# Application of Multiplicative Array Techniques for Multibeam Sounder Systems

Vielstrahler-Systeme mit Anwendung einer Technik von multiplikativen Strahlerzeilen

Application des techniques de réseaux multiplicatifs à des systèmes de sondeurs multifaisceaux

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#### Introduction

Significant improvements in swath sounding system during the last few years have seen multibeam echosounders replacing single beam, and finding wide application in scientific and strategic purposes. The major advantages of the multibeam sounder over other systems serving in the same capacity are its operational economy along with its accuracy and applicability even in rough sea conditions [1]. The existing multibeam sounder is efficiently functioning all over the world, however, there exist certain limitations related to present method of beamforming, i.e. delay sum beamforming or, in other words, "additive beamforming". The operating frequencies for a deep water multibeam are rather low which in turn makes the transducer array bulky [2]. There are other beamforming methods that include complicated signal processing techniques, and provide superior resolution, commonly used for radar and sonar applications. The primary advantage of such techniques lies in the fact that the users obtain better resolution with a small sensor size [3]. If such techniques are used in multibeam sounder applications, high resolution beams may be obtained using small transducer arrays. The complicated signal processing aspects may be carried out in real time by modern computers.

Multiplicative array processing is well known for its narrow beamwidth and low sidelobe level [4, 5], but the array gain is quite low. The effectiveness of such a system becomes lower, especially when the signal to noise ratio is low. Brown and Rowland [6] proposed some modification in terms of additional computation or hardware for improved array gain. The present work is devoted towards the study of a better beamforming method i.e. a multiplicative array technique with some modification proposed by Brown and Rowland. Their method is used for a multibeam sounder system. The modified multiplicative array system consists of 20-elements. The performance of the proposed system is compared with an additive, i.e. delay-sum array of 40-elements of a similar configuration. The performance comparisons are made in terms of vertical beam patterns at different main beam steering angles of 5°, 10°, 20°, 30°, and normal incidence directions. The operating frequency is assumed to be 15 kHz. In addition, the vertical beam pattern of the multiplicative array is also presented for the normal incidence direction.

# Analysis

Fig. 1 shows a linear array in the x-y plane. The array of the multibeam sounder consists of different elements which lie along

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the x-axis. The physical array is placed at the origin of the coordinate system, i.e. symmetrically about the z-axis. The linear additive array which is studied comprises 40-elements. The elements are distributed in two blocks. Each block of the array is considered to be  $0.4 \, \lambda$ , where  $\lambda$  is the design wavelength.

The farfield pattern for a linear array is given by

$$|F(\theta, \phi)| = |f(\theta, \phi)| \cdot |\text{Element pattern}|.$$
 (1)

The array factor  $f(\theta, \phi)$  is presented by Collin and Zucker [7] as:

$$f(\theta,\phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} \exp(j k r_{mn} \cdot \mathbf{R}), \qquad (2)$$

where  $A_{mn}$  is the complex excitation coefficient (which is assumed to be unity) and  $k = 2\pi/\lambda \cdot m$  is equal to the number of blocks used in the array, which is equal to two. n is equal to the number of elements in each block, i.e., 20. In eq. (2), for the above array

$$\mathbf{r}_{mn} = d_{mn}\mathbf{i},\tag{3}$$

where  $d_{mn}$  is the inter-element separation, and i is the unit vector along the x-axis. The polar unit vector R can be expressed as

$$\mathbf{R} = (\sin\theta\cos\phi)\mathbf{i}. \tag{4}$$

The array factor for a linear array can be written as

$$f(\theta, \phi) = \sum_{m=1}^{2} \sum_{n=1}^{20} \exp j k (d_{mn} \sin \theta \cos \phi).$$
 (5)

According to Steinberg [8] the amount of phase delay required between the radiators to steer the beam along the specified direction  $(\theta_0, \phi_0)$  may be expressed as

$$\boldsymbol{\alpha}_{mn} = d_{mn} k (\sin \phi_0 \cos \phi_0) \boldsymbol{i}. \tag{6}$$

The above term is to be applied to the farfield pattern of a uniformly excited linear array of the additive type. The farfield pattern of the steered array may be rewritten as

$$f(\theta,\phi) = \sum_{m=1}^{2} \sum_{n=1}^{20} \exp j k (d_{mn} \sin \theta \cos \phi - d_{mn} \sin \theta_0 \cos \phi_0).$$
 (7)

In eq. (1),

Element pattern = 
$$\frac{\sin(\pi l \sin \theta \sin \phi)}{\pi l \sin \theta \sin \phi},$$
 (8)

where l is the length of the element and is equal to 3.0  $\lambda$ . The final expression for the vertical farfield pattern can be obtained by substituting eq. (7) and eq. (8) into eq. (1).

