



WAVE DIRECTIONAL SPECTRUM FROM ARRAY MEASUREMENTS

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SYNOPSIS : Using the method of Esteva (1976, 1977), which assumes that at the frequency band the waves approach from just a single "mean" wave direction, wave direction has been consistently, accurately and unambiguously evaluated as a function of frequency, from time series observations of wave elevation at a 15-gauge polygonal array, in 8m water depth, operational at the Field Research Facility (FRF) at Duck, North Carolina, USA. It was found that, a maximum of 15 redundant estimates of wave direction has a standard deviation of $<4^\circ$ in the range 14-4 s during 1-6 February, 1994 and that swell of 10 sec came from the east. A developing sea during the time period 01-13 hours on 2 February, was due to a Northeaster, owing to which the wave direction in the wave period range 7-4 s aligned itself with the wind direction, and the significant wave height increased from 0.70 to 1.54m. Estimates of wave direction using Esteva's method were in agreement with those obtained by the FRF using the conventional Iterative Maximum Likelihood Estimation method, which assumes that at a given frequency waves can approach simultaneously from all directions.

INTRODUCTION

Wind generated ocean waves occur in the period range 30-1 sec. Knowledge of wave direction is crucial in the study of littoral transport (Komar and Inman, 1974) and in the design of harbours and coastal structures (Pawka, 1974). Various methods are available for the determination of wave direction. The three chief methods are buoy, radar and array measurements. In array measurements, wave direction is estimated from the simultaneous time series measurements of wave elevation at several gauges placed in either linear or polygonal arrays.

There are two reasons why phase/time/path difference methods for determining wave direction from array measurements have not become popular; the first being the insufficient documentation of Borgman (1974) in case of linear arrays; and the second being the failure of Esteva (1976, 1977) in determining wave direction correctly over the design range 25-7 sec of her 5-gauge polygonal array at Pt. Mugu, California.

Borgman (1974) uses the following formula for determining wave direction from time series measurements of wave elevation at two gauges:

$$D \cos \alpha' = c \phi / (2 \pi f)$$

where D is the distance between the gauges, α' is the wave direction in the range $(0, \pi)$ measured with respect to the line joining the gauges, c is the wave celerity which is a function of frequency f, and ϕ is the phase difference between the arrival of a wave crest at the two gauges and is determined as a function of frequency by cross spectrum analysis.

Often it is convenient to measure wave direction α in the range $(-\pi/2; \pi/2)$ measured with respect to the seaward normal to the shore, by

using the formula, $\alpha = \alpha' - \pi/2 + \delta$, where δ is a small correction to be added to make allowance for the two gauges not being parallel to the shore, which is evaluated from the coordinates of the gauges. Alternatively α can be determined directly using the sine function instead of the cosine function as is done by Munk et al (1963).

Clearly the phase difference ϕ is the same for wave direction α' as well as $2\pi - \alpha'$. Therefore there is an ambiguity within a mirror symmetry in the determination of wave direction by the method of Borgman (1974), i.e. from linear arrays.

By the method of Borgman (1974) just two gauges are sufficient for determining wave direction i.e. the analysis is done in units of gauge pairs with different gauge pairs possible in the linear array giving redundant estimates of wave direction.

Esteva (1976, 1977) derived the following formula for unambiguously determining wave direction α , from simultaneous time series measurements of surface elevation at three non-collinear gauges, i.e. a gauge triad or triangle 123:

$$\alpha = \arctan \frac{\{(x_1 - x_2) \phi_{13} - (x_1 - x_3) \phi_{12}\} / \text{sgn } p}{\{(y_1 - y_3) \phi_{12} - (y_1 - y_2) \phi_{13}\} / \text{sgn } p}$$

where α is the wave direction in the range $(-\pi, \pi)$; (x_i, y_i) , $i = 1, 2, 3$ are the coordinates of the three wave gauges; ϕ_{12} and ϕ_{13} are the phase differences between gauge pairs 12 and 13 respectively and are determined by cross spectrum analysis;

and $p = (x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)$; does not vanish when the three gauges are non-collinear, and by definition $\text{sgn } p = 1$ for $p > 0$ and $\text{sgn } p = -1$ for $p < 0$.

