

AUTOREGRESSIVE SPECTRAL ANALYSIS OF CRANE VESSEL MOTIONS IN SEA WAVES

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SYNOPSIS

The fast Fourier transform technique is often used for the spectral analysis of measured time series of marine vessel motions. Conventional collection of long term vessel responses in random seas requires a large capacity to store the data in either the time or frequency domain formats. The system identification technique, namely autoregressive modelling, can significantly reduce such data storage problems with minimal loss of relevant information. This is demonstrated with measured time series crane vessel motion excited by sea waves to which autoregressive modelling is successfully applied. A comparative study between the fast Fourier transform and autoregressive modelling establishes system identification techniques as a useful tool when applied to time series vessel motions. This is shown by the consistency of the autoregressive derived transfer

function namely response amplitude operator, and those obtained by fast Fourier transform analysis.

INTRODUCTION

The design of offshore structures and marine vessels are based upon theoretical models, and experimental data at model and full scale. In reality, marine vessels experience many unknown phenomena which are yet fully understood in scientific or engineering terms. As an example, WORDEN *et al* (1992) have clearly demonstrated that the wave force calculation by Morison's equation is insufficient. Worden *et al* introduced unknown terms which are estimated using system identification (SI), namely Nonlinear AutoRegressive Moving Average exogenous (NARMAX) techniques. The subsequently modified version of Morison's equation seems to predict real time wave forces more accurately, although without improving our intuitive understanding of the fundamental uncertainties involved.

On many occasions marine vessels are damaged or even capsized in certain marine environmental conditions. In these cases, the design of the vessel's structures may not properly include for such external dynamic forces. Measured full scale vessel motions in sea waves can be used to improve the design. While analyzing such full scale data SI modelling may give some useful information as demonstrated by Worden *et al*.

In this paper autoregressive (AR) algorithm is formulated based on the least squares error approach and applied to the measured crane vessel motions and sea waves in the North Sea. The transfer function (often called response amplitude operator) of the vessel motion in sea waves is defined using AR algorithm. Comparative studies of transfer function estimated from the FFT and AR methods are shown to be consistent. The transfer function estimation shows the influence of waves on vessel motions.

AUTOREGRESSIVE MODEL

The time series $\{y_r\}$ is said to be an autoregressive process, AR(p) of the order p and it is generated from the relationship

$$y_r = - \sum_{k=1}^p a_k y_{r-k} + \sqrt{\rho_r} w_r \quad (1a)$$

Here y_r is the rth sample of the discrete stochastic process and w_r is the Gaussian white noise with variance ρ_r . Equation (1a) can be written in polynomial form as

$$y_r = \left[\frac{\sqrt{\rho_r}}{a(z^{-1})} \right] w_r \quad (1b)$$

where,

$$a(z^{-1}) = 1 + \sum_{k=1}^p a_k z^{-k}$$

The AR(p) model is also called as an *all-pole* model (BURG, 1975). Therefore, the transfer function of the AR(p) process can be expressed as

$$H_{11}(z) = \frac{\sqrt{\rho_r}}{1 + \sum_{k=1}^p a_k z^{-k}} \quad (2)$$

Therefore, the estimated power spectrum, $S_{11}(\omega)$ of the time series y_r can be written as

$$\begin{aligned} S_{11}(\omega) &= |H_{11}(e^{j\omega T})|^2 \\ &= \frac{\rho_r}{|1 + \sum_{k=1}^p a_k e^{-j\omega T k}|^2} \end{aligned} \quad (3)$$

The sampling period, T is generally defined by the cut-off frequency, ω_c , through the Nyquist relationship

$$T = \frac{\pi}{\omega_c} \quad (4)$$

The autocorrelation function, R_k of the finite set of the time series data, i.e., $(0 < Y_k < M)$, can be calculated from the relationships

$$R_k = f(M, \lambda) \sum_{i=1}^i Y_i Y_{i-k} \quad (5)$$

where, $f(M, \lambda) = 1/(M-\lambda)$ for the unbiased sample correlations, and $f(M, \lambda) = 1/M$ for the biased sample correlations.

