



## DIRECTIONAL WAVE SPECTRA OFF SOUTHEAST COAST OF TAMILNADU

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**SYNOPSIS :** Knowledge on directional wave parameter is important for the design of any coastal structure and estimation of nearshore sediment transport. The desired degree of detail may vary from the information on predominant wave direction to the complete wave directional spreading. A well established way to describe the energy content in an irregular wind generated surface wave assumes superposition of linear waves and the two dimensional energy spectrum can be conveniently expressed as a product of the one dimensional energy spectrum and its angular distribution function at each frequency. Directional wave measurements were carried out at 15m water depth off southeast coast of Tamilnadu during November and December 1995. The analysed data shows that predominant wave direction during the study period was from the sector between 50° and 120°. The mean wave direction is found to be the more representative index than the principal wave direction and the mean spreading angle and the directional width showed similar values. It is found that the values of directional spreading parameter 'S<sub>2</sub>' are always more than 'S<sub>1</sub>' and the average value can be used for modelling the directional spectra.

### INTRODUCTION

The directional distribution of ocean wave energy is of great importance for a wide range of engineering and scientific applications (Weigel *et al.*, 1966, Goda *et al.*, 1981 and Deo, 1982). The angular distribution of wave energy around the propagating direction of waves is called the wave directional spreading. In most of the studies short-term directional spreading in shallow water is important and hence is considered in this paper. A well established way to describe the energy content in an irregular wind generated surface wave assumes superposition of linear waves. Accordingly the sea-state is assumed to be composed of a large number of individual wave components, each having a particular frequency (f) and direction of propagation (θ) (Sarpakaya *et al.*, 1981). A two-dimensional theoretical spectrum S(f,θ) may be adopted to describe the real sea state in continuum of frequency and direction regardless of wave numbers. This two dimensional energy spectrum S(f,θ) can be conveniently expressed as the product of the one dimensional energy spectrum S(f) and its angular distribution function D(f,θ) at each frequency (Borgman, 1969).

$$S(f, \theta) = S(f) \cdot D(f, \theta) \quad (1)$$

$$\text{where, } \int_{-\pi}^{\pi} D(f, \theta) d\theta = 1 \quad (2)$$

Many models are available to represent the wave directional spreading (Pierson *et al.*, 1955, Cote *et al.*, 1960, Longuet-Higgins *et al.*, 1963 and Donelan *et al.*, 1985) and the cosine power 2s model originally proposed by Longuet-Higgins *et al.*, (1963) is widely used and is given as below,

$$D(f, \theta) = G(S) \cos^{2s} [(\theta - \theta_0)/2] \quad (3)$$

$$\text{where } G(S) = \frac{2^{2s} \Gamma^2(s+1)}{2\pi \Gamma(2s+1)} \quad (4)$$

and Γ is the gamma function and 's' is spreading parameter. Here the value of the parameter s controls the directional spreading around the mean wave direction. Cartwright (1963) showed that 's' can be related to the Fourier coefficients in two ways as follows;

$$s_1 = \frac{r_1}{1-r_1} \quad \text{or} \quad s_2 = \frac{1+3r_2+(1+14r_2+r_2^2)^{1/2}}{2(1-r_2)} \quad (5)$$

$$\text{where } r_1 = \frac{\sqrt{a_1^2 + b_1^2}}{a_0} \quad \text{and} \quad r_2 = \frac{\sqrt{a_2^2 + b_2^2}}{a_0} \quad (6)$$

where a<sub>0</sub>, a<sub>1</sub>, b<sub>1</sub>, a<sub>2</sub> and b<sub>2</sub> are the Fourier Coefficients.

In this paper various directional wave parameters are estimated based on the data collected in shallow water and discussed.

### DATA MEASUREMENT AND ANALYSIS

The wave measurement was carried out using the Datawell directional waverider buoy, which is spherical in shape and 0.9 m in diameter. It is equipped with a heave-pitch-roll sensor (Hippy-40), a three axis fluxgate compass, two fixed accelerometer and a microprocessor. The roll-pitch information from the Hippy sensor is used to convert the x and y accelerations in the moving buoy reference plane to acceleration along the fixed, horizontal north-west axes. All three accelerations are then digitally integrated to displacements and filtered. The buoy was deployed at 15 m water depth on the south east coast of India in the bay of Bengal. The sea bed contours at that region are straight and parallel to the coast. The wave activity along this coast will be generally more during

the northeast monsoon period (October to January). Hence wave data collection programme was planned during November and December 1995. The sampling interval was 0.78125 s and the data was recorded for 20 minutes duration at every 3 hour interval. The data analysis is carried out by using the analysis technique proposed by Kuik *et al.* (1988). In this method the characteristic parameter of  $D(f, \theta)$  at each frequency are obtained directly from the Fourier coefficients  $a_0, a_1, b_1, a_2$  and  $b_2$  without any assumption of the model. The Fourier coefficients are estimated from the auto- and quadrature spectra of the collected buoy signals.

