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# Inverse Problem of Ocean Acoustic Tomography (OAT)

- A NUMERICAL EXPERIMENT

T.V. RAMANA MURTY, Y.K. Somayajulu, R. Mahadevan\* & C.S. Murty  
National Institute of Oceanography, Dona Paula, Goa-403 004.

\* Ocean Engineering Centre, I.I.T., Madras-600 036.

## ABSTRACT

Acoustic model simulation experiments related to the forward and inverse aspects of ocean tomography have been taken up with a view to estimate the vertical sound speed field by inverting the travel time data. Two methods of inversion have been attempted. These are a) deterministic inverse and b) stochastic inverse. To solve the former, singular value decomposition (SVD) is used to arrive at the model parameter(s). The data kernel consisting of acoustic path lengths in a pre-set number of ocean layers, or grids developed by solving the forward problem of the acoustic model enable build the generalized inverse operator (GIO) that operates on the travel time perturbation data. Resolution matrices obtained through SVD helped to examine the correctness of the solution. In the stochastic inverse case, for random model parameter and data, the vertical structure of the ocean is modelled through the quasi-geostrophic theory while the horizontal structure is assumed to have Gaussian covariance, with length scales of 100 km. In both cases, a good agreement has been observed between the assumed and the reconstructed sound speed perturbation field.

## 1. INTRODUCTION

Ocean Acoustic Tomography (OAT) has been proved to be an important remote sensing tool for collecting synoptic data pertaining to ocean temperature and the mesoscale flow fields from the ocean interior (Munk and Wunsch, 1979). This is carried by propagating sound pulses between a few number of transceivers deployed on moorings. The spaceborne remote sensing together with OAT form a complete system to provide real time data over large

regions of the ocean.

In OAT, we assume a reference ocean stratified in the vertical. Acoustic rays from a source to a receiver are traced and travel times of acoustic pulses along different paths are estimated. These predicted times differ from the measured values in the real ocean due to variations in the prevailing thermal/salinity structures and consequent changes in the sound speed field. The travel time deviations of stable eigen rays, their path lengths in the pre-set number of layers (data kernel) and information on environmental noise are used to estimate sound speed perturbations along a vertical/ horizontal plane. The optimal estimation of sound speed (inverse problem) forms an important first step in the interpretation of the acoustic travel time data obtained from tomographic experiments.

In the present paper sound speed field estimated following the deterministic and stochastic inverse methods has been presented. Simulation experiments carried with an assumed sound speed perturbation field and the results obtained thereof are discussed.

#### INVERSE PROBLEM

##### Deterministic inverse:

The inversion procedure used here is based on singular value decomposition (SVD). Following Chester et al. (1991) the perturbation in travel time could be written as

$$\delta T_i = - \int \frac{\delta C(x,t)}{C_o^2(x,t_o)} ds \quad \dots (1)$$

Model equation could be rewritten as

$$\begin{bmatrix} -R_{ij} \\ C_o^2 \end{bmatrix} \delta C_j = \delta T_i \quad \dots (2)$$

After parameterizing the model, the above equation can be expressed in matrix-vector notation as  $\delta T = A \delta C$  where  $\delta T$  are

the travel time differences between the measurements and those obtained from the ray model;  $A = -R_{ij}/C_0^2$  where  $R_{ij}$  is the path length of ray  $i$  in layer  $j$  and  $C_0$  is the reference sound speed. The travel time perturbation is related to sound speed perturbation through matrix  $A$ . For this analysis, the construction of a generalized inverse operator using SVD employing eigen value technique has been considered (Menke, 1984). According to Jackson (1972), the natural generalized inverse operator  $A_g^{-1}$  always exists and is given by

$$A_g^{-1} = V \Gamma^{-1} U^T \quad \dots (3)$$

where matrices  $V$  and  $U$  are obtained by the coupled eigen value problems

$$A A^T U_j = U_j \quad ; \quad A^T A V_j = V_j \quad \dots (4)$$

In the above equations  $U_j$  and  $V_j$  the column vectors of matrices  $U$  and  $V$  are eigen values of matrix  $A$  arranged in decreasing order. The eigen vectors corresponding to the largest eigen value indicate that large scale factors can best be determined. The selection of the number of eigen values is decided by the closeness ratio approach. The solution of model equation is given by

$$\delta C = [V \Gamma^{-1} U^T] \delta T \quad \dots (5)$$

Calculations of the model resolution matrix and data resolution matrix are needed for the assessment of the resulting sound speed model.

