

A METHOD FOR GEOMETRIC MODELLING OF MAGNETIC ANOMALIES FOR TWO-DIMENSIONAL BODIES

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ABSTRACT

Two-dimensional step and dyke models are extensively used for modelling magnetic anomalies as they are compatible with many planar two-dimensional geological structures. The anomalies for these models are obtained by employing different formulae for bodies of different shapes. A procedure has been evolved to compute the anomalies for all types of step and dyke models from a single formula by suitably reorienting the step model and by redefining its edges and the slope or dip angle. This method also facilitates calculation of the anomalies for a vertical dyke with sloping surface and infinite depth extension.

INTRODUCTION

With the advent of computerisation, the volume of geophysical literature on analysis and interpretation aspects of magnetic data became almost unmanageable. Talwani (1965) presented a computer-oriented method to evaluate anomalies caused by bodies of arbitrary shapes. Self-adjusting procedures using the least squares method or the Marquardt (1963) algorithms were developed by Bosum (1968), Hall (1968), Johnson (1969) and McGrath and Hood (1970) for two-dimensional bodies like cylinders, dykes and steps etc.

Careful examination of the existing methods for computing the magnetic anomalies reveals that different formulae are employed for bodies of different shapes. These have been used to get satisfactory fit for the simple and isolated anomalies. Quite often,

the anomalies recorded on isolated profiles are complex and overlapping and it becomes difficult and time consuming to model anomalies by adopting separate formula for each anomaly. However, this difficulty has been overcome to a large extent in the methods developed for computing the anomalies of arbitrary shaped bodies with polygonal cross-section (Talwani 1965). This method basically requires the closed polygonal body and the anomalies for the two-dimensional models extending infinitely along X or Z-axis could be calculated to some approximation by assigning large values for the infinitely extending corners of the body. However, this method fails to incorporate the dipping dyke extending infinitely downwards. While Hall (1968) and McGrath and Hood (1970) calculated the anomalies for the step

and dipping dyke models, no procedure seems to be available for calculating the anomalies of an infinitely extending vertical dyke with sloping surface. Inverse methods such as the deconvolution method of Warner (1953), later improved by Jain (1976) and Ku and Sharp (1983), and the analytic signal method of Nabighian (1972, 1974), compute the body parameters directly from the observed data. While these methods are extensively used for the analysis and interpretation of magnetic data, they fail to locate the lower edges of the vertical or nearly vertical bodies and could not resolve the edges of the closely-spaced bodies.

Suitable combinations of the two-dimensional step and dyke models that include the step with vertical or sloping surface, the vertical dyke with horizontal or sloping surface and the dipping dyke may be used to approximate many of the two-dimensional geological structures that are usually encountered. An attempt is, therefore, made to evolve a procedure for computing the anomalies of all the above-mentioned two-dimensional bodies from a single formula.

ANOMALY COMPUTATIONS

Magnetic anomalies for different two-dimensional bodies are usually calculated by equations with their origin located over the centre of the bodies. However, if the origin is located arbitrarily to the left of the body, the X-axis is always reckoned positive as was done by Talwani (1965). Adopting the same procedure and a common notation, the equations for the step model of Hall (1968) and Nabighian (1972) as well as for the dipping dyke of Hall (1968) and McGrath and Hood (1970) can be rewritten as :

a) Step Model

$$T(X) = kTC \sin d (2 \sin (2I-d) (\theta_1 - \theta_2) + \cos (2I-d) \ln \frac{r_2^2}{r_1^2}) \quad \text{----(1)}$$

b) Dipping Dyke Model

$$T(X) = kTC \sin D (2 \sin (2I-D) (\theta_1 - \theta_2) + \cos (2I-d) \ln \frac{r_2^2}{r_1^2}) \quad \text{----(2)}$$