The farfield pattern for a multibeam array of 40-elements of the additive type is shown in Fig. 2. The half power beamwidths slowly increase: 3.0°, 3.0°, 3.1°, 3.3° and 3.6°, at angles of 0°, 5°,

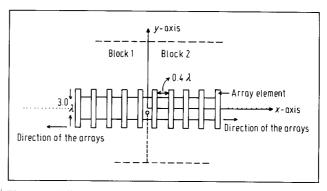


Fig. 1. Multibeam array geometry.

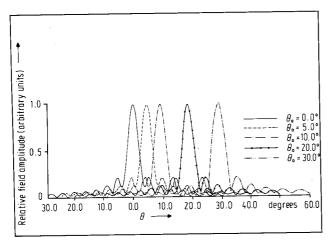


Fig. 2. Vertical farfield pattern for the  $\phi=0^\circ$ -plane for different main beam angles of the additive array.

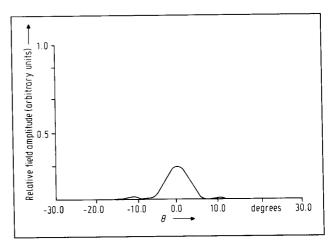


Fig. 3. Vertical farfield pattern for the  $\phi = 0^{\circ}$ -plane for a normal incidence angle for the multiplicative array.

10°, 20°, and 30°. The beamwidths are generally narrow and are useful for high resolution bathymetric applications, but the sidelobe levels are found to be higher, i.e up to  $-13 \, \mathrm{dB}$  for all the angles of incidence.

In order to have suppressed sidelobes with narrow main beamwidths, multiplicative array patterns are computed in the present study. The multiplicative array factor can be rewritten as [9]

$$f_{mn}(\theta,\phi) = \prod_{m=1}^{2} \sum_{n=1}^{20} \exp j k (d_{mn} \sin \theta \cos \phi - d_{mn} \sin \phi_0 \sin \phi_0).$$
 (9)

The above expression along with eq. (8) is used in eq. (1), to calculate the multiplicative array pattern for the  $\theta=0^\circ$  mainbeam directions (Fig. 3). The added output of 20-elements of one block is multiplied with the added output of the other. The normalization factor for each additive block of 20-elements is equivalent to 40. The value of 40 is the normalization factor for the additive array of 40-elements, and for the sake of comparison, the same factor is chosen in each additive block of the multiplicative array before multiplication is carried out. Farfield patterns of such an array reveal that the array gain is reduced to - 12 dB. Though the sidelobes are almost absent, the reduced array gain does not permit such an array to be used for multibeam sounder purposes.

According to Brown and Rowland [6] if the signal to noise ratio of the additive part of the array is higher, then by employing a modification in the signal processing method of the multiplicative array, the array gain can be improved. Aiming for such solutions applicable to a multibeam sounder system, an array size of half of the additive type is considered. Beam patterns are compared with the array size of the linear additive type. The inter-element spacing of the modified array is the same, and the total number of array elements is reduced to twenty, i.e. each block of the array consists of 10-elements.

The expression for the modified multiplicative array pattern may be presented as

$$f'(\theta,\phi) = \sum_{n=1}^{16} f_{mn}(\theta,\phi) \cdot \cos t \,\bar{u},\tag{10}$$

where  $\bar{u}$  is given as

$$\bar{\boldsymbol{u}} = \bar{\boldsymbol{\alpha}} - k \, d \sin \theta \cos \phi \, \boldsymbol{i} \,, \tag{11}$$

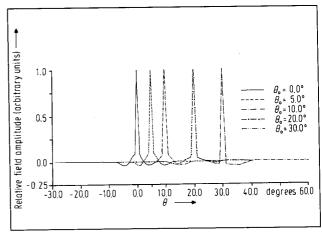


Fig. 4. Vertical farfield pattern for the  $\phi = 0^{\circ}$ -plane for different main beam angles of the modified multiplicative array.

where d is the distance between the two blocks, i.e. the distance from the centre of one block to another. In the present case it is equal to 4.0  $\lambda$ . The factor  $f_{mn}(\theta, \phi)$  has already been defined. Here "m" and "n" vary from 1 to 2 and from 1 to 10 respectively. a is rewritten as

$$\bar{\mathbf{a}} = -dk(\sin\theta_0\cos\phi_0)\mathbf{i}. \tag{12}$$

The notation has already been mentioned, and t is an integer varying from 1 to 16. According to Urick [4], the term t shows the number of power amplifiers required to improve the array gain of the modified array. In order to obtain the farfield pattern of the modified multiplicative array, eq. (10) to eq. (12) and eq. (8) are used in eq. (1). Fig. 4 shows the farfield pattern of the modified arrays at different steering angles. The array gain is substantially higher and similar to the array gain of the additive array. The half-power beamwidths observed at different steering angles are quite narrow, and approximately 1° for all the steering angles. Hence the array is useful for high resolution applications. Its negative sidelobe can be eliminated by using a rectifier [10].