#### Model resolution

The model resolution of a generalized inverse (Menke, 1984) is given by  $R = A_g^{-1} A = V_p V_p^T$ . Here  $p$  indicates the number of factors used in SVD which is less than or equal to the rank of matrix  $A$ . The model parameters will be perfectly resolved if  $V_p$  spans the complete space of the model parameter i.e.  $V_p V_p^T = I$

#### Data resolution

The data resolution matrix is given by  $N = A A_g^{-1} = U_p U_p^T$

The data are perfectly resolved if  $U_p$  spans the complete

space of data. This usually tells the convergence of rays in the given domain. The SVD provides a simple frame work for determining how well the model parameters fit the data and how closely the model parameter estimates the true values. The larger eigen values determine the large scale features of the problem while the small eigen values correspond to small scale or high frequency features.

2b. Stochastic inverse:

In the stochastic inverse method both data 'd' and unknown field  $\delta C(x,z)$  are assumed as random variables having known mean and covariance functions. Then best estimate  $\hat{\delta C}(x,z)$  of the model parameter  $\delta C(x,z)$  can be expressed in the form

$$\hat{\delta C}(x,z) = \sum a_i(x,z) \delta T_i = A(x,z) d, \quad \dots (6)$$

such that the ensemble average of the square of the difference between the estimate of the unknown field and the true field at each point in the medium is a minimum. i.e.  $a_i$ 's are determined such that the square of the deviation (ensemble) is minimum at each point  $(x,z)$ . Using the Gauss-Markov theorem (Liebelt, 1967),  $a_i$ 's are given by,

$$A(x,z) = \langle \delta C(x,z) \delta T_i^T \rangle [\langle \delta T_i \delta T_j^T \rangle]^{-1} \quad \dots (7)$$

where  $\langle \delta C(x,z) \delta T_i^T \rangle$  and  $\langle \delta T_i \delta T_j^T \rangle$  are the model-data and data-data covariance matrices. These are evaluated using the linearized direct problem,

$$\delta T_i = - \int_{\Gamma} \frac{\delta C(x,z)}{C_0^2(z)} ds + \epsilon_i \quad \dots (8)$$

where  $\epsilon_i$  represents noise due to smaller scale phenomena and measurement errors which is assumed to have zero ensemble average and to be uncorrelated with the mesoscale sound speed fluctuation. Using (8) the model-data and data-data covariance matrices become,

$$\langle \delta C(x,z) \delta T_i^T \rangle = \int_{\Gamma} \frac{\langle \delta C(x,z) \delta C(x',z') \rangle}{C_0^2(x')} ds' \quad (9)$$

$$\langle \delta T_i \delta T_j^T \rangle = \langle \delta T_i \delta T_j^T \rangle$$

$$= \int_{\Gamma} ds'' \int_{\Gamma} \frac{\langle \delta C(x', z') \delta C(x'', z'') \rangle}{C_0^2(x', z') C_0^2(x'', z'')} ds' + \gamma_{ij} \dots (10)$$

where  $\gamma_{ij} = \langle \epsilon_i \epsilon_j \rangle$  is the error covariance matrix.  $(x', z')$  and  $(x'', z'')$  are the coordinates of the  $j$ -th and the  $i$ -th rays respectively. In eqns.(9) and (10), the integrations are carried along the eigen rays and the covariance function of the sound speed perturbation field specified in the form.

$$\langle \delta C(x_1, y_1, z_1) \delta C(x_2, y_2, z_2) \rangle = \sum_{i=1}^2 \beta_i H_i(r) F_i(z_1) F_i(z_2) \quad (11)$$

where  $r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$  and  $H_i(r)$  are the horizontal covariance functions. Here the covariance is expanded in terms of sound speed perturbation eigenmodes.  $F_i(z)$ , and  $\beta_i$ 's are the weights to be assigned to different modes. In this expansion only two terms are generally considered.