Clearly by the method of Esteva just three non-collinear gauges are sufficient for determining wave direction unambiguously, i.e. the

analysis is done in units of gauge triads, with the different gauge triads possible in the polygonal array giving redundant estimates of wave direction. Esteva used the above formula for obtaining ${}^5C_3 = 10$ redundant estimates of wave direction as a function of frequency from time series measurements at her 5-gauge polygonal array at Pt. Mugu, California, where swell 16 and 8 sec was observed. Esteva using "consistency" within the 10 redundant estimates of wave direction as a criterion for accuracy, reported success in determining direction of 16 sec swell, and failure in case of 8 sec swell.

Fernandes et al (1988) provided the necessary documentation in case of linear arrays, and repeating the computations of Esteva for her polygonal array and were successful in consistently, accurately and unambiguously determining the direction of both 16 sec as well as 8 sec swell. Their success was due to their adroit use of the criterion of Barber and Doyle (1956), which stipulates that for ensuring correct directions, the distances between the gauges 12 and 13 in the semi-ordered gauge triad 123, should both be less than half a wave length for the particular frequency band.

Thus Fernandes et al (1988) established that the method of Esteva for determining wave direction from polygonal arrays works in case of computer simulated data. The object of the present study is to establish that the method of Esteva works in case of measured wave data also.

DATA

The measured wave data used in this study comes from a 15-gauge polygonal array of bottom mounted pressure gauges operational at

the Coastal Engineering Research Center's Field Research Facility (FRF) at Duck, North Carolina, USA. The array comprises of 10 alongshore gauges (operational since March 1987) and 5 intersecting cross-shore gauges (added in 1990) placed at ≈ 8 m depth (see Fig. 1). Code names for the 15 wave gauges are as follows; 0 through 9 and A through E. Gauge coordinates, mean water depth at the gauge locations, the height of the pressure sensor above the bottom, are given in Table 1.

The data used consisted of 47 three hourly records of time series of wave elevation simultaneously measured at the 15 wave gauges during 16 February, 1994. Simultaneous time series for barometric pressure, wind speed and direction at the site were also available. At each gauge the time series record consisted of 5×4096 points sampled at 2 Hz, i.e. 0.5 sec. Starting time of the records were 01, 04, 07, 10, 13, 16, 19, and 22 hours of each day. The record for 10 hours on 1 February was not available. The data was kindly supplied by Dr. C. E. Long of the FRF on two 150 MB cartridge tapes. Earlier Dr. Long had supplied 2 days data (2- 3 February, 1994) on a 1/2" magnetic tape alongwith plots of wave directional spectra for the same using the Iterative Maximum Likelihood Estimation method, based on which we prepared a preliminary report (Fernandes et al., 1995).

Long and Olzman-Shay (1991), who discuss the directional characteristics of the waves at the FRF at Duck using the then 10-gauge linear array, present detailed information about the pressure sensors used, data collection procedure, coastline characteristics, wind and wave climate etc. The coastline in the vicinity of the FRF is nearly

straight and the depths contours are nearly parallel. The coastline is oriented such that the seaward shore normal makes an angle of 70° with true North. According to Leffler et al (1989) swell generally comes from the east to the south east; and the most common wave generating winds are Northeasters.

Table 1 : Gauge coordinates, water depth and height of gauge above bottom for the 15 gauges at North Carolina, U.S.A. All distances and heights are in metres; d is water depth at the site of the gauge ; b is the height of the gauge above the bottom.

Gauge	Gauge Coord		d	b
	x	y		
1	914.5	825.7	7.90	0.40
2	914.1	816.1	7.90	0.40
3	914.9	800.5	7.90	0.34
4	914.5	795.3	7.90	0.43
5	914.0	760.9	7.90	0.50
6	914.2	735.4	7.90	0.48
7	914.3	930.7	7.90	0.24
8	914.3	956.1	7.90	0.40
9	915.2	990.2	7.90	0.40
0	919.1	816.0	7.91	0.48
A	834.7	800.4	7.44	0.46
B	875.0	800.0	7.65	0.49
C	904.5	800.8	7.82	0.27
D	935.0	800.4	8.00	0.34
E	954.5	800.6	8.13	0.52

CROSS SPECTRUM ANALYSIS

Cross spectrum analysis lies at the heart of phase/time/path difference methods. Cross spectrum analysis means the computation of two parameters, viz. phase and coherence between two time series, as a function of frequency (phase spectrum and coherence spectrum).

