

HEAT TRANSFER IN MHD UNSTEADY STAGNATION POINT FLOW WITH VARIABLE WALL TEMPERATURE

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An analysis of MHD heat transfer in hyperbolic time-variation flow near a stagnation point of a heated blunt-nosed cylinder whose wall temperature varies as Ax^N is presented. Temperature profiles are shown graphically for different values of N and the numerical values of the rate of heat transfer ($-\theta'(0)$), Δ_1^* the thermal boundary layer thickness and Δ_2^* the convection thickness are entered in a table. It is observed that $(-\theta'(0))$ increases with increasing N but Δ_1^* , Δ_2^* decreases with increasing N .

1. INTRODUCTION

Lighthill¹ studied the effects of free-stream oscillations on the flow of an incompressible viscous fluid past a semi-infinite body. The small amplitude phenomenon was studied. The unsteady motion of a semi-infinite plate in a viscous fluid was also studied by Cheng and Elliott², Cheng³. The hyperbolic time-dependent flow near a stagnation point of a body was studied by Yang⁴ in isothermal case. Soundalgekar *et al.*⁵ studied the heat transfer aspect of the hyperbolic time-dependent flow near a stagnation point by considering the variable wall temperature. In Refs. (4, 5), similarity solutions were presented by considering the hyperbolic time-variational free-stream in the form $u_\infty = x/(L - \alpha u_0 t)$ where t is the time, α the constant and u_0 the reference velocity (see Yang⁴ and Soundalgekar *et al.*⁵). Also L is a characteristic length of the cylinder.

The effects of transversely applied magnetic field on the hyperbolic time-dependent stagnation flow were recently considered by Soundalgekar and Murty⁶. In this paper, the temperature of the wall was considered as constant. The effects of transversely applied magnetic field on the heat transfer when the wall temperature is varying as Ax^N were not studied in the literature. It is now proposed to study this aspect of the problem. In section 2, the mathematical analysis is considered and in section 3 the conclusions are set out.

2. MATHEMATICAL ANALYSIS

MHD flow of an incompressible viscous fluid near a forward stagnation point of a blunt-nosed two dimensional body is considered. Neglecting induced magnetic field, the unsteady flow is governed by the following equations :

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_{\infty}}{\partial t} + u_{\infty} \frac{\partial u_{\infty}}{\partial x} + v \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (u_{\infty} - u) \quad \dots(1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots(2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} \quad \dots(3)$$

The initial and boundary conditions are :

$$\left. \begin{aligned} u = 0, V = 0, T = T_w(x) & \quad \text{at } y = 0 \\ u = u_{\infty}(x, t), T = T_{\infty} & \quad \text{at } y \rightarrow \infty \end{aligned} \right\} \text{for } t > 0$$

$$u = U_s(x, y), V = V_s(x, y), T = T_s(x, y) \text{ for } t \leq 0. \quad \dots(4)$$

Here B_0 is the applied magnetic field and all the other qualities have their usual meaning except that σ is the scalar electrical conductivity of the fluid.

Assuming $T_w - T_a = AX^N$, we can reduce these equations to the following form

$$f'' + ff'' - f'^2 + 1 = \alpha \left(\frac{1}{2} \eta f'' + f' - 1 \right) - M(1 - \alpha t^*) (1 - f') \quad \dots(5)$$

$$\theta'' + Pr \left(f - \frac{\alpha}{2} \eta \right) \theta' - NP_r f' \theta = 0 \quad \dots(6)$$

where the non-dimensional quantities are defined as follows :

$$\eta = y \left[\frac{u_0}{\nu(L - \alpha u_0 t)} \right]^{1/2}, \quad \psi = (\nu x u_{\infty})^{1/2} f$$

$$t^* = \frac{u_0 t}{L}, \quad Pr = \mu c_p / k, \quad m^2 = \frac{\sigma B_0^2 L^2}{\mu}$$

$$R = \frac{u_0 L}{\nu}, \quad M = m^2 / R.$$

Here η is the modified Blasius variable, ψ the stream-function, t^* non-dimensional time, m the Hartmann number, R the Reynolds number and M the ratio of the Hartmann number and the Reynolds number.

