

Arc arrays: studies of high resolution techniques for multibeam bathymetric applications

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In this paper a theoretical study is initiated to observe the utility of directional spectral estimation techniques for 'arc array' geometries. We examine the suitability of a 15° arc transducer geometry for multibeam bathymetric applications. This geometry is tested using the Bartlett method for varying arc and linear arrays of 30-elements. We also examine 'high resolution techniques' such as the Maximum Likelihood (ML) method and the Maximum Entropy (ME) methods (different orders), for 16-element arrays of 15° arc. The superiority of the high resolution methods is seen by examining the beam patterns of the 15° arc array under multiple source/interference conditions, i.e. the situation for rough terrain or artefact-creating conditions. Furthermore, the 15° arc array geometry is found to be useful for multifrequency applications at different operating wavelengths, which is useful for underwater bathymetric applications.

Keywords: arc arrays; bathymetry; underwater applications

Multibeam echo-sounding surveying of the seafloor is a well-known method of obtaining high resolution bathymetry with greater area coverage¹. Linear arrays of transducers are used for such purposes to obtain directive beam-patterns for transmission or reception. Transducer arrays with geometries that depend on angular parameters have been studied extensively, and are expected to be frequency independent². Our study of arc geometry was carried out to observe the suitability of such an array for multibeam bathymetric applications. Moreover, the usefulness of arc arrays is examined for multi-frequency sounding applications, e.g. varying seabed feature determination³.

The beam-forming method, which is extensively used for multibeam applications, is known as the 'delay-sum' method. The difficulty with such a method is the requirement of a large array size, especially for high resolution use. By using a smaller transducer array, a superior beam-forming method, known as a 'high resolution' technique, may be applied, which requires complicated signal processing to obtain higher resolutions. The 'high resolution' techniques are also known as 'directional spectral estimation' methods⁴.

The directional spectral estimation method, known as the Maximum Likelihood (ML) method was proposed by Capon⁵. When a set of observed signals (signals across array elements) is considered, and their related parameter

of interest (direction of signal arrival) is desired, the ML method assigns the direction of arrival based upon the computed maximum log likelihood function. Similarly, the Maximum Entropy (ME) method⁶, is based on selecting one of the array elements as a 'reference'. A computation is then carried out to determine weights to be applied to the remaining elements and to obtain their sum with a minimum mean square fit to the receiver⁷. Chakraborty and Schenke⁸ examined a high resolution method for a 16-element linear array and their study emphasized the application of such methods for bathymetric studies. More importantly, Johnson⁹ emphasized the necessity of studying high resolution techniques for arrays of varying geometry. Because the location of each sensor within the array is hidden in the formulation of the spectral estimate, the use of high resolution techniques on the 'arc arrays' will also provide some interesting information related to the behaviour of different algorithms for varying array geometry.

In this paper, a 15° arc geometry is selected as a suitable geometry after studying the theoretical Bartlett beam patterns of 30-element (e.g. 120°, 60°, 15°) arcs and equivalent linear arrays. The beam patterns of the 16-elements' arc of 15° are then examined for different signal-to-noise ratios for varying directions of arrival using the ML and ME methods (the bottom is assumed to be flat). Although the study is carried out for 15° arc arrays, it can be extended for any arc geometries. The advantage of arc arrays over linear arrays is well known. By using arc arrays, multibeam beam-steering may be

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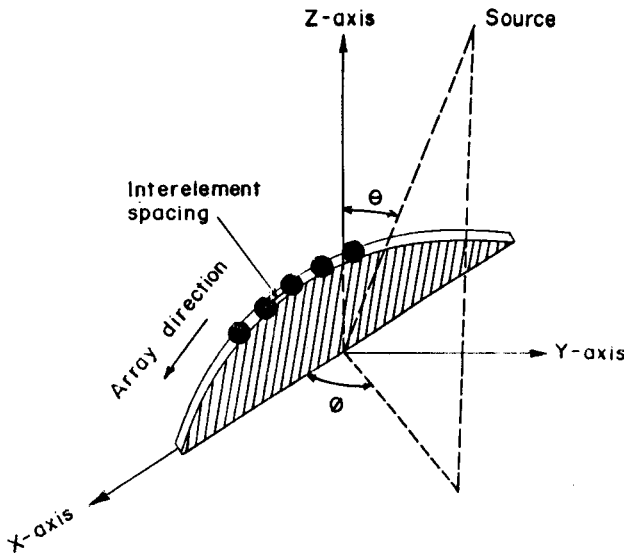


Figure 1 Geometry of the arc array

achieved for any direction by adding more radiators within the array. This is difficult for linear arrays beyond a certain angle of steering.

