound field computations made using this software.

*Case i*—A subsurface cold core eddy identified in the north western Bay of Bengal based on CTD observations<sup>11</sup> made during July 1984 is used in the present study to examine the eddy effect on acoustic propagation. For sound speed computations, the empirical relations developed by Chen & Millero<sup>12</sup> were employed. Sound speed profiles derived along the section through the eddy (Fig. 1a) provided the necessary input for the sound field computations. The source frequency chosen was 400 Hz (commonly used in ocean acoustic tomography studies for monitoring the meso scale features) having the required bandwidth capability to resolve the acoustic signal arrivals. Though the IFD program can handle the case of a variable bathymetry, for the present computations a uniform bottom (absorbing type) at 3000 m is considered since the changes in the bottom topography in the study area are minute. The source depth and propagation range used are 150 m (coinciding with the depth of the center of the eddy) and 650 km respectively. Choice of these input parameters for computation enabled to scan the oceanic layer which is influenced by the eddy.

*Case ii*—The annual mean temperature and salinity data from climatological atlas<sup>13</sup> were utilised for computing sound speeds and thereby constructing the sound speed profiles along the geodesic path-great circle on a spheroidal earth<sup>14</sup> - between Cape Leeuwin, Australia and Madras (Fig. 1b). A hypothetical source at 1000 m depth operating at a central frequency of 70 Hz (to achieve ranges of this order) is chosen for numerical simulation of sound propagation in the SOFAR channel over ranges up to 5000 km. During SOFAR propagation, focussing of most of the energy takes place in the vicinity of sound channel axis and this

energy usually survives to long ranges. In order to examine this effect along the proposed track without loss of generality, a flat absorbing bottom at 3500 m is considered. Plots of the results for cases (i) and (ii) (Figs 2b and 3b) were prepared in a similar way in the form of contour maps of acoustic intensity levels higher than -35 dB with a contour interval of 1 dB. As the range and depth dimensions of both the cases differ considerably (case ii is about 10 times larger than case i), Fig. 3b appears like a scatter plot.

The source pattern was assumed to be Gaussian centered at the source depth. The integration step size which depends on the source frequency (wavelength of the signal) was 1 m and 5 m respectively for the above two cases. The IFD program was run on a PC AT 386 system for the above computations. Output of the IFD program consisted of acoustic intensity at uniform depth and range intervals covering the entire vertical domain of the transect. Vertical sound speed structure along the sections used in this study are presented in Figs 2a and 3a while the acoustic intensity level in dB (after removing the component due to cylindrical spreading loss) are plotted and shown in Figs. 2b and 3b respectively. Plots of Figs 2a and 2b were presented for the upper 500 m as the effect of the eddy was limited to that depth interval.

## Results and Discussion

The vertical section of sound speed structure upto 500 m depth has been shown in Fig. 2a to depict the oceanic variability associated with the cold core eddy. In the vicinity of the cold core (cyclonic) eddy, the isopleths showed an upward tilt of about 150 m to shallower depths as seen from the dome shape of the contours between ranges 200 km to 400 km. The sound speed showed a reduction of about 10 m. sec<sup>-1</sup>. This feature extends to depths up to the SOFAR channel axis<sup>15</sup>.

To understand the effect of the eddy on acoustic propagation, the computed intensity levels at uniform depth and range intervals were used to prepare a contour map in the vertical domain up to 500 m depth (Fig. 2b). The intensity levels above -35 dB were plotted and shown in this diagram. The intensity levels showed variations between -20 dB to -35 dB. Higher losses were encountered in the region between 250 and 500 km range as seen

by the -35 dB contour. In an earlier study<sup>15</sup>, acoustic intensity along steep ray paths passing through the core of the eddy were reported to undergo about 20-25 dB additional losses. The presence of the eddy in the range interval between 200-400 km brings in a lens like effect which causes defocussing of energy. Thus isolines of intensity exhibited deviations between 250 and 500 km range, resulting in a shadow zone like feature centered at a range of 325 km and another region of focussing of energy around 450 km range which extended from 200 m to the deeper layers as seen in Fig. 2b.

The sound speed structure in the vertical along the geodetic path for a range of 5000 km is shown in Fig. 3a. The depth of the SOFAR channel axis shows variations from 1000 m at southern point (off Cape Leeuwin, Australia) to about 800 m at the equator and then deepens to about 1500 m in the Bay of Bengal. The scattered plot of intensity (Fig. 3b) showed concentration of points close to the depth of the sound channel axis. In other words, the maximum insonified region existed in the depth interval 800 to 1500 m. This indicates that the SOFAR propagation undergoes least attenuation losses. In this mode of propagation, acoustic energy mostly confines in the sound channel and is unaffected by the changes in the upper ocean.

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