The phase function gives the phase difference (lead/lag) between the two series. The coherence is a measure of the consistency or stability of the relative phase difference between the two series, and like the regression coefficient is defined in the range (0,1). A coherence of zero indicates that the phase difference has large variance (see Halpern, 1973), so that the average phase difference does not make any physical sense.

For performing cross spectrum analysis we have used the Fast Fourier Transform approach as given in Bendat and Piersol (1971). For details please see Fernandes et al (1985).

The following methodology was followed for computing the cross spectrum analysis of the measured data at Duck:

- (i) The time series of 5×4096 points sampled at $h = 0.5$ sec, was divided into, $M = 80$, ensembles of, $N = 256$ points
- (ii) Blackman window in time domain was used. (Other choices of windows available in the software were Rectangular, Hanning, Hamming and Cosine Taper). The power spectral densities were scaled up by a factor of 1/0.303 to compensate for energy loss due to windowing.
- (iii) The wave amplitudes were also amplified to account for attenuation of waves with depth using the formula:

$p(\kappa) = \text{Cosh } \kappa d / \text{Cosh } \kappa b$; where p is the amplitude amplification factor, κ is the wave number, d is the water depth at the gauge site and b is the height of the gauge above the bottom. To avoid amplification of noise at the higher frequencies, the attenuation correction for frequencies above 0.2 Hz were reckoned as equal the attenuation correction for 0.2 Hz.

(iv) The coherence squared c^2 as a function of frequency for all frequencies below the Nyquist is computed following Thompson (1979). The threshold values of c^2 , which are significant at α significance level are given by $c^2 = 1 - \alpha^{1/(M-1)}$, from which it is clear that for 80 ensembles the threshold of coherence squared at 0.01 significance level is 0.057, indicating that if the observed coherence squared is larger than 0.057, then in just one case out of 100 the two time series are likely to be not coherent. If the coherence squared at a frequency is statistically significant the corresponding phase is statistically reliable.

(v) The phase spectrum ϕ for all frequencies below the Nyquist frequency is uniquely determined in the interval $(-\pi, \pi)$ from the co-spectrum C and the quadrature spectrum Q by taking into consideration the signs of the numerator and denominator in the formula: $\phi(f) = \text{arc tan} [-Q(f)/C(f)]$

A positive value of ϕ indicates that the time series x leads over the time series y , i.e. a wave crest first approaches the gauge x and then only the gauge y .

Due to the circular nature of the angular (phase) scale, the above formula gives the "apparent" phase. The "true" phase may be one of the infinite values $\phi + 2j\pi$, where j is an integer (positive negative or zero). One way of removing this ambiguity in phase is to restrict

the "true" phase also to the interval $(-\pi, \pi)$. This can be done by insisting that $|D \text{Cos } \alpha'| < \lambda/2$. But since α' is the unknown parameter to be determined the more stringent condition, $D < \lambda/2$, i.e. the criterion of Barber and Doyle (1956) has to be imposed as has been done by Fernandes et al (1988) for correctly determining wave direction, in case of computer simulated data.

The above strategy can be used in case of measured data also, but it is wasteful, as it treats perfectly good signal at the higher frequencies as noise and discards it. An inspection of cross spectra from measured data suggests an alternative strategy to determine the "true" phase $\phi + 2j\pi$ from the "apparent" phase ϕ . Generally when the distance between the gauges is $\approx 30\text{m}$, the gauges are coherent for all frequencies less than about 0.25 Hz and the phase spectrum ("apparent" phase) in the above frequency range is a continuous function of frequency evolving from zero phase at zero frequency, except for a few jumps (generally a maximum of two jumps) across the $-180^\circ / +180^\circ$ pseudo discontinuity arising due to the circular nature of the angular scale. Such phase spectra may have either positive or negative jumps and the "true" phase is derived from the apparent phase by adding $\pm 2\pi$ for every positive/negative jump (see Fig 2). The computation of the "true" phase is terminated at the high frequency end at the lowest frequency for which the coherence squared falls below 0.057.