The directional parameters obtained are;

$$\text{directional width, } \sigma = \sqrt{2(1 - m_1)} \quad (7)$$

$$\text{skewness, } \gamma = \frac{-n_2}{[2(1 - m_1)]^{3/2}} \quad (8)$$

$$\text{kurtosis, } \delta = \frac{6 - 8m_1 + 2m_2}{[2(1 - m_1)]^2} \quad (9)$$

where  $m_1, m_2$  and  $n_2$  are estimated from the Fourier coefficients.

The two representative wave directions defined by Longuet-Higgins *et al.* (1963) as follows are also estimated.

$$\text{The mean wave direction, } \theta_0 = \arctan(b_1/a_1) \quad (10)$$

$$\text{The principal wave direction, } \theta_p = 0.5 \arctan(b_2/a_2) \quad (11)$$

Here the directional width is an index of the directional spreading while the skewness and the kurtosis give information about the shape of the directional spreading function i.e., it indicates whether the directional distribution is uni-modal or symmetric. To get an estimation of directional spreading Goda *et al.* (1981) defined the long crestedness parameter ( $\tau$ ) and the mean spreading angle ( $\theta_k$ ) as follows.

$$\tau = \left[ \frac{1 - \sqrt{a_2^2 + b_2^2}}{1 + \sqrt{a_2^2 + b_2^2}} \right]^{1/2} \quad (12)$$

$$\theta_k = \arctan \left[ \frac{\sqrt{0.5 b_1^2 (1 + a_2) - a_1 b_1 b_2 + 0.5 a_1^2 (1 - a_2)}}{a_1^2 + b_1^2} \right] \quad (13)$$

During the period of wave data collection an Aanderaa self recording weather station was installed on the open coast at 10m height above the mean sea level and the wind speed and direction were recorded at every 3 hour interval.

## RESULTS AND DISCUSSION

The probability distribution of the significant wave height, maximum wave height, zerocrossing wave period and the wave period corresponding to maximum wave height are shown in Fig. 1. It shows that the significant wave height predominantly varied