For each point in a given region, coefficients  $a_i(x, z)$  are determined from the eigen rays and operated on the travel time perturbations to objectively estimate the model parameter. This procedure permits a continuous representation of the unknown field. In this simulation experiment, the horizontal covariance function used for both modes is assumed homogeneous, isotropic and Gaussian with a horizontal length scale of 100 Km. That is

$$H_1(r) = H_2(r) = \exp \left\{ - \frac{|x_2 - x_1|^2}{100^2} \right\} \dots (12)$$

The vertical modes,  $F_i(z)$ , are computed based on quasi-geostrophic theory (Pedolosky, 1980). The data-data and model-data covariance matrices are determined for the reference sound speed distribution with a suitably selected source-receiver configuration. Using these matrices sound speed field of the real ocean is obtained for the given travel time perturbations.

### 3. SIMULATION EXPERIMENTS

#### a) Deterministic case

In this study the locations for acoustic source and receiver are chosen in the Arabian Sea along  $12.5^{\circ}\text{N}$ , at  $67^{\circ}\text{E}$  and  $69.5^{\circ}\text{E}$  in the SOFAR channel at depths of 1535 and 1685m respectively, separated by a distance of 270km. The annual mean profiles of temperature, salinity and sound speed for this region are presented in Figs. 1. Fig. 2 shows ray travel time versus emergence angle. It can be seen that all the near axial rays with flat angles reach the receiver simultaneously complicating identification of individual rays. Hence only selected rays which are identifiable by their arrival times are used in the analysis. The eigen rays computed for the reference case are shown in Fig. 3. The travel time versus intensity of eigen rays are depicted in Fig. 4.

The computed travel times of eigen rays for the reference (climatic mean) sound speed profile and the ones obtained for the assumed (winter mean) profile have been used to generate possible perturbations in travel times which are operated by the GIO to get the model parameter perturbation (Fig.5). The plot indicates that for a six layer model, five eigen vectors can reproduce the features adequately as seen by the points close to the assumed perturbation (continuous curve).

To make the inverse solution complete, the model and data resolution matrices are analysed (Chester et al., 1991). The diagonal elements of model resolution depict how well the individual model parameters are resolved. A value below unity indicates less resolution while unit value corresponds to good resolution.

For the deterministic case, we have demonstrated the use of GIO through SVD for the simple case of 1-dimensional model. This could be extended to the 2-dimensional case also. But this method has the limitations in the sense that at the layer/grid boundaries, discontinuities exist besides the difficulty in

choosing the optimum layer/grid size. This may lead to integration errors while solving the forward problem for obtaining the data kernel. In the real ocean sense, the model parameter and data are random in nature wherein an alternate method called the stochastic method has been applied (Cornuelle, 1983). The stochastic inverse procedure permits a continuous representation of the unknown field.

b) Stochastic case

Stochastic inverse requires the specification of the covariance function of sound speed perturbations. We used two horizontal modes derived by Cornuelle (1983) which are assumed to be homogeneous, isotropic and Gaussian with a length scale of 100 km. For the vertical modes either the statistical modes (EOFs) derived from historical data or dynamic modes computed from the reference ocean could be used. In the present study dynamic modes derived using quasi-geostrophy theory are used. For stochastic inverse, the dynamic modes in terms of the vertical modes of sound speed perturbation are needed. The following relation (Desaubies, 1990) has been used

$$F_{c_i}(z) = F_i(z) \gamma(z) \quad \text{where } \gamma(z) = (\partial C_p / \partial z) \quad \dots (13)$$