Where

T(X) = Total magnetic field intensity anomaly,

k = effective magnetic susceptibility,

T = intensity of earth's main magnetic field,

C = $1 - \cos^2 i \sin^2 A$,

i = inclination of the earth's magnetic field,

A = the angle between the magnetic north and the positive X-axis,

d = slope angle of the face of the step,

D = dip of the dyke,

I = $\tan^{-1} (\tan i / \cos A)$

$\theta_1 = \tan^{-1} ((X(I) - BX_1) / BZ_1)$,

$\theta_2 = \tan^{-1} ((X(I) - BX_2) / BZ_2)$,

$$r_1^2 = (X(I) - BX_1)^2 + BZ_1^2$$

$$r_2^2 = (X(I) - BX_2)^2 + BZ_2^2$$

$BX_1, BX_2 = X$ co-ordinates of the body corners 1 and 2,

$BZ_1, BZ_2 = Z$ co-ordinates of the body corners 1 and 2 and

X(I) = X co-ordinate of the observations prints on the X-axis

The parameters $\theta_1, \theta_2, r_1, r_2, d$ and D for

the step and the dipping dyke models are shown in Figs. 1A and 1B respectively. In Fig. 1 and in subsequent figures the body corners are numbered 1 and 2 for convenience of their representation and further discussion.

(or vice versa) by suitably re-orienting the corners of the body so that the step model in Fig. 2A can be transformed into the dyke in Fig. 2C. This could be accomplished by rotating the step model in Fig. 2A by 90° in

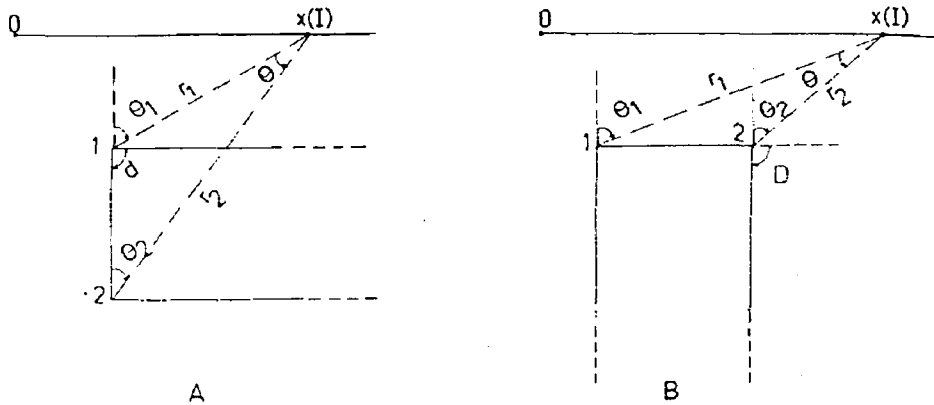


Fig. 1 A) Vertical step and (B) Dipping Dyke

The X-axis is positive towards right and the Z-axis is positive downwards.

Dipping Dyke :

In equations 1 and 2, all the parameters are identical except d and D , which correspond to

clockwise direction. This would make the Z co-ordinates of both the corners equal and the throw of the step model would become the width of the vertical dyke model ($D = 90^\circ$). However, the corner 2 of the body is located to the left of corner 1 and is 180° out of phase. Therefore, the body in Fig. 2B has to

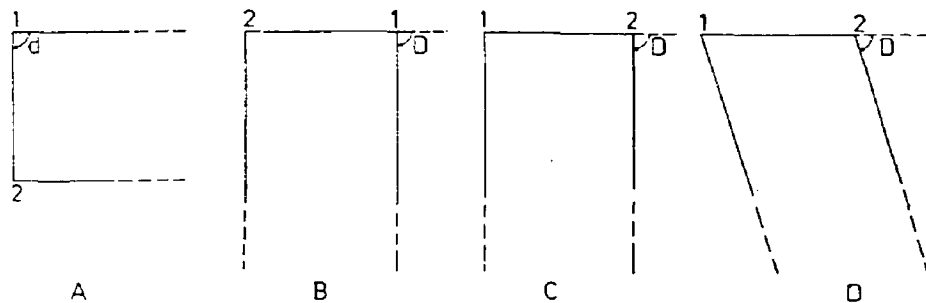


Fig. 2 A) Vertical step, (B) Vertical dyke obtained after rotating the step by 90°, (C) Vertical dyke after being flipped and (D) Dipping dyke.

