

Monte Carlo Simulation of Diffuse Attenuation Coefficient in Presence of Non Uniform Profiles

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ABSTRACT

This paper presents a Monte Carlo simulation of the vertical depth structure of the downward attenuation coefficient (K_d), and the irradiance reflectance (R) for a given profile of chlorophyll. The results are in quantitative agreement with other models.

INTRODUCTION

In a non-homogeneous ocean with a variable pigment distribution, a light beam travelling from surface to depth z obeys the exponential law:

$$E_d(z, \lambda) = E_{0-} \exp \left[- \int_0^z K_d(z, \lambda) dz \right] \quad \dots (1)$$

where E_d is the downward irradiance at z , E_{0-} is its value at the surface, and K_d is the diffuse attenuation coefficient for downward irradiance. In general, the quantities E_d and K_d are functions of depth z and wavelength λ . They are known as the apparent optical properties because they are dependent on the radiance distribution of the incident solar radiation, and are affected by a changing sky state, sun angle, winds and waves.

Downward irradiance has been widely measured by different workers, and as a result there is a large body of data on K_d in oceanographic data banks. Various relations have been derived linking K_d to the inherent optical properties a and b , the absorption and scattering coefficient which are difficult to measure *in situ*. The most well known of these is the approximation of the exact expression as in [10]:

$$K_d(z) = (a + b_b) / \langle \mu_d \rangle \quad \dots (2)$$

where b_b and $\langle \mu_d \rangle$ are the backward scattering coefficient and mean downward cosine at depth z respectively. In the above approximation K_d derives its spectral dependence (to a large extent) from the absorption co-

efficient 'a'. There is no analytic expression that describes the depth and spectral dependence of K_d for a given pigment profile. The way out is either to measure K_d *in situ* or simulate it through Monte Carlo methods.

There is one important application area where a knowledge of the depth and spectral dependence of K_d can be useful, and that is in the satellite derived estimate of total chlorophyll. It has been shown in [1] that the remotely sensed concentration of chlorophyll is given by:

$$C_{sat} = \frac{\int_0^z c(z) \cdot g(z) \cdot dz}{\int_0^z g(z) \cdot dz} \quad \dots (3)$$

where $c(z)$ is chlorophyll concentration at depth z measured by fluorometry or by discrete sampling, and $g(z)$ is a weighing function described by:

$$g(z) = \exp \left[-2 \int_0^z K_d(z, \lambda) dz \right] \quad \dots (4)$$

As a first order approximation, K_d is assumed to be constant with depth (which may not be true for stratified waters). This can result large errors in the estimation of water column primary production and photic depth [9].

In this paper we shall use the Monte Carlo procedure followed in [4] and [8] to study the vertical structure of K_d for measured vertical chlorophyll profiles under different solar altitudes. The software has been designed also to look at other apparent properties namely, the irradiance reflectance (R), the mean cosines, and the upward attenuation coefficient ($> K_u$), and the scalar irradiance (E_s).

METHOD

In a Monte Carlo (MC) simulation, the history of a large number of photons ($\sim 10^7$) is tracked with a computer program to study the average behaviour of the total photon flux [4]. Under given assumptions (see later), the outputs of the MC simulation correspond to those quantities that would be experimentally measured in the field. Kirk [4] has used this method to good effect to study diffuse attenuation K_d , the irradiance reflectance R , the mean cosines etc., and has been able to derive several useful empirical relations between K_d and the absorption (a) and scattering (b) coefficients. However, the MC studies to-date, have for simplicity assumed that the water column is homogeneous described by constant absorption and scattering coefficients.