where  $C_p$  is the potential sound speed. We computed vertical modes  $F_{c_i}(z)$  for the given profiles of Brunt-Vaisala frequency and adiabatic sound speed. These plots are shown in Fig. 6. Using the dynamic modes (Fig. 7) and considering only the first term in the expansion (Eq. 11), covariance of sound speed perturbation is defined. With this, data-data and model-data covariance matrices are computed. A sound speed perturbation

$$\delta C(x, z) = 10 \lambda \sin(2\pi/L) F_{c_i}(z) \quad \dots (16)$$

is assumed where  $L=100\text{km}$ . For this distribution, the travel time perturbations for the six eigen rays are computed. This forms the input for the computer program which reconstructs the sound speed anomalies. The assumed and reconstructed anomalies are shown in Fig. 8. A good agreement can be seen between them.



References

H.H. and Wunsch, C.:1979, 'Ocean Acoustic Tomography: a tool for large-scale monitoring', Deep-Sea Research, Vol.26A, pp. 126-161.

Steinberg, D.B., Rizzoli, P.M. and De Ferrari, H. Acoustic Tomography in the Straits of Florida, J. Geophys. Res. , vol.96 (1991), 7023-

Wax, G.W. Geophysical data analysis:Discrete Inverse theory. Academic Press, New York (1984).

Wax, G.W. and Monod, D.D. (1972) Geophys. J. Roy. Astro. Soc., vol.28, 97.

Soltis, P.B., 1967, 'An introduction to optimal estimation', Addison-Wesley publishing company, London, pp. 273.

St. Pierre, B.D., 1983, 'Inverse methods and results from the 1981 ocean acoustic tomography experiment', Ph.D Thesis, WHOI/MIT Joint program, WHOI, USA.

Stoker, J., 1980, 'Geophysical Fluid Dynamics', Springer - Verlag, New York. 624pp.

Desaubies, Y. 1990, Ocean Acoustic Tomography in 'Oceanographic and Geophysical Tomography', (Eds.) Desaubies, Y., Tarantola, A and Zinn-Justin, J., Elsevier Science Publishers.

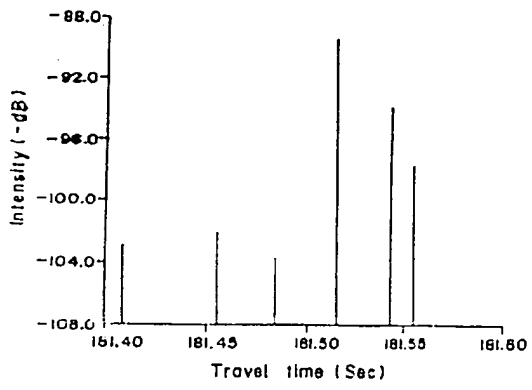
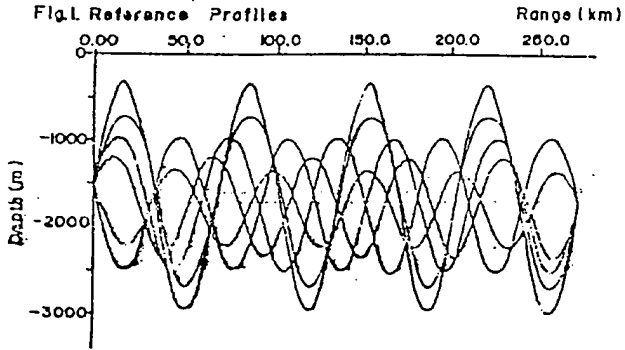
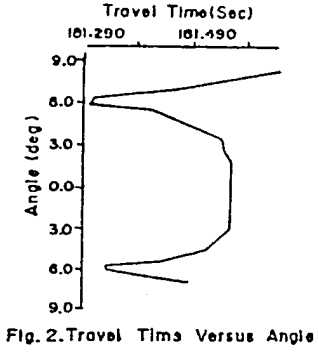
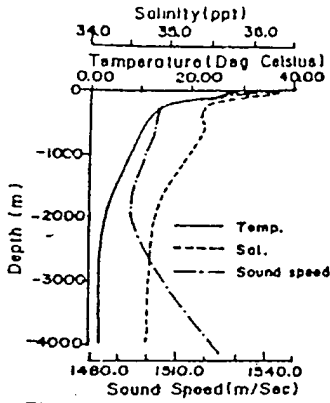