The corresponding boundary conditions are given by

$$\left. \begin{aligned} f(0) = f'(0) = 0, \quad f'(\infty) = 1 \\ \theta(0) = 1, \quad \theta(\infty) = 0. \end{aligned} \right\} \dots(7)$$

The case of $\alpha = 0$ corresponds MHD stagnation point flow. When $\alpha > 0$, (but $\alpha t^* \ll 1$) it is an accelerating flow and $\alpha < 0$ then corresponds to a decelerating flow.

Due to the explicit appearance of t^* in (5), it is clear that similarity solution is not possible. It is possible when the applied magnetic field satisfies $M(1 - \alpha t^*) = \text{constant}$ i.e. when B_0 (the applied magnetic field) is not uniform but varies with time as $B_0(t) \rightarrow (1 - \alpha t^*)^{-1/2}$.

Also the solution is non similar and valid for small time.

Then for $t^* \ll 1$, we can expand f and θ in powers of t^* as

$$f = f_0 + t^* f_1 \text{ and } \theta = \theta_0 + t^* \theta_1 \dots(8)$$

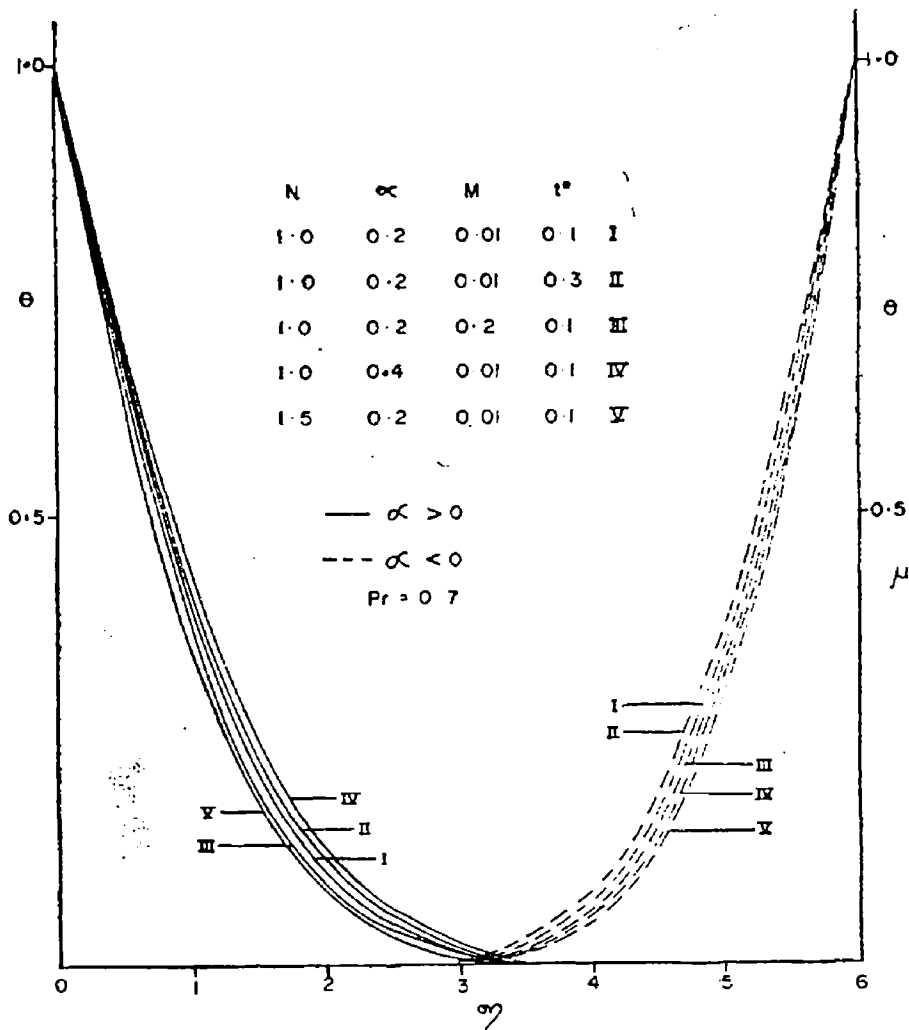


FIG. 1. Temperature profiles.