In the Monte Carlo method a photon is selected, and its pathlength P is determined from cumulative distribution $C(P)$ for absorption or scattering up to P :

$$C(P) = 1 - \exp(-c \cdot P) \quad \dots (5)$$

where $c = (a + b)$, and the random number r between 0 and 1 is equal to $C(P)$. A check is made on its direction of travel, given by the sign of its zenith angle. If the photon is moving 'up', a check is made on whether it has crossed a depth marker. If yes, the 'up' photon counter is updated from start to end markers, and its angular interval is assigned. Further checks are made on whether the photon has escaped from the surface or reflected. A 'down' photon counter is similarly updated, and checks are made on whether the photon has reached the bottom, and hence been absorbed. After these counter updates have been made, the program decides the nature of interaction of a new event by a random number toss r' :

if $r' < a/(a + b)$ then absorption event is true

if $r' > a/(a + b)$ then scattering event is true.

An absorption event results in the selection of a new photon from the bag. A scattering event means random selection of pathlength for the same photon. In this way, Monte Carlo accounts for multiple scattering in the ocean by tracking individual photon histories to the final demise of the particle.

ASSUMPTIONS IN THE METHOD

The following assumptions have been made in the calculation:

- 1) A flat ocean surface without waves, and a direct solar beam at a given sun angle are assumed.
- 2) We have adopted Petzold's volume scattering function measured at San Diego Harbour [see 4,7].
- 3) Vertical chlorophyll profiles are described by a Gaussian distribution fitted to experimental data expressed as:

$$C(z) = C_0 + (h/\sigma\sqrt{2\pi}) \exp[-(z - z_m)^2/(2\sigma^2)] \quad \dots (6)$$

where C_0 is the background chlorophyll concentration, h is the excess concentration above C_0 , σ is the spread of Gaussian, and z_m is the geometric depth from the surface to the profile peak.

4) The absorption coefficient $a(\lambda)$ and the scattering coefficient $b(\lambda)$ for a given chlorophyll concentration depend on the relative concentration of particle and detritus in the seawater. They are computed using following relations as described in refined bio-optical model given by Gordon [3].

$$a(\lambda) = a_w(\lambda) + [ac^*(\lambda) + ad^*(\lambda) * D/C] * C \quad \dots (7)$$

$$b(\lambda) = b_w(\lambda) + Bc * C^{0.62} + [f(c) + (1 - f(c)) * 550/\lambda] \quad \dots (8)$$

where $ac^*(\lambda)$ and $ad^*(\lambda)$ are the normalized specific absorption and scat-

tering coefficients, $f(c)$ is the ratio of plankton concentration (C) to the total concentration of plankton and detritus ($C+D$), a_w and b_w are the absorption and scattering coefficients of pure seawater, and B_c is the scattering constant with an average value of 0.30.

The Monte Carlo code takes care of three different type of scattering events namely due to water molecules, particle and detritus with associated volume scattering functions (VSF). Molecular volume scattering function is fairly uniform for all angles with a back scattering probability of 0.5, detritus VSF is chosen to be Petzold type and particle VSF is a modified Petzold type VSF to account for strong forward scattering observed in case of particles as described by Gordon [3].

5) The azimuth angle of photon is not taken into account in the calculation.

6) The bottom of the ocean is assumed to be completely absorbing.

7) In Eq. (5), if r is the random number for the path length determination then by rearrangement.

$$P.(a+b) = -\ln(1-r) \text{ or } P.c = \text{constant} \quad \dots (9)$$

RESULTS

The Monte Carlo C-program written for the purpose of simulating the vertical structure of K_d and diffuse reflectance R was run using measured vertical chlorophyll profile shown in Fig. 1 (Fig. 4a of Platt *et al.* [9]) with the parameters $h=29$, $\sigma=5$, $z_m=21$, $C_0=0.45$, of a normal distribution as in Eq. (6).