The situation is more complicated when multibeam systems are operated over seamounts, slopes or ridge areas. The multiple source/interference arrivals for different beam directions are well known. This condition significantly affects the resolution of the array¹⁰. In view of these problems, a study is conducted to understand the performance of the 15° arc array geometry. It is also assumed that such conditions are occurring simultaneously. The present study includes the cases for closely and widely separated sources of different strengths. The cases of coherent and incoherent sources are also examined.

In addition, a study is also initiated to observe the multifrequency performance of 15° arc arrays when high resolution techniques are used. The ML and ME beam-forming methods are carried out at operating wavelengths of 0.5λ, 1.5λ and 2.5λ respectively. λ is assumed to be the design wavelength.

Analyses

We assume that the linear array of 30-elements is oriented along the x-axis and element separation is 0.5λ. In order to make a comparison of different arc arrays with linear arrays, we also assume comparable radii of 8.37λ, 14.50λ and 55.54λ for 120°, 60° and 15° arc arrays respectively. These arcs, or segmented circular arrays, are kept in the xy-plane (Figure 1). The spatial correlation matrix of the sensor output for spatially uncorrelated white noise is given by

$$R = [R'(m, n)] + \Sigma_n^2 I \tag{1}$$

where Σ_n^2 is the white noise power level at each element, m and n are element numbers, and $R'(m, n)$ is given as

$$R' = \Sigma_s^2 \exp[-jk d_{mn} \sin \theta_1] \tag{2a}$$

where Σ_s^2 is the source power level, which is identical across each sensor element. θ_1 is the source angle. We

also assume that the source direction is changing within the XZ-plane, i.e. $\phi = 0^\circ$. The wave-number, $k = 2\pi/\lambda_0$, where λ_0 is the operating wavelength. d_{mn} is the separation between the transducer elements and may be expressed as

$$d_{mn} = 2a \sin\{(m - n)\pi \text{arc}/360N\} \tag{2b}$$

where N is total number of array elements, the arc is measured in degrees and a is the radius of the arc. Using the Bartlett method, the beam-formed output for a known look angle is rewritten as⁹

$$P_{BM} = [E'(\theta)RE(\theta)] \tag{3}$$

where E represents the plane-wave propagating in the look direction θ

$$E'_m = \exp(-jk d_m \sin \theta) \tag{4}$$

d_m is the distance from the origin to different elements within the array geometry. E'_m is the complex conjugate of the transpose of E . Under the assumption of a flat bottom, only a single beam is formed for each direction. Equation (3) is used to compute the Bartlett pattern for a 30-element array. Figure 2 shows the Bartlett beam patterns for 120°, 60° and 15° arc arrays and the equivalent linear array size at source angles of 0° and 45°. The signal-to-noise ratio across each element is assumed to be 0 dB. For the 0° source direction, the half power beam widths are found to be the same (3.3°) for all given arrays, although the sidelobe levels are significant for 120° arc arrays. However, for the 45° source direction the beam degradation and sidelobe levels are high for 120° and 60° arc arrays, which makes them unsuitable for multibeam applications. Even though the beamwidths are higher (5°) for 15° arc and linear arrays, the performance of a 15° arc array is such that it seems to be suitable for multibeam applications. Therefore, we will examine the performance of a 15° arc array to observe its effectiveness as a 'high resolution array', as well as its possible multifrequency application.

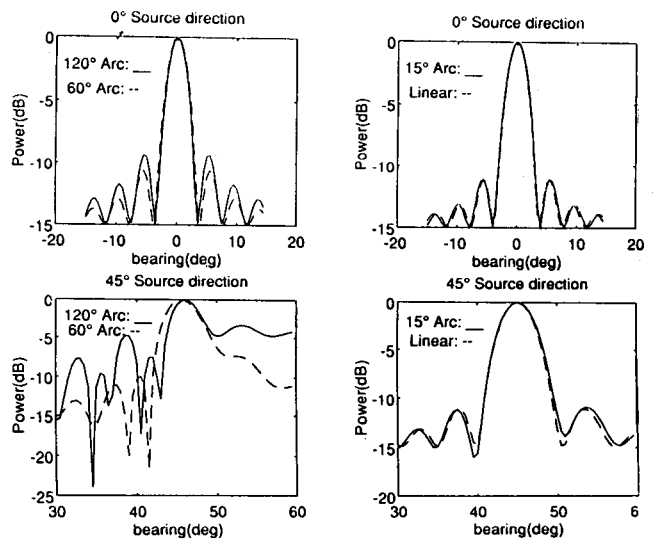


Figure 2 Bartlett beam patterns of different arc and linear arrays of 30-elements