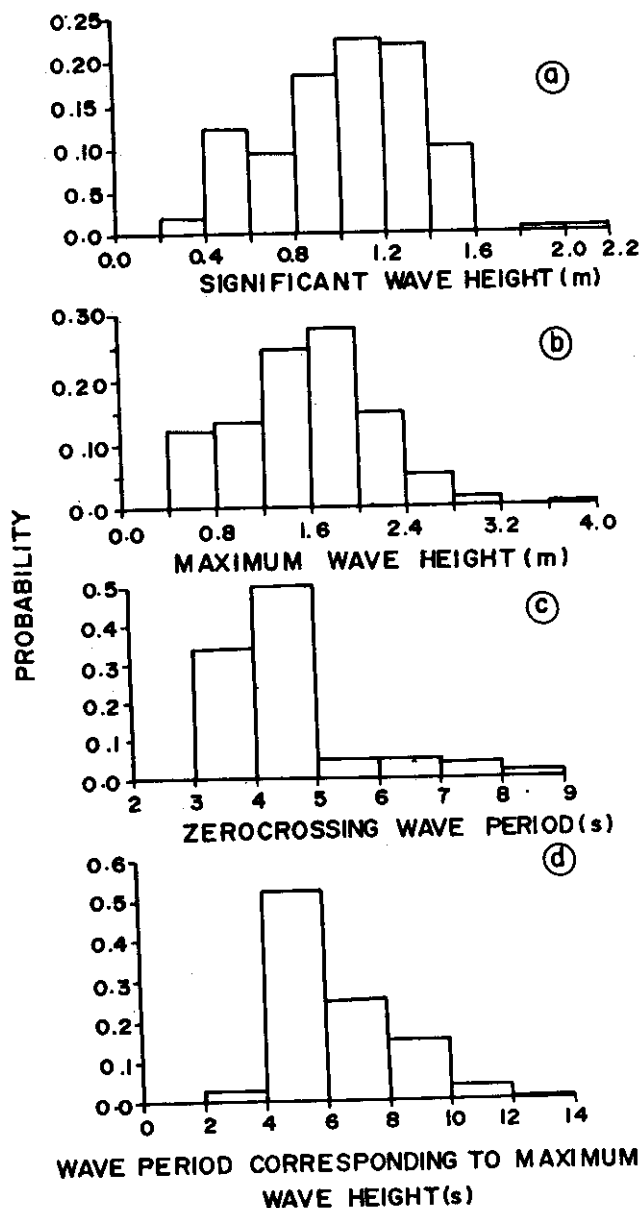


Fig. 1. Distribution of significant wave height, maximum wave height, zerocrossing wave period and wave period corresponding to maximum wave height.

from 0.4 to 1.6 m during the study period. The maximum wave height predominantly varied from 0.4 to 3.2 m. The zerocrossing wave period predominantly varied from 3 to 5 s and the period corresponds to maximum wave height predominantly varied from 4 to 10 s. Fig. 2 shows the variation of significant wave height with wind speed. The wind speed varied from 0.5 to 7 m/s. The maximum wave height determined from each 20 minutes record shows that it is 1.5 times the significant wave height (Fig. 3). Based on Rayleigh distribution for an average wave period of 4 to 5 s, the maximum wave height has to be between 1.6 to 1.7 times

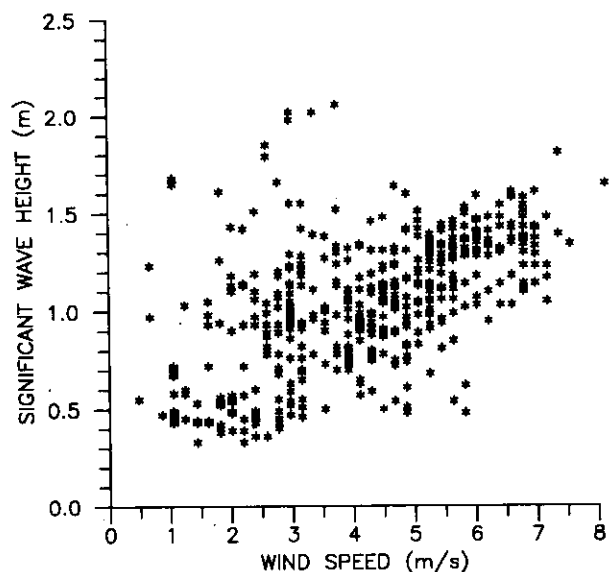


Fig. 2. Comparison of significant wave height and wind speed.

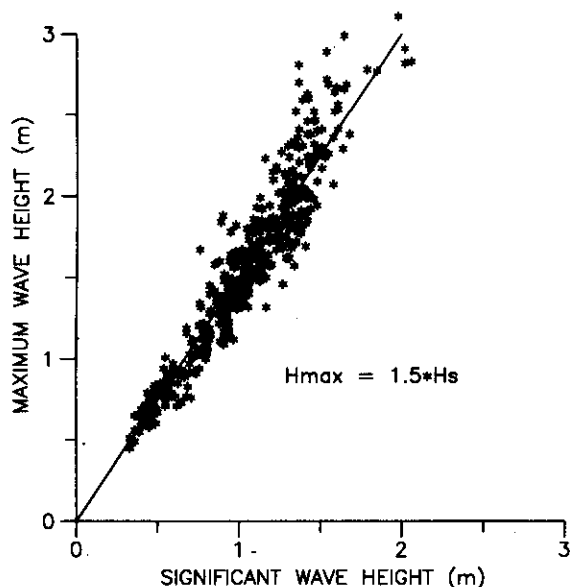


Fig. 3. Comparison of significant wave height and maximum wave height

the significant wave height. The difference may be due to the assumption of linear, Gaussian, narrow band free surface and of independent consecutive wave in shallow water.

