

Heat Transfer in Flow of an Elastico-Viscous Fluid past a Semi-Infinite Plate with Variable Temperature

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Abstract

An analysis of heat transfer in boundary layer flow of an elastico viscous fluid (Walters' liquid B') past a semi-infinite flat plate has been carried out by assuming the plate temperature to vary such that $T_w - T_\infty = Ax^N$, where N is a constant. Here T_w is the temperature of the plate and T_∞ is the temperature of the free-stream. Series solutions have been given and the resulting ordinary differential equations have been integrated numerically. It has been observed that an increase in K (elastic parameter) leads to an increase in the temperature and a fall in the rate of heat transfer, whereas an increase in N leads to a fall in temperature and a rise in the rate of heat transfer.

Keywords: Forced Convection, Boundary Layer Flow, Elastico-Viscous Fluid.

Introduction

Heat transfer in boundary layer flow of a Newtonian fluid past a semi-infinite plate was studied by Eckert (1942) by assuming the variable plate temperature. This is discussed in Schlichting (1968). In modern technology, many new fluids were discovered after World War II, which are not Newtonian in character and hence these are not governed by the Navier-Stokes equations. The constitutive equation of such fluids, not obeying Newtonian-laws were given by different researchers like Oldroyd (1949) and Beard and Walters (1964) etc. Walters' theory is based on the concept of vanishing memory in the fluid, i.e. fluids which exhibit small elastic property and these are known as Walters' liquid B'. The boundary layer flow of Walters' liquid B' near a stagnation-point was analysed by Beard and Walters - and later on Soundalgekar and Vighnesan (1980) studied the heat-transfer aspect of this problem. The heat-transfer aspect of boundary layer flow of Walters' liquid B' past a semi-infinite flat plate was also studied by Soundalgekar and Murty (1981) by assuming constant plate temperature. However, this assumption is rather restrictive in practice. If the plate temperature varies as some power of the distance from the leading edge, how the rate of heat transfer is affected in the boundary layer flow of Walters' liquid B', past a semi-infinite flat plate? This is the motivation for studying this physical situation.

Mathematical Analysis

We consider the flow of Walters' liquid B', with vanishing memory, past a semi-infinite flat plate with the x-axis measured from the leading edge of the plate and the y-axis measured normal to the plate. The constitutive equations characterising the elastico viscous liquid B' are

$$p_{ik} = -pg_{ik} + p'_{ik} \tag{1}$$

$$p'^{ik}(x, t) = 2 \int_{-\infty}^t \Psi_1(t-t') \frac{\partial x^i}{\partial x'^m} \frac{\partial x^k}{\partial x'^r} c_{mr}^{(l)}(x', t') dt' \tag{2}$$

where p_{ik} is the stress-tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed coordinate System x^i, x^i the position at time t' of the element which is instantaneously at the point x^i at time t , e'_{ik} the rate of strain tensor and

$$\Psi_1(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau} \exp[-(t-t')/\tau] d\tau$$

$N(\tau)$ being the distribution function of relaxation time τ . Walters (1964) has shown that in the case of liquids with short memories (i.e. short relaxation times), the equation of state can be written in a simplified form as

$$p^{,ik} = 2\eta_0 e^{(l)ik} - 2K_0 \frac{\partial e^{(l)ik}}{\partial t} \quad (3)$$

where $\eta_0 = \int_0^{\infty} N(\tau) d\tau$ is the limiting viscosity at small rates of shear $K_0 = \int_0^{\infty} \tau N(\tau) d\tau$ and

$\frac{\partial}{\partial t}$ denotes the convected differentiation of a tensor quantity, which for any contravariant tensor b^{ik} is given as

$$\frac{\partial b^{ik}}{\partial t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - b^{im} \frac{\partial v^k}{\partial x^m} - b^{mk} \frac{\partial v^i}{\partial x^m} \quad (4)$$

where v^i is the velocity vector.

From these equations, Beard and Walters have derived the boundary layer equation which is given as follows:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\eta_0}{\rho} \frac{\partial^2 u}{\partial y^2} - \frac{K_0}{\rho} \left[u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right] \quad (5)$$

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

and the energy equation, on neglecting viscous dissipation heat, is

$$u \frac{\partial T}{\partial X} + v \frac{\partial T}{\partial Y} = \alpha \frac{\partial^2 T}{\partial Y^2} \quad (7)$$

where $\alpha = K/(\rho c_p)$

The boundary conditions are

$$\begin{aligned} u=0, v=0, T=T_w(x) \quad \text{at } y=0 \\ u=U_0, v=0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (8)$$

All the physical variables are defined in the nomenclature.

