

Richardson Number, stability and turbulence -- a coherent view

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As turbulence in water is governed by vertical mobility controlled by static stability and horizontal mobility controlled by currents, the Richardson Number should give a measure of turbulence also. It is argued in this note that inverse of the Richardson Number (Ri^{-1}) is proportional to a process dependent vertical eddy mixing coefficient in water bodies away from surface, bottom and density discontinuities.

Brunt-Vaisala Frequency is defined in the upper ocean as,

$$N = [(-g/\rho_o) d\rho/dz]^{\frac{1}{2}} \quad \dots (1)$$

where, g = gravity acceleration, ρ_o = mean density for the layer, ρ = spot density and z = depth.

Richardson Number is defined as,

$$Ri = N^2/(c\rho/dz)^2 \quad \dots (2)$$

where, c = resultant current. Now, density,

$$\rho = \delta(t,s,p) \quad \dots (3)$$

Hence, in a barotropic ocean N would be a function of pressure (p) only and is universal for all oceans. As the distribution of temperature (t) and salinity (s) vary from Equator to poles and oceans to oceans, N would vary from region to region. Inhomogeneities in temperature and salinity and external wind driving are the three significant factors which cause the field of motion in oceans. Now, considering the wide variations of oceanic currents Ri can be understood to vary depthwise and from place to place. The field of motion consists of various space and time scales, varying from seconds to hours and millimeters to tens of metres. These different modes of motions in relation to vertical eddy coefficient (Kz) can be seen in a unified way in Fig.1 constructed using historical data¹ on Kz . Here the different wavy domains, A1, A2 etc., specify some ranges of Kz in relation to different processes, depth ranges, boundaries etc.

Various reviewers have pointed out the large variations in Kz from different environments¹⁻³. Sverdrup *et al*'s² summary showed a range from 0 to 7500 $g.cm^{-1}.sec^{-1}$ obtained using different tech-

niques from very different environments. Pollard's¹ summary showed a range of 20-500 $cm^2.sec^{-1}$ in the surface layer alone. Kullenberg⁴ and Buch⁵ studied vertical eddy coefficient in shallow fords as a function of buoyancy frequency, wind and current using dye injection techniques for short periods. Kullenberg (loc cit) could only find that Kz decreases with increasing Ri . Jankowski⁶ found good correlations for Kz with N and Ri . Davies⁷ reviewed different methods of estimating Kz in surface boundary layers and found that Kz increases rapidly with wind speed.

Gargett³ after reviewing the literature on vertical eddy diffusivity in the interiors of oceans suggested an 'N'-dependent relationship as,

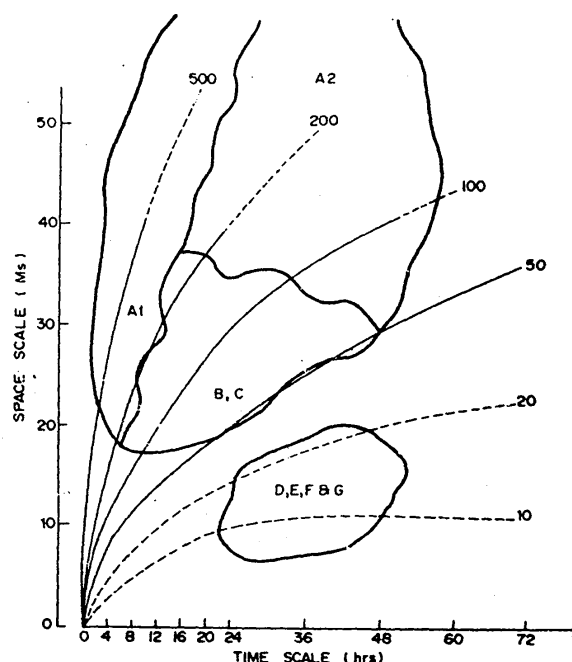


Fig. 1—Historical data on Kz distributed on space-time domains

$$Kz = a_0 N^{-q} \text{ with } q \sim O(1) \quad \dots (4)$$

Gargett (loc cit) arrived at $O(1)$ as the power of N from a range of 0.72 to 1.2 found by various workers. Varkey⁸ taking the cue from Gargett assigned $q = 1$ and calculated Kz for the seasonal thermocline in the Bay of Bengal using seasonal mean data. The vertical Kz profile with a peak at the pycnocline showed good agreement with vertical profiles of dc/dz and N . These results point to a strong possibility that in nature Kz and stability are related just inversely, i.e. $q = 1$. Let us examine some physical aspects involved in the mixing processes.