Let ϵ be the error between the target spectrum, $S(\omega)$, and the estimate power spectrum of the AR output, $S_{yy}(\omega)$, is expressed as

$$\epsilon = \frac{b_1^2}{2\pi} \int_{-\pi}^{\pi} \frac{S(\omega)}{S_{yy}(\omega)} d\omega \quad (6)$$

Substituting $S_{yy}(\omega)$ [equation (4)] in above equation gives

$$\epsilon = \frac{1}{2\pi} \int_{-\pi}^{\pi} S(\omega) \left| 1 + \sum_{k=1}^p a_k e^{-jk\omega} \right|^2 d\omega \quad (7)$$

By minimizing ϵ , the parameters a_k are determined. Therefore, minimisation of error, ϵ , brings the following important relationship

$$\frac{d\epsilon}{da_k} = 0$$

$$\sum_{k=1}^p a_k R_{|k-1|} = -R_k \quad (8)$$

Equation (8) can be written in Toeplitz matrix form as

$$\begin{bmatrix} R_0 & R_1 & R_2 & \dots & R_{p-1} \\ R_1 & R_0 & R_1 & \dots & R_{p-2} \\ R_2 & R_1 & R_0 & \dots & R_{p-3} \\ \dots & \dots & \dots & \dots & \dots \\ R_{p-1} & R_{p-2} & R_{p-3} & \dots & R_0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \dots \\ a_p \end{bmatrix} = - \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ \dots \\ R_p \end{bmatrix} \quad (9)$$

These are also known as Yule-Walker equations. Using equations (9), when the parameters $a_1, a_2, a_3, \dots, a_p$ and noise variance ρ , are determined, the time series y_t can be generated digitally by using recursive equation (1). Here one has to obtain optimal

spectral estimates for the values of a_1 and p such that AIC or MDL between the target and estimated spectra must be minimum.

Based on BRIGGS and VANDIVER (1982) the transfer function of vessel motion in random sea is expressed as

$$S_{v,1}(\omega) = |H_{v,1}(\omega)|^2 S_s(\omega) \quad (10)$$

where $S_{v,1}(\omega)$ is the power spectrum of vessel motion, $S_s(\omega)$ is the power spectrum of sea waves, and $H_{v,1}(\omega)$ is the transfer function. Using equations (1) to (9) the AR power spectra of vessel motions and sea waves are estimated and then equation (10) yields the transfer function of a particular type of vessel motion in sea waves.

APPLICATIONS

An offshore monitoring unit developed by the Dept. of Mechanical Engineering, University College London, was installed on a mono-hull crane vessel in the North Sea in April 1992 and recorded the vessel's motions at different headings and sea conditions. Here one set of such time series of 1024 data points in head seas is analyzed. The measured data consists of pitch, roll, strap down vertical (heave) acceleration, transverse acceleration, longitudinal acceleration and wave elevation sampled at 5 Hz. The present study considers pitch, roll, heave acceleration and sea waves (Figures 1a-d). FFT power spectra calculated (using block of 128 data points and 8 averaged) show that these are narrow banded spectra. The autocorrelation lags are very high up to which the process is convergent. Since these time series are narrow banded, a low order AR model can be used to represent the time series. The appropriate AR model orders are selected based on the best estimator as described by MARPLE (1987) and MANDAL (1992).

First the sea wave data is modelled using autocorrelation and least squares error (LSE) techniques. It shows that even though the actual model order seems to be very high, the best fit between the measured wave and estimated AR spectra shows that a model order of 24 seems to adequately represent the wave process. While

examining the LSE (Figure 2a) there is not much variation between the AR(20) and AR(24) models (Figures 2b-c) which are considered for estimation of vessel's transfer functions. The AR spectra are compared with the FFT spectra. Based on autocorrelation and LSE methods pitch [AR(16) model], roll motion [AR(29) model], and heave acceleration [AR(13) model] spectra are estimated and compared with their equivalent FFT spectra as shown in Figures 3, 4, and 5 respectively. The above comparisons of spectral estimates show that the estimated AR models are in good agreement.

The transfer functions, $H_{ii}(\omega)$ estimated by AR modelling for each type of vessel motions in random sea waves are compared with the transfer functions determined by the FFT technique which shows good consistency with the AR derived transfer functions. Figures 6a,b show the comparison between the pitch transfer functions estimated by the AR modelling and FFT technique. Similarly Figures 7a,b and Figures 8a,b show the comparison of the roll and heave transfer functions respectively. All are consistent. This study of transfer function further confirms that the estimated lower order AR models can be used to represent vessel motions with significant reduction in data storage requirements.

DISCUSSION

Even though ocean dynamic systems are random in nature, the confused ocean surface elevations can be characterised by their power spectral estimation using system identification techniques (SPANOS, 1983 and MANDAL *et al*, 1992). Similarly offshore vessel motions can be expressed by their responses. Sometimes for vessel motions the correlation lag up to the stage in which the process is convergent is of very high order owing to its sharp peaked nature. Hence the Toeplitz matrix [equation (9)] can become illconditioned. For such a high correlation lagged dynamic system the AR model needs to be properly selected. The lower order AR model can generally be used to describe the sharp peaked spectrum of dynamic system. The monohull crane vessel motions show

examples of such cases. Using autocorrelation, Akaike information criterion, or minimum description length methods the selected model order of the crane vessel motion yields a very high value for which the Toeplitz matrix in equation (9) becomes illconditioned. However, the lower order AR model can fairly represent the above dynamic systems. For the time series data of the vessel motion, one should make first examinations at autocorrelation lags up to which the process is convergent. If it is too high, one can start from a lower order and then estimate the proper AR model which yields a better representation of the vessel motions.