Fig. 4. Travel Time Versus Intensity

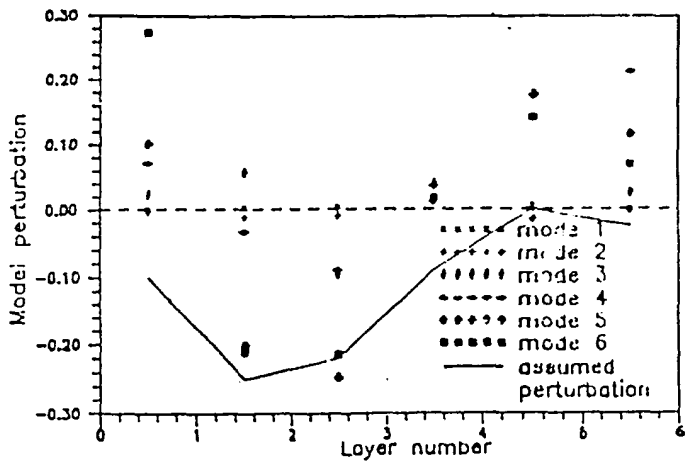


Fig. 5. Assumed and reconstructed model perturbations with different energetic modes for six layer case.

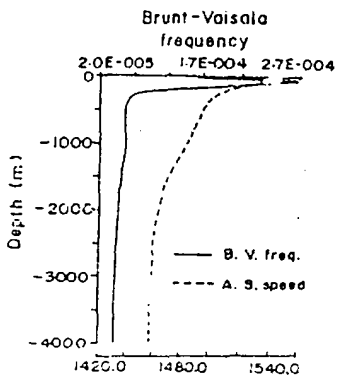


Fig. 6 Reference Profiles

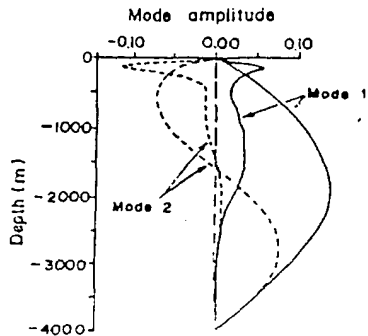


Fig. 7 Vertical Dynamic modes and  
Sound speed Perturbation.

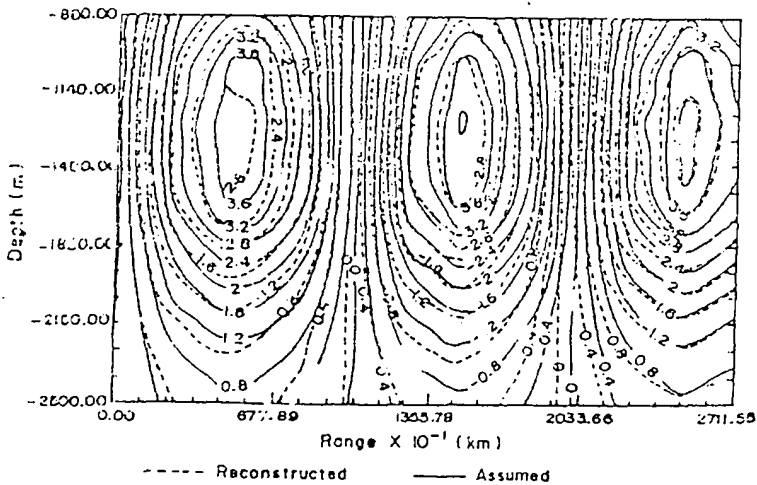


Fig. 8 Sound Speed Perturbation. (m/sec)