Fig. 1 is, in part, a depiction of Pollard's¹ summary of data on Kz . It can be seen from this figure that high Kz values are associated with a wide range of space scales (or wave lengths) and a low range of time scales (or periods). As the time scale increases, the range of Kz and its variations decrease considerably with low range of space scales. It is known that Kz is process dependent and hence model specific. In other words, in the field, turbulence processes (generating agencies including internal waves, dissipation etc) would have definite associations with depth-ranges, types of water bodies and boundary situations which affect energy input from outside. The depth-range association can be better understood from Fig. 2. It is known that the turbulence-processes would be very different in the upper mixed layer (a_1) compared to the lower mixed layer (a_2) and the pycnocline (b). Similarly for other lower layers also. Considering only the surface layer, the turbulence processes can be very different in a strait, fjord, shelf waters and open sea. Again, different boundaries like sea surface, deep sea bottom, sill depths and bottom regions of shelf waters

also have a controlling role on the nature of the turbulence. Hence it would be possible to delineate some Kz -domains wherein the turbulence processes can be characteristically different (A1, A2 etc. in Fig. 1). Now, consider the depth-range a_1 in Fig. 2. Here, the main generating agencies are wind, surface waves, currents and convection. The direct effects of wind and surface waves would be limited to a very shallow layer of about 25 m. In this layer different agencies would have dominant roles depending upon time of day, seasons and weather. Here Kz would show high values and wide scatter as shown in Fig. 1 (domain A1). (Hoerber noted¹ that eddy viscosity varied enormously with wind speed from 400 to 4000 $\text{cm}^2 \cdot \text{sec}^{-1}$ for an increase from 7 to 8 $\text{m} \cdot \text{sec}^{-1}$). Below ' a_1 ', in ' a_2 ', convection, currents and internal wave breaking can be the dominant agencies. Again, this depth-range would be influenced by total thickness of the mixed layer (i.e. ' a_1 ' + ' a_2 '). In an area where the mixed layer is about 100 m, the mixing processes can be very different from those of an area with a mixed layer of 40 m. In the seasonal thermocline range ('b' and 'c'), shear and internal waves are the main generating agencies. In the layer 'd' (~ 400 to ~ 2000 m) sigma-t is distributed linearly with $\log(z)$. Bennette⁹ also observed a similar pattern in the Arabian Sea on mean data profiles and postulated mean Kz to be equal to $0.02 \times D$ (in metres), i.e., $K_{400} = 8 \text{ cm}^2 \cdot \text{sec}^{-1}$ and $k_{1200} = 24 \text{ cm}^2 \cdot \text{sec}^{-1}$. In the interior of oceans ('e' and 'f') the energy content of the processes would be much less and Kz can be expected to be low. Varkey⁸ estimated Kz as $7 \text{ cm}^2 \cdot \text{sec}^{-1}$ in the Bay of Bengal for the layer 1750-2250 m (i.e., 'e' layer in Fig. 2). Wyrki¹⁰ for computing vertical advection in deep sea basins of the South East Asian waters (depth-range 'g' in Fig. 2) assigned $Kz = 4$. These low values fall close to the Kz -domain G. Since Kz is always related to stability inversely, the constant a_0 (in Eq. 4) would be controlled by factors like generating agencies, depth-range and the measuring or averaging techniques used depending upon range of wave lengths and periods of the processes. In the deep interiors of the oceans (say 2000 m), $a_0 \cdot N^{-1}$ can give a good estimate if the data is seasonal mean or 10^6 square mean. But, if Kz is to be estimated over short time scales and in regions wherein currents are strong, the appropriate formula seems to be $Kz = a_0 \cdot R^{-1}$. Hence, away from surface and bottom boundaries and density discontinuities (in D, E F in Fig. 1), an inverse relationship of N or Ri would give a very good estimate of Kz provided the time and space scales of the processes are properly considered during measuring and processing of the data and quantified in a_0 .

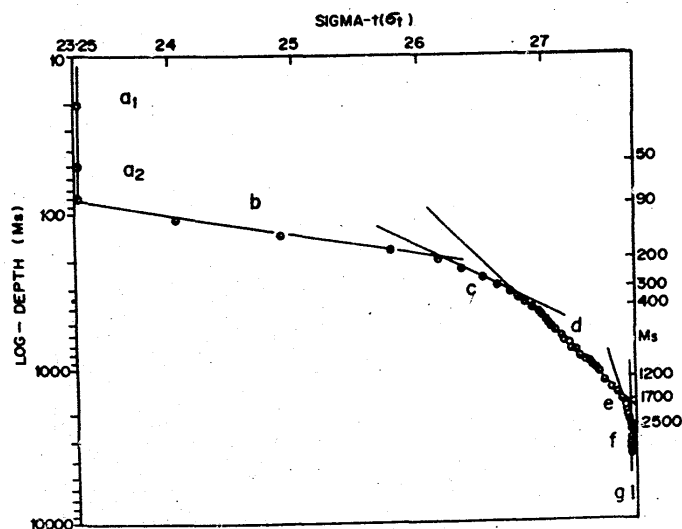


Fig. 2—Sigma-t against log-depth using CTD data from a deep station in the Arabian Sea

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