the slope angle of the step and the dip angle of the dyke respectively. Therefore, the anomalies for the dyke model could be computed from the equation for step model

be flipped so that the X co-ordinates of the corners 1 and 2 increase on the positive X-axis as shown in Fig. 2C. Having done this, the anomalies for the vertical dyke with

horizontal surface could be calculated by substituting the corresponding angle ($d = D = 90^\circ$), the depth to the top of the dyke ($BZ_1 = BZ_2$ or vice versa) and BX_2 is equal to BX_1 plus the vertical throw of the step in equation 1. Anomalies for the dipping dykes (Fig. 2D) could be computed by substituting appropriate dip angles D in equation 1.

the anomalies for the vertical dyke with sloping surface (Fig. 3B) could be computed. However, the corners of the vertical dyke model in Fig. 3B are 180° out of phase with reference to the origin. In order to place the body corners in proper direction as shown in Fig. 3C, the model in Fig. 3B has to be flipped. This involves change in sign of the

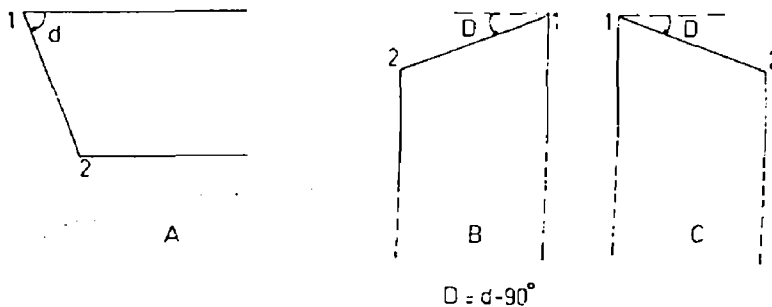


Fig. 3 A) Step with sloping surface, (B) Vertical dyke after rotating the sloping step by 90° and (C) Vertical dyke after being flipped.

Anomalies for the vertical or dipping dykes with horizontal surface, could also be calculated directly by substituting the corresponding X and Z co-ordinates of the two corners of the dyke model and the dip angle (Fig. 2D) in equation 1 without having to rotate dyke with sloping surface.

With the same procedure of anomaly computations and the same notation for the parameters as in Figs. 1 and 2, the equation 1 for the sloping step model (Fig. 3A) could also be employed to calculate the anomalies for the vertical dyke of infinite extension with sloping surface (Fig. 3C), which is a widely used model. When the sloping step model in Fig. 3A is rotated clockwise by 90° , it becomes a vertical dyke with sloping surface as shown in Fig. 3B, and the angle D between the sloping surface of the dyke and the horizontal will be equal to $d - 90^\circ$. By substituting $d - 90^\circ$ in place of d in equation 1,

anomalies computed for model in Fig. 3B.

Anomalies are computed for the infinite dipping dyke for dip angles of 30° , 60° and 120° after substituting the transformed co-ordinates in equation 1 and also directly from equation 2. The anomalies computed are found exactly the same in both the cases and are plotted in Fig. 4. Anomalies are also calculated for the infinite and the finite vertical dyke with sloping surface after substituting the transformed co-ordinates in equation 1 as well as the method by Talwani (1965). These are plotted in Fig. 5. While the anomalies are exactly same in case of finite dyke they differ by a constant factor of about 0.122 gammas in case of infinite dyke which could be due to the finite value of 999 assigned for the infinitely extending corners of the body in Talwani method. Anomalies for the finite dyke model are calculated from equation 1 following the conventional

procedure by subtracting the anomalies of one infinite STEP model from another infinite STEP model.