and substitute in eqns. (5) and (6), equate the coefficients of different powers of r^* , neglecting the coefficients of r^{*2} , and we have,

$$f_0'' + f_0 f_0'' - f_0'^2 + 1 = \alpha \left(\frac{\eta}{2} f_0'' + f_0' - 1 \right) - M(1 - f_0') \quad \dots(9)$$

$$f_1'' + f_1 f_0'' - f_0 f_1'' - 2f_0' f_1' = \alpha \left(\frac{\eta}{2} f_1'' + f_1' \right) + M(f_1' - \alpha f_0' + \alpha) \quad \dots(10)$$

$$\theta_0'' + Pr \left[\left(f_0 - \frac{\alpha}{2} \eta \right) \theta_0' - N f_0' \theta_0 \right] = 0 \quad \dots(11)$$

$$\theta_1'' + Pr \left[\left(f_0 - \frac{\alpha}{2} \eta \right) \theta_1' + f_1 \theta_0' - N (f_0' \theta_1 + f_1' \theta_0) \right] \quad \dots(12)$$

and the boundary conditions are now given by

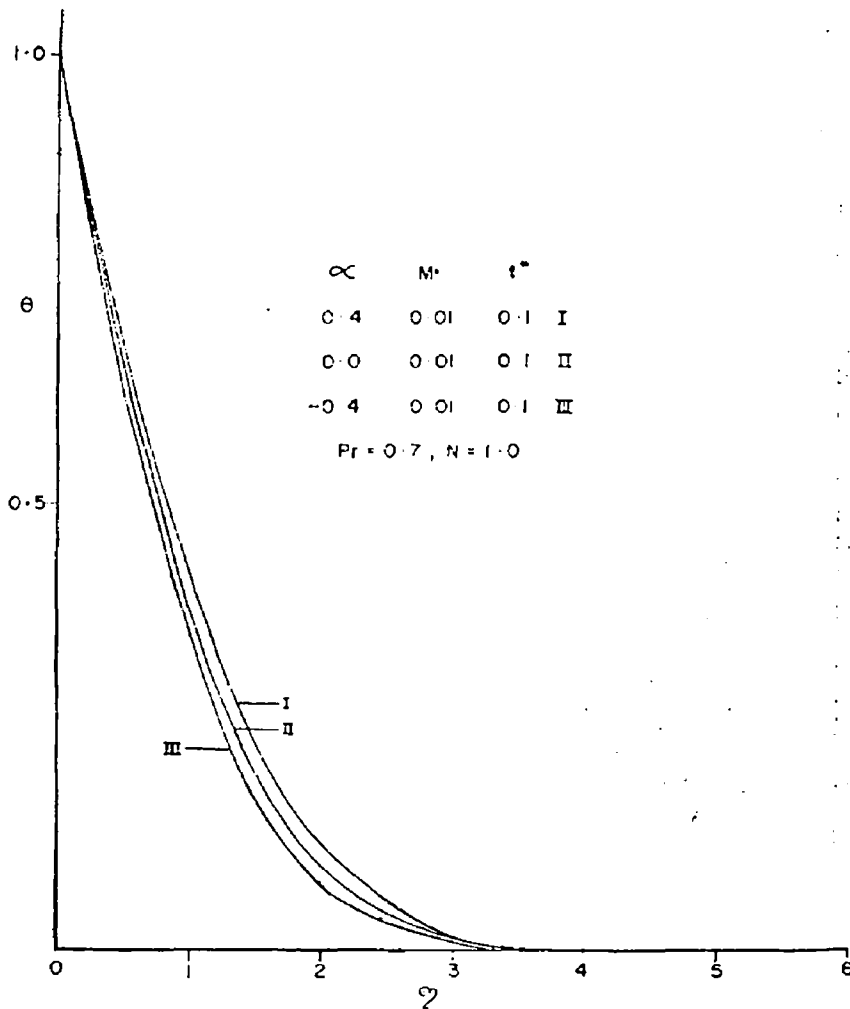


FIG. 2. Temperature profiles.

$$\left. \begin{aligned}
 f_0(0) = f'_0(0) = f_1(0) = f'_1(0) = 0 \\
 f'_0(\infty) = 1, f'_1(\infty) = 0 \\
 \theta_0(0) = 1, \theta_1(0) = 0; \theta_0(\infty) = \theta_1(\infty) = 0.
 \end{aligned} \right\} \dots(13)$$

Equations (9)-(13) are solved numerically on a computer for $Pr = 0.71$ and the temperature profiles for different values of N and α are shown in Figs. 1 and 2.