A comparison of mean wave direction and the principal wave direction close to the peak frequency region is carried out (Fig. 4). It shows that the mean wave direction and the principal wave direction close to peak frequency is same. But the mean wave direction and the principal wave direction corresponds to higher and lower frequencies are not same. Since the principal wave

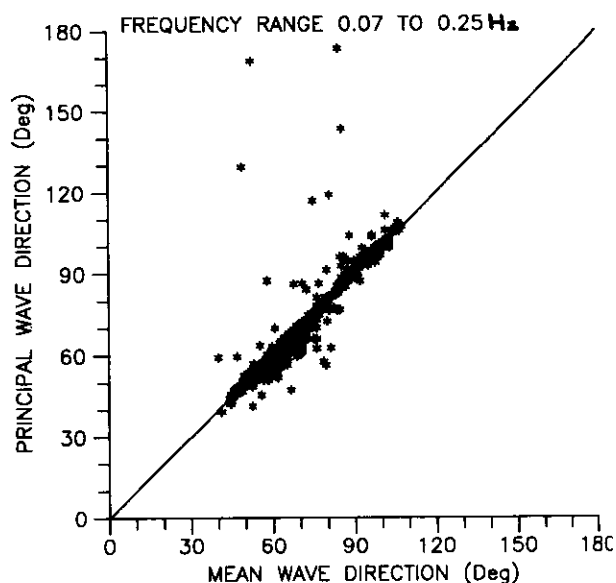


Fig. 4. Correlation between principal wave direction and mean wave direction

direction is obtained only in the range 0 to  $\pi$  where as the mean wave direction is obtained in full range (0 to  $2\pi$ ), the mean wave direction only is considered in further studies and the same can be considered as the most representative index.

The mean wave direction corresponds to the peak frequency varied from  $50^\circ$  to  $120^\circ$  with respect to north whereas the wind direction recorded at the coast shows that it varied predominately between  $0^\circ$  to  $90^\circ$  and between  $270^\circ$  to  $360^\circ$  (Fig. 5). The orientation of the coastline at this region is  $90^\circ$  with respect to north and is a straight and long open coast. This shows that the waves are

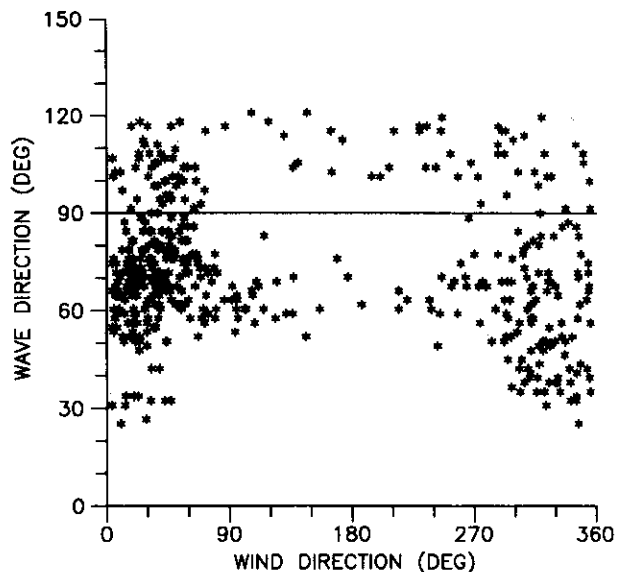


Fig. 5. Correlation between wave direction and wind direction.

generated in the deep water and the transformed wave characteristics are only recorded by the buoy, since the waverider buoy was close to the coast at 15 m water depth, about 5 km away from the coast. So care has to be taken while assuming the mean wave direction to be the same as the wind direction in modelling of shallow water waves. Donelan *et al.* (1985) also shown that waves with frequencies near the spectral peak can travel at off wind angles in non-stationary or fetch limited conditions. They have observed a value up to 50 degrees in Lake Ontario.

Fig. 6 shows the values of ' $s_1$ ' and ' $s_2$ ' estimated based on the collected data. Here the values of ' $s_1$ ' is found to be smaller than ' $s_2$ ' which is consistent with the observations of Cartwright (1963), Mitsuyasu *et al.* (1975), Hasselman *et al.* (1980), Tucker (1987) and Wang (1992). The difference may be due to bimodality, noise in the system (Tucker, 1987) and limitation of the resolution of the buoys. Therefore the parameter ' $s$ ' can be taken as the average of ' $s_1$ ' and ' $s_2$ ' (Cartwright, 1963). The effect of noise on ' $s_1$ ' and ' $s_2$ ' by numerical simulation was studied by Ewing and Laing (1987) and made the conclusion that ' $s_1$ ' is more sensitive to ' $s_2$ ' and recommended to use ' $s_2$ '. But here the difference in ' $s_1$ ' and ' $s_2$ ' is not large hence the average value of these can be taken for modelling the directional wave distribution.