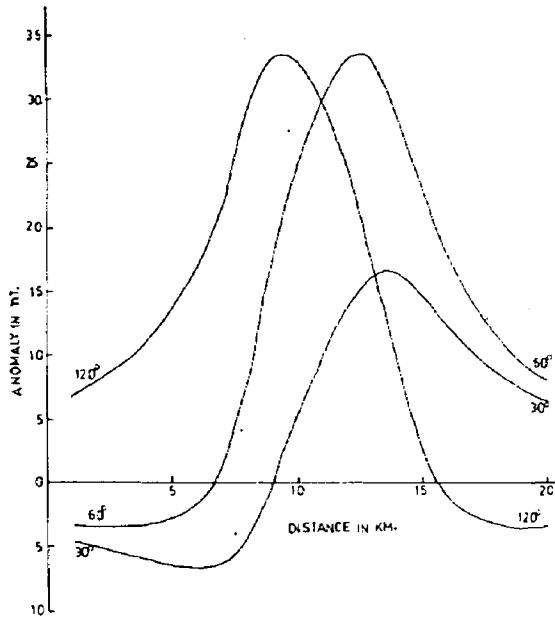


Fig. 4 Anomaly for Dipping dyke, 30°, 60°, 120° .

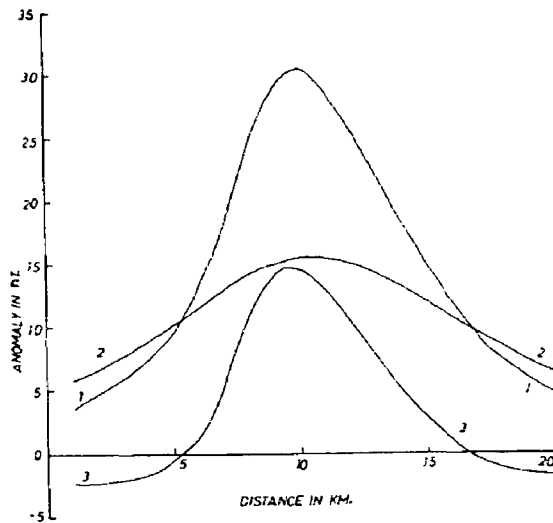


Fig. 5 Anomaly for infinite dykes (1,2) and finite dyke (3).

The listing of the computer programme "STEPD" developed for this method along with some test data is given in APPENDIX 1.

CONCLUSIONS

Though equations 1 and 2 for a step model and a dipping dyke model are separately given, only equation 1 is employed in all the computations because the dyke model is derivable from the step model by substituting $(d-90^\circ)$ for D . As such, the necessity of adopting different equations of different types of bodies is obviated. By using the basic step model with two corners only and with appropriate substitutions in equation 1, keeping the corners 1 and 2 in proper positions, it has become possible to compute anomalies of different types of step models and dyke models. It has also been possible to compute, by the present method, the anomalies for the infinitely extending vertical dyke with sloping surface. Combinations of bodies of appropriate shapes that could be approximated to two-dimensional geological structures can also be derived without necessity of assigning values for the infinitely extending sides of the bodies.