On introducing the following usual boundary layer transformations,

$$\eta = y \sqrt{\frac{U_0}{\nu x}}, \Psi = \sqrt{U_0 \nu x} f(\eta)$$

$$u = U_0 f', V = \frac{1}{2} \sqrt{\frac{\nu U_0}{x}} (\eta f' - f) \quad (9)$$

$$\theta = \frac{T - T_\infty}{T_w(x) - T_\infty}, T_w(x) = T_\infty + Ax^N$$

$Pr = \eta_0 c_p / K$ (the Prandtl number)

$K = K_0 U_0 / \rho \nu x$ (elastic parameter)

in Eqs. (5) to (7), we get

$$f_{\eta\eta\eta\eta} + \frac{1}{2} f_{\eta\eta} = \frac{K}{2} [f_{\eta\eta}^2 - 2f_\eta f_{\eta\eta\eta} - f f_{\eta\eta\eta\eta}] \quad (10)$$

$$\theta'' + \frac{1}{2} Pr f \theta' - N Pr f' \theta = 0 \quad (11)$$

with the following boundary conditions:

$$f = 0, f' = 0, \theta = L \quad \text{at } \eta = 0$$

$$f' = 1, \theta = 0 \quad \text{as } \eta \rightarrow \infty \quad (12)$$

Here $f_\eta = \frac{\partial f}{\partial \eta}$

Equation (10) is still not an ordinary equation as the parameter K is a function of x . As K is a small quantity for these fluids, we have $K \ll 1$. This also helps us to overcome the mathematical difficulty viz. a fourth order equation with three boundary conditions. Hence we now assume

$$f = f_0 + K f_1, \quad \theta = \theta_0 + K \theta_1 \quad (13)$$

Table 1 Values of $\{-\theta'(0)\}$

Pr	K/N	0	1	2
2	0	0.4223	0.6856	0.8329
2	0.05	0.4190	0.6799	0.8256
2	0.1	0.4158	0.6741	0.8184
5	0	0.5766	0.9325	1.1317
5	0.05	0.5719	0.9243	1.1216
5	0.1	0.5671	0.9161	1.1115

Substituting (13) in Eqs (10) to (12), equating the coefficients of different powers of K , neglecting those of K^2 , we get a set of ordinary differential equations which are solved numerically on a computer. The numerical values of $\theta = \theta_0 + K\theta_1$ are shown graphically for different values of Pr and N (≥ 1) and $K = 0.05, 0.1, 0.2$.