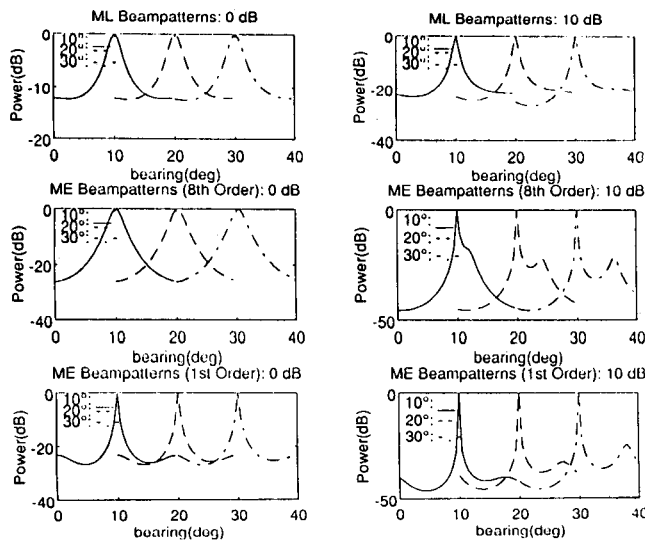


Figure 3 ML beam patterns and ME beam patterns for the first and eighth-order arc array of 16-elements

High resolution arrays

Using the ML method, the beam-formed output of a steered beam is given by

$$P_{ML} = [E'(\theta)R^{-1}E(\theta)]^{-1} \quad (5)$$

The power output of the ME beam is expressed as

$$P_{ME} = [E'(\theta)cc'E(\theta)]^{-1} \quad (6)$$

where c is a column of the R^{-1} matrix. Under the assumption of flat bottom conditions, only a single beam is formed for each direction. Using Equations (5) and (6), the ML and ME beam patterns were generated, and are presented in Figure 3. Chakraborty and Schenke⁸, demonstrated the utility of a 16-element linear array using the 'high resolution' technique. In this paper a similar study is carried out for 16-element arc arrays of 15°. The source directions are chosen to be 10°, 20° and 30°. The signal-to-noise ratio across each element is assumed to be 0 and 10 dBs respectively. Also, a comparison between the 'high resolution' ME beam patterns is made by selecting column matrices of the order of 1 and 8. No spurious peaks are seen for different directional beams using signal-to-noise ratios of 0 dB. The half-power beamwidths are narrowest for ME beam patterns of first order. The beamwidths are comparatively narrower for a signal-to-noise ratio of 10 dB than for 0 dB. However, spurious peaks are observed in the ME beam patterns. The level of the spurious peaks is less significant for ME beam patterns of first order than for the eighth order. The presence of such spurious peaks makes the ME method less effective for multibeam bathymetric applications.

Multiple source conditions

Under the assumptions of flat bottom conditions, a single beam is formed at a particular instant of time. When terrain is rough, however, multiple beams are formed at a particular instant of time. Such conditions frequently occur while surveying seamounts, continental slopes or ridge areas. The formation of multiple beams at a particular instant of time may be considered as the arrival

of multiple sources or interferences from different directions towards the array. The performance of the array is seriously affected under such situations.

Error-causing phenomena affecting multibeam systems include sidelobe interference, bottom tracking gate malfunctions, and external interferences from other sounding devices¹⁰. External sounding devices can generate signals that may appear at different beams and act as different sources. Similarly for slope areas, the multiple source arrival condition is observed for a particular direction, because of sidelobe interference from transmission beams towards the receiving beams, and this is known as the 'Omega effect'. Signal saturation for the normal incidence beam is common in flat bottom areas. This beam sometimes acts as a sidelobe to the adjacent beams. This is known as a 'Tunnel effect'. The omega effect, tunnel effect, and external interferences may affect ML and ME beam-forming methods.

Incoherent sources

External interference appearing at the array elements may be assumed to come from incoherent sources. Cox¹¹ has examined the utility of the ML method to resolve two equal energy sources (uncorrelated). For multiple incoherent sources (i.e. for two sources) the correlation matrix (Equation (2)) is rewritten as

$$R'(m, n) = \Sigma_1^2 \exp[jk d_{mn} \sin \theta_1] + \Sigma_2^2 \exp[jk d_{mn} \sin \theta_2] \quad (7)$$

Σ_1^2 and Σ_2^2 are source powers. θ_1 and θ_2 are source directions. Other notations were already described. To compute the beam patterns of the ML and ME algorithms of first order, the above equation is used along with Equations (5) and (6).