RESULTS AND DISCUSSION

Software in FORTRAN language, constructed on the "black-box" principle, has been developed to determine wave direction from measurements of surface elevation at a 15-gauge polygonal array at

North Carolina, USA using phase/time/path difference methods of Esteva(1976, 1977) in case of polygonal arrays and Borgman(1974) in case of linear arrays, with two modifications. One modification is that as already stated we use the "true" phase instead of the "apparent" phase; the other modification is that we accept a wave direction estimate as valid only when the relevant gauges are statistically coherent. The latter modification places an upper limit on the frequency for which wave directions can be correctly computed. Henceforth in this paper when we mention Esteva's method, the reference will be to the modified Esteva's method. Before applying the software to the measured data, the software was applied to computer simulated wave fields at the 15-gauge array and was found to consistently yield directions correctly in case of both linear and polygonal arrays.

Using the method of Esteva, we have made 15 redundant estimates of wave direction as a function of frequency for each of the 47 records of measured data at North Carolina. The following 15 gauge triads: 012, 34C, D03, D24, C1D, 4C5, D0E, CB1, CB2, CB0, D04, D0C, 02D, 41D and C24; satisfying $D < \lambda/2$ for wave periods above 9 sec were chosen. Two gauge triads with small gauge separations D , viz. 012 and 34C satisfied the same criterion for periods above 3.7 sec. It was found that, a maximum of 15 redundant estimates of wave direction has a standard deviation of $< 4^\circ$ in the range 14-4 sec during 1-6 February 1994 (Fig. 3). During this time swell of ≈ 10 sec came from the east and the energy contour ridge line during 01-13 hours on 2 February is indicative of a developing sea generated by a Northeaster of ≈ 10 m/sec, owing to which the wave direction in the period range 7-4 sec aligned itself with the wind direction, and the

significant wave height increased from 0.70 to 1.54 m (see Fig 4). It is generally understood that surf beat is generated at the shore and propagates away from the shore. On the contrary we note that surf beat of ≈ 25 sec came from offshore when low wave conditions prevailed during 5-6 February.

Estimates of wave direction using Esteva's method were in agreement with those obtained by the FRF using the conventional Iterative Maximum Likelihood (IMLE) method (Fig. 5), which assumes that at a given frequency waves can approach from all directions - in case of all the 16 records for which the IMLE analysis by the FRF was available.

Using the modified method of Borgman, we have made 10 redundant estimates of wave direction as a function of frequency for each of the 47 records of measured data at North Carolina. It was found that the mean of these 10 redundant estimates were indistinguishable from the mean of 15 redundant estimates using Esteva's method, in the period range 14-4 sec.

CONCLUSIONS

Phase/time/path difference methods of Esteva(1976, 1977) and Borgman(1974) can be successfully used for computing wave direction as a function of frequency from actual field measurements using polygonal and linear arrays respectively.

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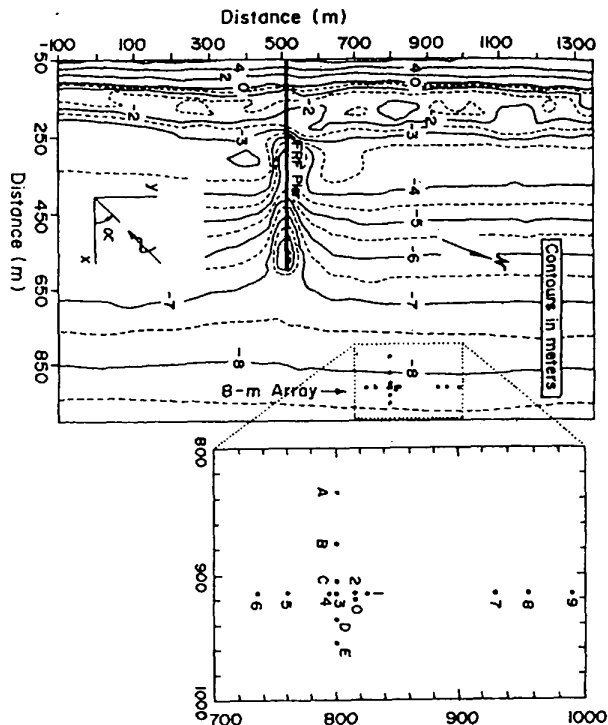


Fig. 1 15-gauge array at Duck, North Carolina, USA.

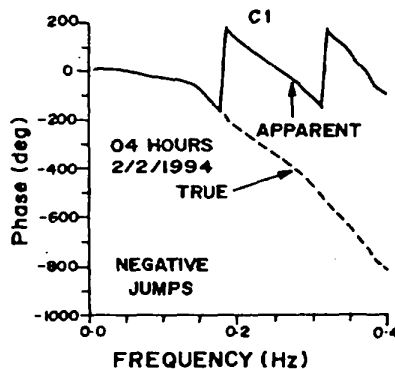


Fig. 2 Example showing the derivation of the "true" phase from the "apparent" phase.