The results in Fig. 2A show the simulated K_d vertical structure at a wavelength of 440 nm at sun altitude 0, 45 and 90 degrees to the zenith. In general, the K_d curves at 440 nm mimic the shape of the chlorophyll

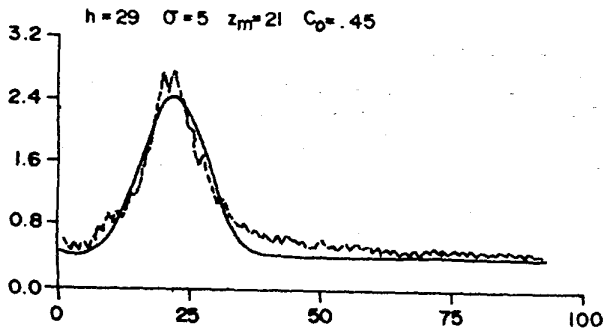


Fig. 1—Vertical distribution of chlorophyll used for simulations {taken from Fig. 4b of Platt *et al.* [9]}

profile. The peak of K_d simulations coincide with the depth of the chlorophyll maximum. The magnitude of peak value of $K_d = 0.2 \text{ m}^{-1}$ for a sun angle of 0° agrees with corresponding $K_d = 0.230 \text{ m}^{-1}$ estimated from Morel's modeling of K_d in terms of the total chlorophyll concentration [5,6]. The sun angle dependence of K_d in Fig. 2A shows increasing values with increasing sun angle due to the fact that the probability of photon absorption is higher at angles directed away from the vertical.

Corresponding vertical structure of irradiance reflectance R is also shown in Fig. 2B. The simulations show that R decreases as the chlorophyll maximum is approached, which is consistent with *in situ* radiometric observations and predictions of Gordon's refined biooptical model [3].

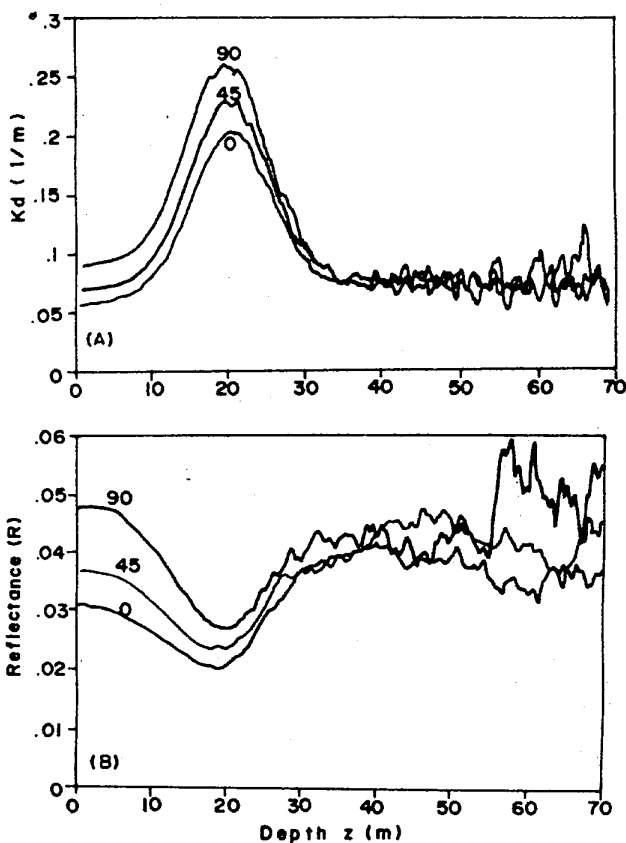


Fig. 2—Monte Carlo simulations of A) K_d at 440 nm and B) diffuse reflectance R at 440 nm for sun altitude 0° , 45° and 90° based on chlorophyll profile of Fig. 1.

The sun angle dependence of R is similar to that of K_d showing increasing value of R with that of sun angle. In both cases the sun angle dependence of K_d and R are non linear.

CONCLUSION

A Monte Carlo simulation of the optical properties of a continuously stratified ocean defined by a known pigment profile has been used to predict the vertical structure of the diffuse attenuation coefficient for downward irradiance K_d and reflectance R . The outputs of the simulation show qualitative agreement with experimentally known changes of these properties.

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