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APPENDIX 1

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C      PROGRAM STEPED.FOR
C      A      MAGNETIC INCLINATION
C      B      PROFILE DIRECTION CLOCKWISE FROM GEOGRAPHIC NORTH
C      D      DECLINATION
C      KTOT   TOTAL NUMBER OF POINTS
C      BX1,BX2 X-COORDINATES OF THE BODY
C      BZ1,BZ2 Y-COORDINATES OF THE BODY
C      SUS    SUSCEPTIBILITY
C      IF MODEL IS 1, THE BODY IS VERTICAL DYKE WITH SLOPING
C      SURFACE, THE ANGLE (DIP) IS MEASURED FROM THE VERTICAL
C      SIDE OF THE DIKE AT ITS TOP EDGE.
C      IF THE MODEL IS OTHER THAN 1, THE BODY IS STEP MODEL,
C      THE ANGLE (DIP) IS MEASURED DOWNWARDS FROM THE POSITIVE
C      X-AXIS.
C      F      TOTAL MAGNETIC FIELD INTENSITY
C      NOTE:  THE TOP EDGE OF THE BODY SHOULD ALWAYS BE
C      COUNTED AS THE FIRST CORNER OF THE BODY.
C      NN     NUMBER OF BODIES
C      KK     ORIGIN TO BE RECKONED FROM
C      KP     NUMBER FIELD POINTS AT WHICH ANOMALIES ARE
C            TO BE CALCULATED
C      DD     DIP OF THE BODY
C      DIMENSION PX (100), THET (100), R (100), T (100), TI(100)
C      OPEN (5, FILE='STEPEDIN', ACCESS='SEQUENTIAL')
C      OPEN (6, FILE='STEPDOUT',ACCESS='SEQUENTIAL')
C      DO 64 I = 1, 100
64      TI (I)=0.0
C      READ (5,*)NN
C      DO 30 II=1, NN
C      READ (5,*)BX1,BZ1,BX2,BZ2
C      READ (5,*)F,SUS,A,B,D,KK,KP,DD,MODEL
    
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WRITE(6, 15) BX1,BZ1,BX2,BZ2,MODEL
WRITE(6, 190) F,SUS,A,B,D,KK,KP,DD
190  FORMAT (2X,'F=',F10,2, 'SUS=',F10.6, 'INCLI=',F8.1,PROF.
DIR=' ,F5.0, 'DECLI=' ,F5.0, 'ORIGIN=' ,13, 'NO.POINTS=' ,
13, 'DD='F5.0)
PI=3. 1415927
DD=DD*PI/180.
KTOT=KK+KP
DO 10 I=KK, KTOT
PX (I) = I
THETA=(PX(I)-BX1)/BZ1
THETB=(PX(I)-BX2)/BZ2
THET (I)=ATAN(THETA)-ATAN (THETB)
THET (I)=-THET (I)
R1=SQRT ((PX(I)-BX1) **2+BZ1**2)
R2=SQRT ((PX(I)-BX2) **2+BZ2**2)
R(I)=ALOG(R1/R2)
R(I)=-R(I)
10  CONTINUE
BZ=BZ1-BZ2
BX=BX1-BX2
IF (BX) 71, 72, 73
71  IF (BZ) 83, 76, 81
73  IF (BZ) 81, 76, 83
72  DIP=PI/2.0
GO TO 60
81  DIP=-(PI-ATAN(BZ/BX))
IF (MODEL.EQ.1) GO TO 78
GO TO 60
83  DIP=ATAN (BZ/BX)
IF (MODEL.EQ.1) GO TO 78
GO TO 60
78  DIP=DIP-PI/2.0
GO TO 79
76  DIP=DD
79  DO 50 I=KK,KTOT
THET(I)=-THET(I)
R(I)=-R(I)
50  CONTINUE
60  AM=F*SUS
B=B-D
A=A*PI/180.
B=B*PI/180.
DELTA=1-COS (A) *COS(A) *SIN(B) *SIN(B)
AA=ATAN(SIN(A)/(COS(A) *COS(B)))
PHI=2.0*AA-DIP-PI/2.0
DO 20 I=KK, KTOT
T (I)=2.0*AM*DELTA*(SIN(DIP)) *THET(I) *COS(PHI) +R(I) *SIN(PHI)
TT (I)=T(I)+TT(I)
20  CONTINUE
30  WRITE (6,14) (T(I),I=1,KTOT)
WRITE (6,901)
901  FORMAT (5X,///)
WRITE (6, 14) (TT(I),I=1, KTOT)
14  FORMAT (10F12.3)
15  FORMAT (5X, ' BX1=' ,F8.3, 'BZ1=' ,F8.3, 'F8.3, 'BZ2=' ,F8.3,
12X, 'MODEL=' , 15,/)
STOP
END