We observe from these figures that an increase in N or M leads to a decrease in the temperature for both $\alpha \geq 0$. Also for constant value of N , there is a rise in the temperature when the flow is accelerating and a fall in the temperature in case of a decelerating flow.

TABLE I
 Values of $\{-\theta'(0)\}$, Δ_1^* , Δ_2^* , $G = \Delta_1^* / \Delta_2^*$

α	M	t^*	N	$\{-\theta'(0)\}$	Δ_1^*	Δ_2^*	G
0	0.01	0.1	0	0.496157	1.1880	0.708412	1.67355
			1.0	0.709417	0.935437	0.506669	1.84625
			1.5	0.786384	0.862127	0.449333	1.91868
0.2	0.01	0.1	0	0.466002	1.231670	0.752527	1.63671
			1.0	0.690984	0.962454	0.534270	1.80144
			1.5	0.771358	0.883620	0.472066	1.87181
0.2	0.01	0.3	0	0.465993	1.231690	0.752510	1.63677
			1.0	0.690968	0.962470	0.534250	1.80151
			1.5	0.771338	0.883630	0.472050	1.87189
0.2	0.20	0.1	0	0.470218	0.223390	0.759010	1.61181
			1.0	0.698387	0.955150	0.539460	1.77058
			1.5	0.779929	0.876620	0.476840	1.83837
0.4	0.01	0.1	0	0.431157	1.306830	0.823960	1.58603
			1.0	0.671871	0.997090	0.569110	1.75202
			1.5	0.756002	0.909650	0.499090	1.82262
-0.2	0.01	0.1	0	0.532148	1.106280	0.633040	1.74757
			1.0	0.728573	0.900970	0.471820	1.90955
			1.5	0.801741	0.836660	0.422650	1.97953
-0.2	0.01	0.3	0	0.532158	1.106270	0.633050	1.74750
			1.0	0.728691	0.900950	0.471830	1.90946
			1.5	0.801762	0.836640	0.422660	1.97943
-0.2	0.20	0.1	0	0.536745	1.098590	0.641070	1.71367
			1.0	0.736912	0.893670	0.478240	1.86868
			1.5	0.811387	0.829510	0.428540	1.93563
-0.4	0.01	0.1	0	0.563427	1.054080	0.583010	1.96875
			1.0	0.745150	0.873750	0.443810	1.96875
			1.5	0.816693	0.815520	0.400030	2.03866

We now study the effects of variable temperature on the thermal boundary layer thickness Δ_1^* , and the convection thickness Δ_2^* which are given by

$$\Delta_1^* = \int_0^{\infty} \theta \, d\eta, \quad \Delta_2^* = \int_0^{\infty} f' \theta \, d\eta. \quad \dots(14)$$

The rate of heat transfer expressed in terms of the Nusselt number is given by

$$Nu \, R^{-1/2} = \left(\frac{hL}{k} \right) \left(\frac{U_0 L}{\nu} \right)^{1/2} = \frac{\{-\theta'(0)\}}{(1 - \alpha t^*)^{1/2}}. \quad \dots(15)$$

We have calculated numerically the values of $\{-\theta'(0)\}$, Δ_1^* , Δ_2^* for different values of N , α , M and t^* and these are entered in Table I.

We observe from this table that an increase in N leads to an increase in the value of $(-\theta'(0))$ for both $\alpha \geq 0$ and for all values of M . But Δ_1^* , Δ_2^* decrease with increasing N and for all $\alpha \geq 0$.

3. CONCLUSIONS

Due to an increase in plate temperature, there is an increase in the rate of heat transfer at the plate and a decrease in the thermal boundary layer thickness Δ_1^* and the convection thickness Δ_2^* .

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