Fig. 3a Standard deviation of a maximum of 15 redundant estimates (gauge triads) of wave direction (degrees) at Duck during 1-6 February 1994.

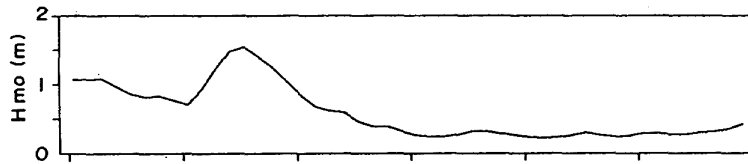


Fig. 3b Significant wave height, H_{mo} (m).

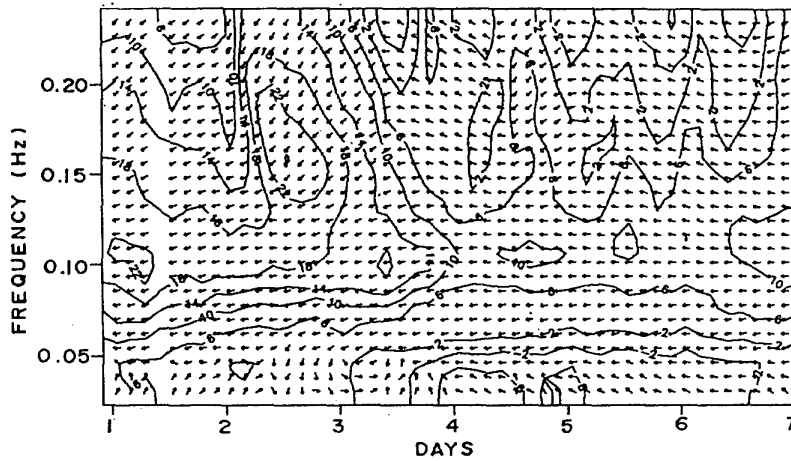


Fig. 4a Wave directional spectrum at Duck during 1-6 February 1994. Wave directions are a mean of a maximum of 15 redundant estimates (gauge triads). Energy contours are in decibels (cm^2).

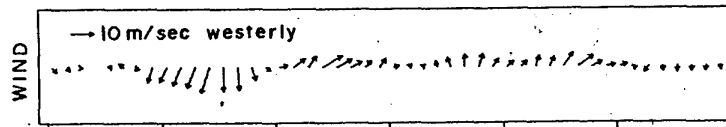


Fig. 4b Wave wind speed and direction.

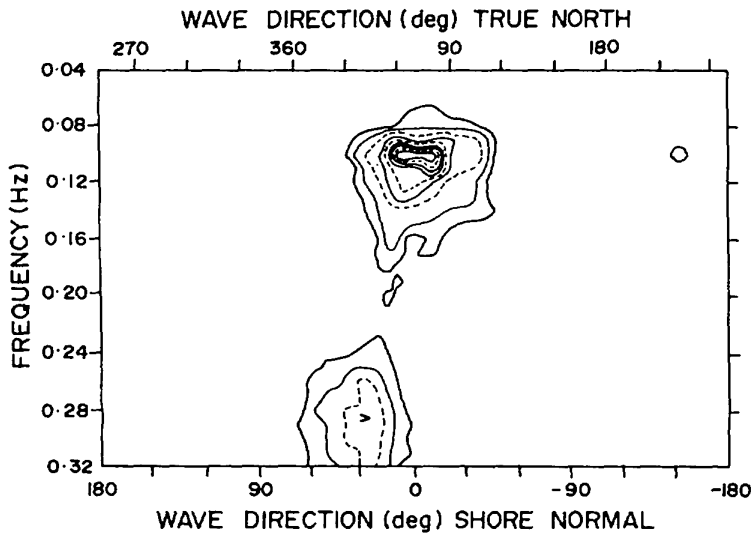


Fig. 5 Wave directional spectrum at Duck on 2 February 1994 at 01 hours obtained using the Iterative Maximum likelihood method. Contours of energy are at 5% and then tenths of maximum energy (based on figure provided by the FRF at Duck).