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TEST DATA AND RESULTS

1. DIPPING DYKE

INPUT PARAMETERS :

$BX_1 = 8.0, BX_2 = 14.0, BZ_1 = BZ_2 = 2.0$

$SUS = 0.0025, F = 42600, A = 18^\circ, B = 90^\circ, DD = 30^\circ, 60^\circ, 120^\circ,$

$D = 0.0, KK = 1, KP = 20, Model = 1$

TABLE -1

Distance in km	D = 30°		D = 60°		D = 120°	
	From Eq. 1	From Eq. 2	From Eq. 1	From Eq. 2	From Eq. 1	From Eq. 2
1	-4.571	-4.571	-3.293	-3.293	7.126	7.126
2	-4.965	-4.965	-3.373	-3.373	8.150	8.150
3	-5.414	-5.414	-3.373	3.373	9.495	9.495
4	-5.906	-5.906	-3.199	-3.199	11.322	11.322
5	-6.391	-6.391	-2.635	-2.635	13.903	13.903
6	-6.676	-6.676	-1.180	-1.180	17.668	17.668
7	-6.182	-6.182	2.247	2.247	23.040	23.040
8	-3.789	-3.789	8.194	8.194	29.194	29.194
9	0.669	0.669	17.489	17.489	32.971	32.971
10	5.588	5.588	24.835	24.835	32.905	32.905
11	9.995	9.995	29.984	29.984	29.984	29.984
12	13.658	13.658	32.905	32.905	24.835	24.835
13	16.151	16.151	32.971	32.971	17.489	17.489
14	16.491	16.491	29.194	29.194	8.914	8.914
15	14.611	14.611	23.040	23.040	2.247	2.247
16	12.172	12.172	17.668	17.668	-1.180	-1.180
17	10.147	10.147	13.903	13.903	-2.635	-2.635
18	8.614	8.614	11.322	11.322	-3.199	-3.199
19	7.454	7.454	9.495	9.495	-3.373	-3.373
20	6.558	6.558	8.150	8.150	-3.373	-3.373

2. INFENITE AND FINTE DYKE WITH SLOPING SURFACE

INPUT PARAMETERS

a) For equation 1

$BX1 = BX2 = 8.0$, $BX2 = BX4 = 14.0$, $BZ1 = 2.0$,
 $BZ2 = 5.0$, $BZ3 = 6.0$, $BZ4 = 9.0$, other parameters
 same as in case of Dipping Dyke

TABLE - 2

Distance in Km	Infinite Dyke		Finite Dyke			
	Equation 1 (1)	Talwani Method	Equation 1 (1)	Talwani Method		
1	3.832	3.710	3.832	6.066	-2.234	-2.234
2	4.673	4.551	4.473	6.944	-2.271	-2.271
3	5.825	5.702	5.825	7.968	-2.143	-2.143
4	7.455	7.333	7.455	9.141	-1.678	-1.687
5	9.853	9.731	9.853	10.446	-0.593	-0.593
6	13.491	13.369	13.491	11.827	1.664	1.664
7	18.884	18.762	18.884	13.181	5.703	5.703
8	25.338	25.216	25.338	14.362	10.976	10.976
9	29.691	29.569	29.691	15.220	14.471	14.471
10	30.406	30.284	30.406	15.644	14.762	14.762
11	28.696	28.574	28.696	15.599	13.097	13.097
12	25.718	25.596	25.718	15.125	10.593	10.593
13	22.136	22.014	22.136	14.309	7.827	7.827
14	18.412	18.290	18.412	13.259	5.153	5.153
15	14.922	14.800	14.922	12.082	2.840	2.840
16	11.915	11.793	11.915	10.869	1.046	1.046
17	9.480	9.358	9.480	9.689	-0.209	-0.209
18	7.582	7.460	7.582	8.586	-1.004	-1.004
19	6.126	6.004	6.126	7.585	-1.459	-1.459
20	5.012	4.890	5.012	6.695	-1.683	-1.683

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