For the present study, we considered two combinations of sources (10 dB, 10 dB) and (0 dB, 0 dB). These are considered for a close beam separation of 4° and wide beam separation of 20°. The beam patterns are shown in Figure 4. It is observed that for wide separations, the sources are well resolved for the 15° arc array using the

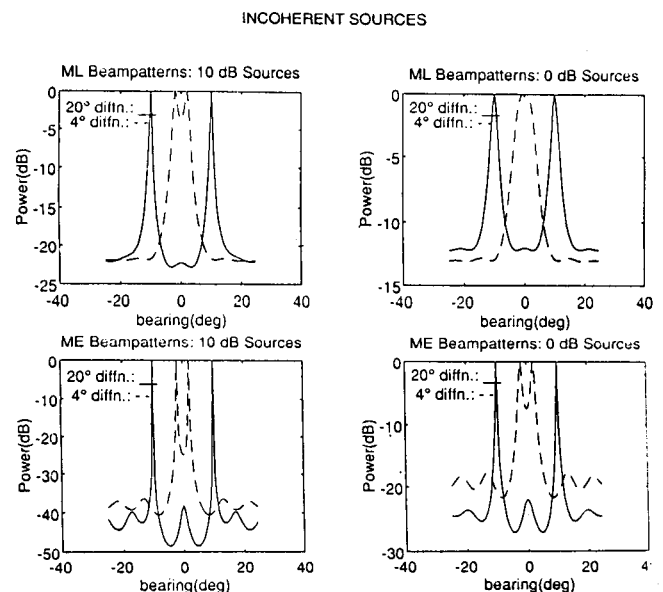


Figure 4 ML and ME (first order) beam patterns of incoherent sources

COHERENT SOURCES

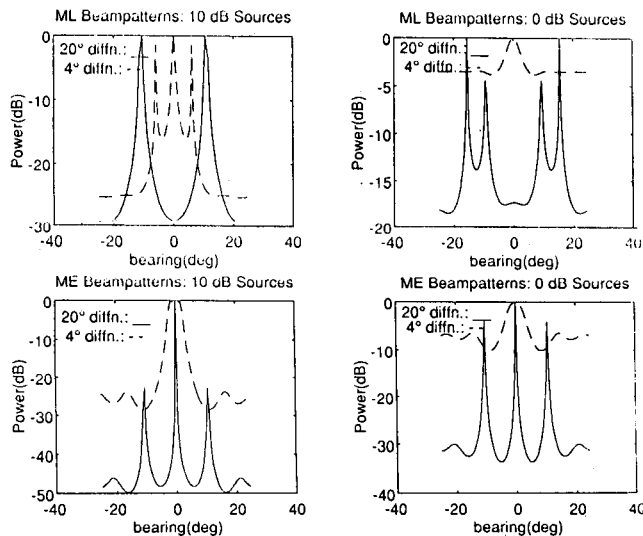


Figure 5 ML and ME (first order) beam patterns of coherent sources

ML and ME methods. The resolution is slightly better for the ME of first order than for the ML method. Using the ML method, the sources are unresolved at source separations of 4° for 0 dB sources.

Coherent sources

Two sources are said to be coherent if one is a scaled and delayed replica of the other¹². If these signals share the same centre frequency and have a constant phase relationship, then the coherence magnitude will be one¹³. Therefore, the interferences resulting from the ‘omega’ and ‘tunnel’ effects or the interference from rough terrain may be considered to be coherent interference, because such sources share the same frequencies and have fixed phase differences.

The correlation matrix for the two coherent sources may be expressed as¹⁴

$$R' = \Sigma_1^2 \exp[jk d_{mn} \sin \theta_1] + \Sigma_2^2 \exp[jk d_{mn} \sin \theta_2] + 2 \Sigma_1 \Sigma_2 E \{ \cos[(k/2)(m+n)d \Delta u - \phi] \} \times \exp[jk d_{mn} u_m] \quad (8)$$

$E\{\dots\}$ is the expected value of the function inside the curly brackets: d is the closest spacings between the elements, i.e. d_{mn} for $m=2$ and $n=1$ in Equation (2a); Δu is represented by $(\sin \theta_1 - \sin \theta_2)/\lambda$; ϕ is the phase difference, for the present study it is assumed to be 0; and u_m is rewritten as $(\sin \theta_1 + \sin \theta_2)/2\lambda$. The beam patterns computed using the ML method for a 10 dB signal-to-noise ratio across each element do not show any degradation for widely separated sources, whereas for closely separated sources (4°) a spurious peak is seen in the 0° direction (Figure 5). For a 0 dB signal-to-noise ratio, spurious peaks are observed for widely separated sources but the sources are unresolved for closely located sources. This situation is relatively better than the ME methods. For higher signal-to-noise ratios across each sensor element and widely separated source conditions, the ML method performs better than ME. Spurious peaks are seen in ME beam patterns at widely separated sources

of both the signal-to-noise ratio conditions, whereas for closely separated sources the sources are unresolved.