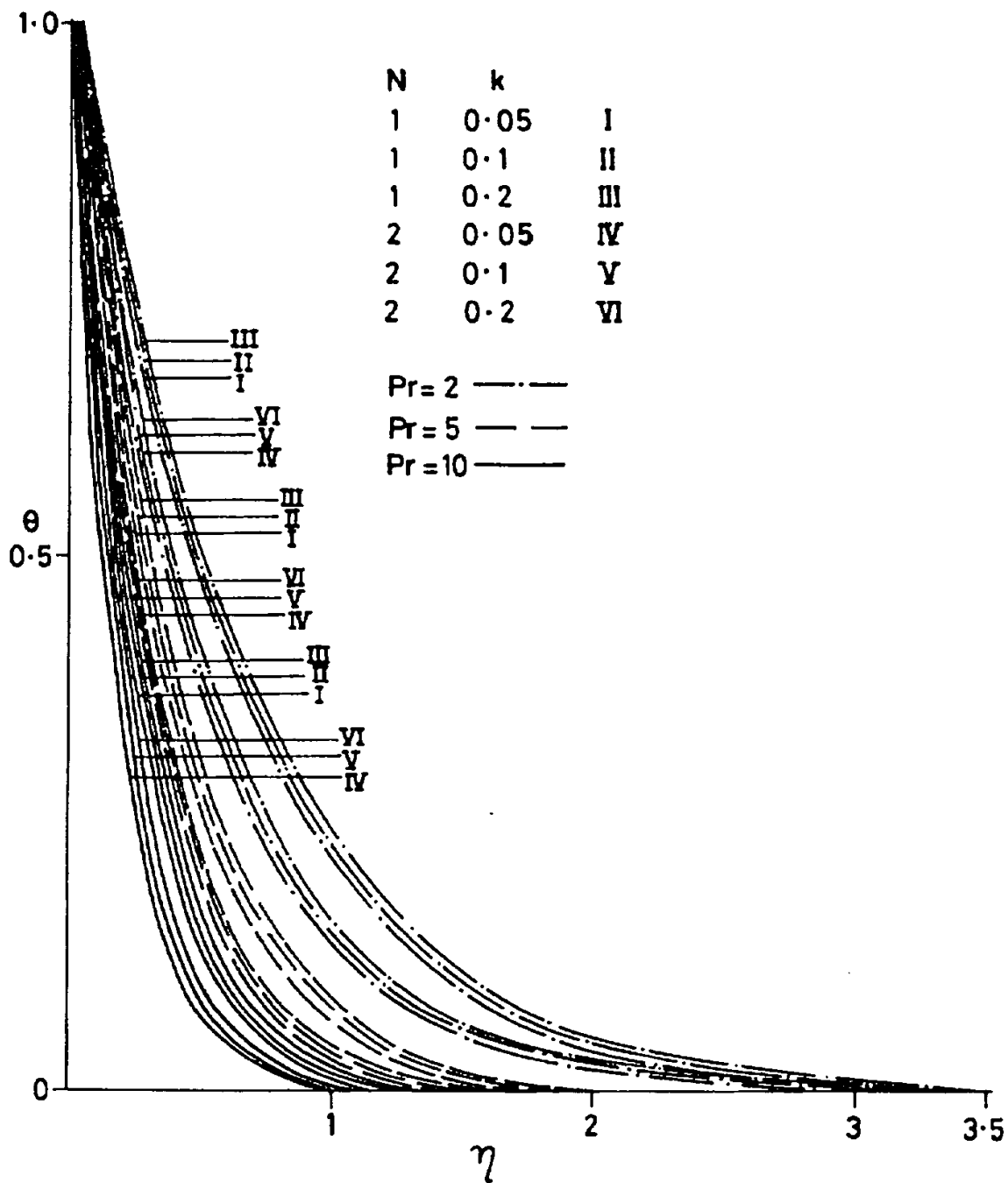


Fig. 1 Temperature Profiles

Knowing the temperature field, we now study the rate of heat transfer. It is given by

$$q = -K \frac{\partial T}{\partial Y} \Big|_{y=0} \tag{14}$$

where K is the thermal conductivity of the fluid. In view of the transformations (9), Eq. (14) reduces to the following:

$$q(x) = -K \sqrt{\frac{U_0}{\nu x}} (T_w(x) - T_\infty) \cdot \theta'(0) \quad (15)$$

The numerical values of $\{-\theta'(0)\}$ are entered in Table 1.

Results and Discussions

We observe from Fig. 1 that an increase in K leads to an increase in the temperature of the fluid when $N (\geq 1)$. But as N increases, the temperature of the fluid falls when Pr and K are constant. Also, an increase in the Prandtl number Pr leads to a fall in the temperature of the elasto-viscous fluid. †

Also, we conclude from Table 1, that for Walters' liquid B', due to the presence of the elastic property in the fluid, the rate of heat transfer decreases as compared to the ordinary Newtonian fluid, and its decreases more when the elastic parameter K is increased. However, the rate of heat transfer increases with increasing N or the Prandtl number Pr .

Conclusions

An increase in K leads to an increase in the temperature but a fall in the rate of heat transfer.

There is a fall in temperature and a rise in the rate of heat transfer due to increasing N .

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Nomenclature

p_{ik} = stress tensor	f^{ik} = contravariant tensor	T_w = temperature of the plate
p = an arbitrary isotropic pressure	v^i = velocity vector	T_∞ = temperature of the fluid far away from the plate
g_{ik} = metric tensor	u, v = velocity components along x, y directions	q = rate of heat transfer
e'_{ik} = rate of strain tensor	X, Y = coordinate axes	ψ = stream-function
$N(\tau)$ = distribution function of relaxation times τ	ν = kinematic viscosity	θ = non-dimensional temperature
η_0 = limiting viscosity defined in Eq. (3)	T = temperature of the fluid	Pr = prandtl number
κ_0 = small rates of shear	α = thermal diffusivity	κ = non-dimensional elastic parameter
	U_0 = fluid velocity outside the boundary layer	