The transfer function [equation (10)] estimation by the AR model shows consistent results. This study also confirms the validation of AR modelling of crane vessel motions in random seas. The comparative study between the FFT and AR methods shows that relatively few values which are the AR parameters can be used to represent the time series data. Therefore, one can use SI techniques for frequency domain analysis of vessel motions as an alternate method to FFT technique.

CONCLUSIONS

AR algorithms has been formulated based on the least squares error method. It is noted that the correlation lag up to which the process is convergent is very high in some cases of the crane vessel motions. In these types of processes one should be careful about proper selection of the model order. It is observed that the lower order AR models can generally be used to express such types of time series. These lower order AR transfer functions show consistency with those estimated by the FFT method. The approach presented herein provides a suitable alternate method of frequency domain analysis to the FFT method, with the benefit that relatively few values are required to establish the AR model for representing time series of vessel motions and consequently transfer functions.

ACKNOWLEDGEMENTS

The authors would like to gratefully acknowledge the support of the sponsors of the Integration of Design Techniques Managed Research Programme at University College London and Dr. B. U. Nayak, Deputy Director, National Institute of Oceanography, Goa.

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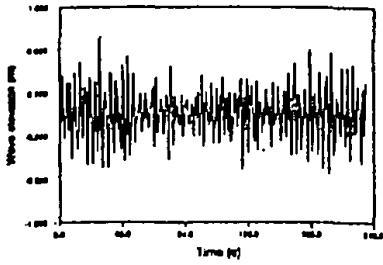


Figure 1a Time series waves measured (near crane vessel) at North Sea.

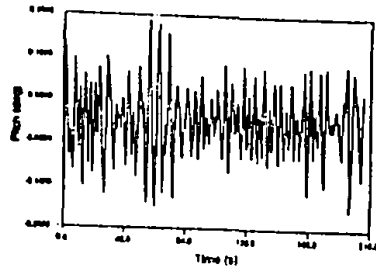


Figure 1b Time series pitch motion of crane vessel at North Sea

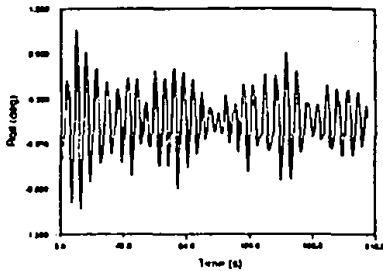


Figure 1c Time series roll motion of crane vessel at North Sea

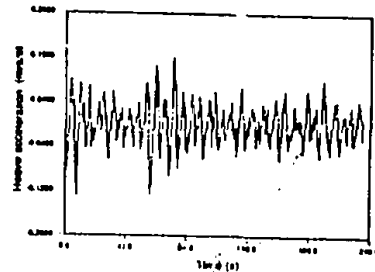


Figure 1d Time series heave acceleration of crane vessel at North Sea

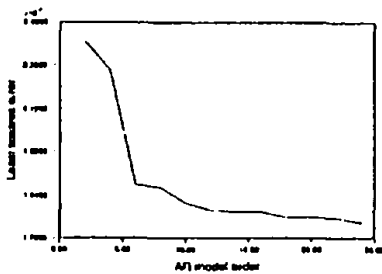


Figure 2a Least squares error variation for North Sea waves

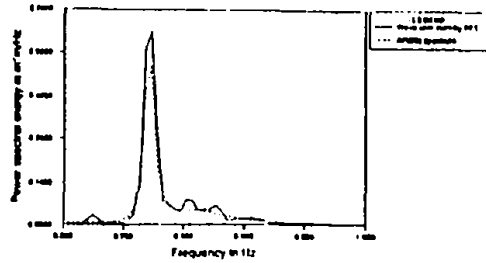


Figure 2b North Sea waves (near crane vessel) spectral estimates by FFT and A1(20) model

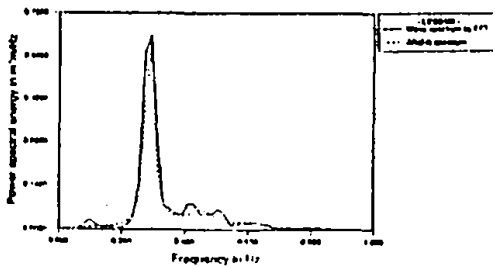


Figure 2c North Sea waves (near crane vessel) spectral estimates by FFT and A1(24) model