#### Conclusions

The expressions for the farfield patterns of additive, multiplicative, and modified multiplicative arrays are developed. The narrow main beamwidths with suppressed sidelobes, and the higher array gain for a modified multiplicative array of reduced array length (half of the linear additive array) shows its usefulness for multibeam sounder applications.

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# References

[1] Moustier, C. D., State of the art in swath bathymetric survey system. International Hydrographic Review 65 [1988], 25-54.

# Characteristics of the Lamb-type Peripheral Waves $S_1$ and $S_4$ in the Problem of Scattering of an Obliquely Incident Plane Acoustic Wave by a Circular-Cylindrical Shell

Eigenschaften der Lamb-Oberflächenwellen S<sub>1</sub> und S<sub>4</sub> bei der Streuung einer beliebig einfallenden ebenen Welle an einer kreiszylindrischen Schale

Caractéristiques des ondes périphériques de type Lamb  $S_1$  et  $S_4$  dans la diffusion d'une onde acoustique plane en incidence oblique par une coque cylindrique à section circulaire

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#### 1. Introduction

Results of computation of the partial mode resonance components of the Lamb-type peripheral waves  $S_1$  and  $S_4$ , generated in a shell by an obliquely incident plane acoustic wave, are presented. The influence of the angle of incidence on the phase velocity, amplitude and Q-factor of these waves is investigated.

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- [2] Jantti, T. P., Trials and experimental results of the Echoes-XD multibeam echosounder. IEEE Trans. Ocean Eng. OE-14 [1989], 306-313.
- [3] Owsley, N. L., Sonar Array Processing. In: Array Signal Processing (Haykin, S., Ed). Prentice-Hall Inc., New Jersey 1985, 115-193.
- [4] Urick, R. J., Principles of Underwater Sound. McGraw-Hill, New York 1975, 60-63.
- [5] Collin, R. E., Zucker, F. J., Antenna Theory. McGraw-Hill, New York 1969, 580-680.
- [6] Brown, J. L., Rowland, R. O., Design of directional arrays. J. Acoust. Soc. Am. 31 [1959], 1638-1643.
- [7] Collin, R. E., Zucker, F. J., Antenna Theory. McGraw-Hill, New York 1969, 141.
- [8] Steinberg, B. D., Principles of Aperture and Array system design. John Wiley and Sons, New York 1976, 78-80.
- [9] Chakraborty, B., Coaxial circular array. Studies related to sidelobe suppressed beampatterns for underwater transducer application. Proc. IEEE 76 [1988], 1374–1376.
- [10] Roebuck, I., The interaction between adaptive and multiplicative processing. Proceedings of the conference held at Loughborough University of Technology, England on 14th June 1977.

### 2. Formulation of the problem

Let an infinite circular-cylindrical shell be immersed in an acoustic medium. A plane acoustic wave strikes the shell and is scattered by it. The direction of propagation of incident wave makes an angle  $\alpha$  with the normal  $\eta$  to the longitudinal axis of the shell  $\xi$ . The problem is considered as a steady-state one. The incident wave generates in the shell creeping and peripheral waves which, revolving around the shell on helical paths, produce a secondary (scattered) acoustic field [1, 2]. The exact solution in the series form of the considered problem is given in [3]. We shall use it below, as well as the notation accepted there. The computations have been carried out for an observation point situated in the plane  $(\xi, \eta)$  on the angle  $\alpha = 0$  in the far field, for an aluminium shell immersed in water. The following physical parameters are

aluminium:  $\varrho_1 = 2790 \text{ kg/m}^3$ ,  $c_1 = 6380 \text{ m/s}$ ,  $c_1 = 3100 \text{ m/s}$ , water:  $\varrho = 1000 \text{ kg/m}^3$ , c = 1470 m/s.

The relative thickness of the shell is h = (a - b)/a = 1/10, where a is the outer radius of the shell and b is its inner radius.

#### 3. Dispersion curves

One family of the peripheral waves, generated in the shell by the incident wave, is similar to the Lamb waves in a plane layer. For the considered situation of an aluminium shell immersed in water, when the relative impedance is small ( $\varrho \, c/\varrho_1 \, c_1 \ll 1$ ), the ambient liquid weakly affects the positions of the resonance frequencies, i.e. the resonance frequencies of the shell immersed in liquid practically coincide with those in a "dry" (without any contact) shell. For the case of normal incidence (with  $\alpha=0$ ), the dispersion curves of the phase velocities of the Lamb-type peripheral waves in the shell almost do not differ from those generated in a plane "dry" layer.