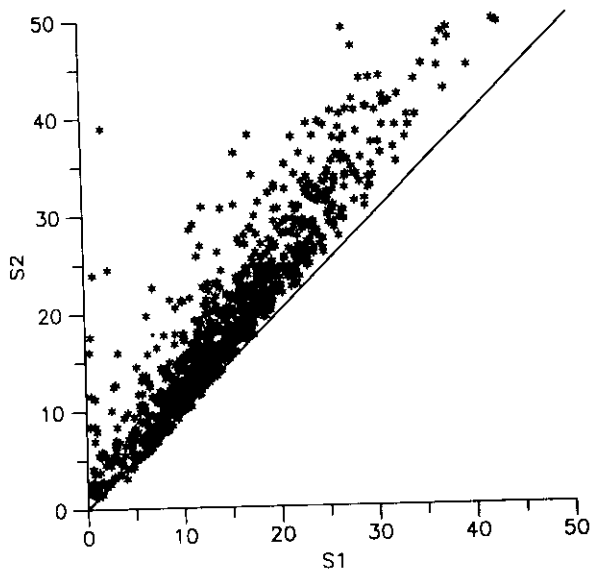


Fig. 6. Correlation between  $S_1$  and  $S_2$ .

A wave spectrum recorded during the study period alongwith the variation of mean direction, principal direction, directional width and ' $s$ ' parameter with frequencies are presented in Fig. 7. It is already seen that a two dimensional spectral model can easily be represented by the product of a unidirectional spectrum and a directional spreading model. For the west coast of India, Scott spectrum (Scott, 1965) has been recommended by Dattatri *et al.* (1977), Narasimhan and Deo (1979) and Sanil Kumar *et al.* (1994) by analysing the data collected off Mangalore, Bombay High and Karwar respectively. Here the spectrum computed from measured

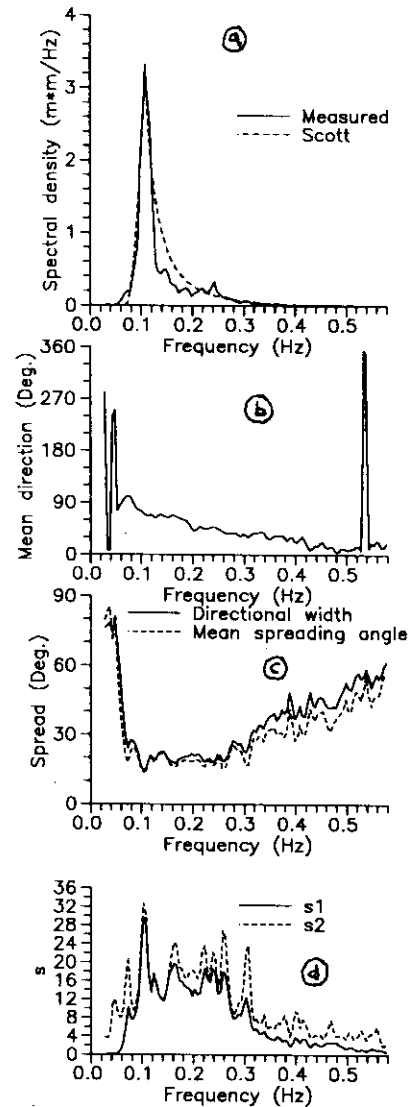


Fig. 7. Variation of spectral energy, wave direction, directional width, mean spreading angle and  $s$  with frequency.

data along with the Scott spectrum is presented in Fig. 7. When sea and swell are present there will be a secondary peak indicating the influence of direct wind input. This multi-peakedness seems to become predominant when the waves approach the coast due to a combination of swells and locally generated wind sea. The above recommended spectra will not model the multi-peakedness. But most of the designs are based on extreme conditions. At higher sea states the tendency of spectra with more than one peak is significantly reduced. Hence for practical applications the sea state can be represented by a single peak spectra and can be represented by the Scott spectra.

The study indicates that the mean spreading angle and the directional width shows similar values (Fig. 7c). Goda *et al.* (1981) and Besnard *et al.* (1994) also observed similar results.

## CONCLUSIONS

The maximum wave height was found to be 1.5 times the significant wave height. Since the waves were recorded in the nearshore, the waves were almost parallel to the coast. It is found that the wind direction deviates from the wave direction recorded during the same period. The mean wave direction is found to be the most representative index than the principal wave direction. The mean spreading angle and the directional width showed similar values.

The directional spreading parameter is found to have larger values near the peak frequency and it was less towards the lower and higher frequencies. It is found that the values of ' $s_2$ ' is always more than ' $s_1$ ' and the average value can be used for modelling the directional spectra and the unidirectional spectra can be modelled using the Scott spectra.

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