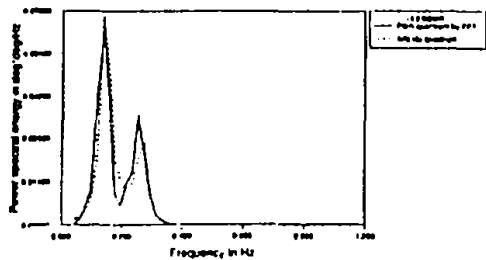


Figure 3 Pitch spectral estimates by FFT and A1(16) model

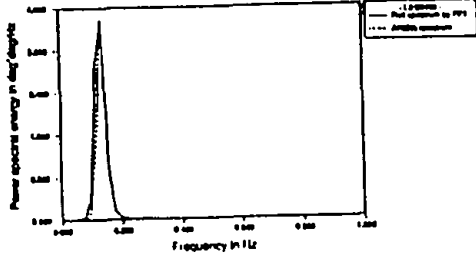


Figure 4 Roll spectral estimates by FFT and AR(28) model

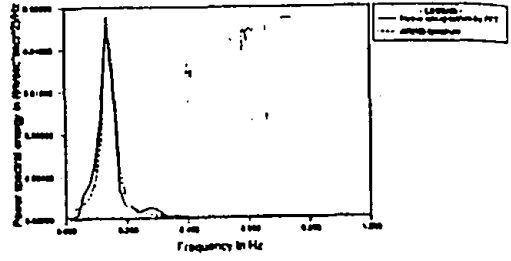


Figure 5 Heave acceleration spectral estimates by FFT and AR(13) model

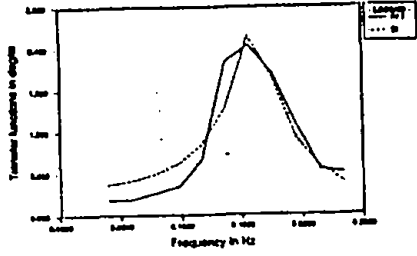


Figure 6a Pitch transfer functions estimated by FFT and SI models with 38 parameters (from Figures 3 and 2b)

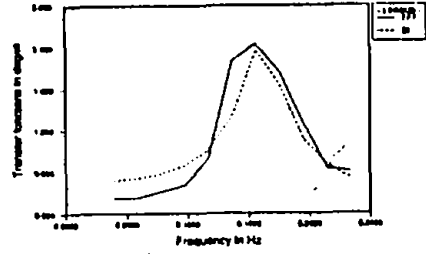


Figure 6b Pitch transfer functions estimated by FFT and SI models with 40 parameters (from Figures 3 and 2c)

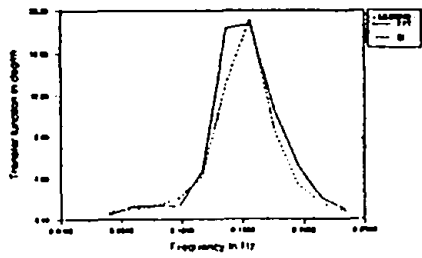


Figure 7a Roll transfer functions estimated by FFT and SI models with 49 parameters (from Figures 4 and 2b)

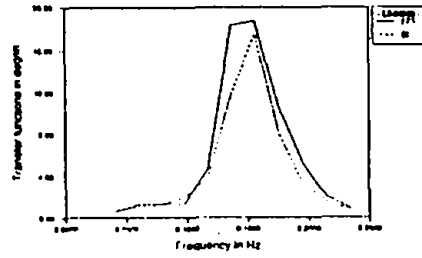


Figure 7b Roll transfer functions estimated by FFT and SI models with 63 parameters (from Figures 4 and 2c)

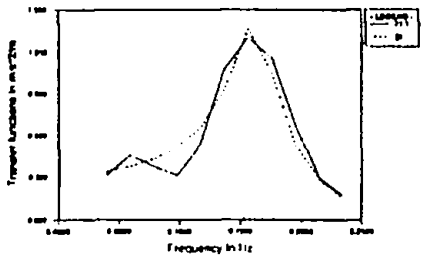


Figure 8a Heave acceleration transfer functions estimated by FFT and SI models with 33 parameters (from Figures 6 and 2b)

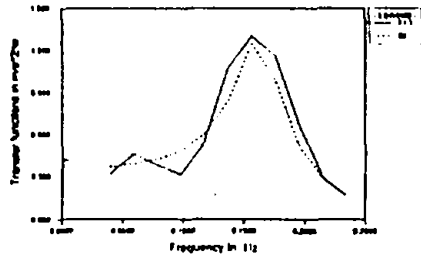


Figure 8b Heave acceleration transfer functions estimated by FFT and SI models with 37 parameters (from Figures 5 and 2c)