Multifrequency applications

In order to study the wideband performance of a 15° arc array, the ML and ME directional spectral estimation techniques are used at operating wavelengths of 0.5λ , 1.5λ and 2.5λ . These beam patterns may be compared with the beam patterns of operating wavelengths of 1.0λ . If the design frequency of the 15° arc array is assumed to be 15 kHz, then the operating wavelengths of 0.5λ , 1.5λ and 2.5λ correspond to 30 kHz, 10 kHz and 6 kHz respectively. The velocity of sound in water is considered to be 1500 ms^{-2} .

The high resolution ML and ME beam patterns are computed using Equations (5) and (6) for a single source with a 0 dB input signal-to-noise ratio. The source direction is assumed to be 0° and 30° and the bottom is assumed to be flat. The results are presented in Figure 6. For the ML method of 0° direction, the half power beam widths increase from 0.5λ to 2.5λ . For the 30° source direction the beamwidths are found to be similar to the 0° direction, and no significant changes in beam patterns are observed. The beam patterns are found to be sharpest for the ME method of 0° and 30° directions, and variations of operating wavelengths are found to be insignificant. Although an increase in the beamwidth is seen with higher operating frequencies for ML beam patterns, the affect of changing source directions is not observed. No spurious peaks are observed for either high resolution method, and no bias in the source direction is seen for both methods, even at low signal-to-noise ratios across the sensor element. Both high resolution methods are frequency independent under the single source condition. The changes in beamwidth with operating frequencies are slightly higher for the ML method than the ME method. This study demonstrates the effectiveness of an arc array for high resolution multibeam bathymetric use.

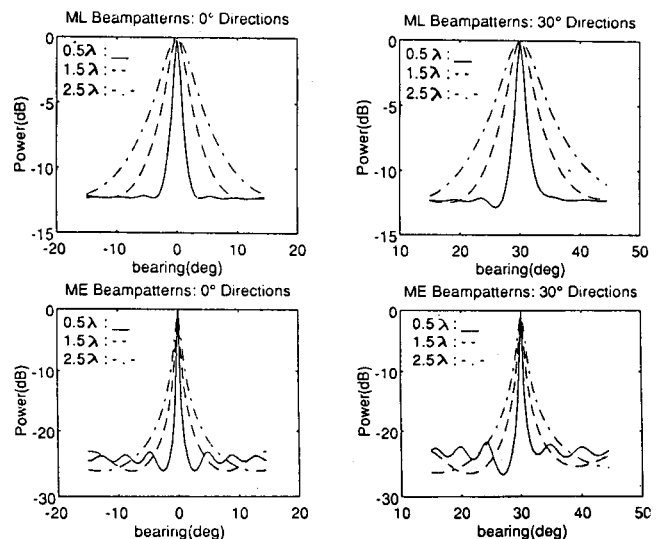


Figure 6 ML and ME (first order) beam patterns at different operating wavelengths

Conclusions

In this paper a theoretical study is initiated to observe the utility of high resolution beam-forming methods for arc arrays. Examination of single source Bartlett beam patterns of different 30-element arc geometries confirms the suitability of 15° arc arrays for multibeam bathymetric applications. The application of high resolution studies for 16-element arrays of 15° arc shows the usefulness of eighth order ME methods in terms of narrower beamwidths and lower level of spurious peaks. This study also incorporates the results of 'high resolution' techniques by examining an arc array under multiple source conditions, i.e. rough terrain as well as artefact creating conditions.

Except for closely separated (4°) incoherent sources (0 dB sources) using ML methods, all the sources are resolvable. The result is completely different for coherent sources¹³. Except for widely separated (20°) coherent sources using the ML method (10 dB sources), the beam patterns are severely degraded in terms of the unresolvability of sources or the appearance of spurious peaks. Generally, the performance of the high resolution ME method is found to be more suitable, except for multiple-coherent source conditions. For multifrequency use, the examination of single-source beam patterns for the 0° direction for high resolution techniques suggests the superior performance of the ME method over the ML one for 15° arc arrays.

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