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M₂ Tidal Currents on the Shelf off Goa, West Coast of India

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ABSTRACT

Current meter records collected during three different months from a site off Goa (15° 08'N, 73° 16'E) over the western continental shelf of India have been used to describe the M₂ tidal structure in a water column of depth about 100 m. Tidal ellipse parameters and Greenwich phases were estimated using the least squares fit technique. M₂ tidal currents have shown significant variations from one season to another (15 cm/s during May 1984 to 3.7 cm/s during November 1986). The baroclinic M₂ tides have been interpreted in terms of linear internal wave dynamics. The results show that the M₂ baroclinic tide is dominated by the first mode. The horizontal wavelength over the shelf predicted by this method is about 22 km. This is smaller than the shelf width of about 110 km. The group velocity for the first baroclinic mode is about 0.45 m/s, which means that energy takes approximately two and a half days to cross the shelf. Evidence presented here suggests that stratification over the continental shelf and the adjacent slope has influenced the amplitude of internal tidal currents. The bottom slope switches from being supercritical to subcritical in the shelf break region during May and remains subcritical all over the shelf and the slope during November.

INTRODUCTION

The tides off the west coast of India are of mixed dominant semi-diurnal type. The largest tidal constituent is the principal lunar or M₂. Tidal models on global scales, based on Laplace's tidal equations, are available to predict the elevations and phases in this region with reasonable accuracy [10,13]. The M₂ tidal elevation thus predicted is of the order of 50 cm with phase propagating from north to south. While ample information is available on the tidal elevations and phases, either from mathematical models or from the analysis of tide gauge data [4], information on tidal currents in this region is sparse. During three separate field studies in May 1984, March 1986 and November 1986, moorings with five current meters each were deployed over the shelf off Goa (Fig. 1). Although the data records are short

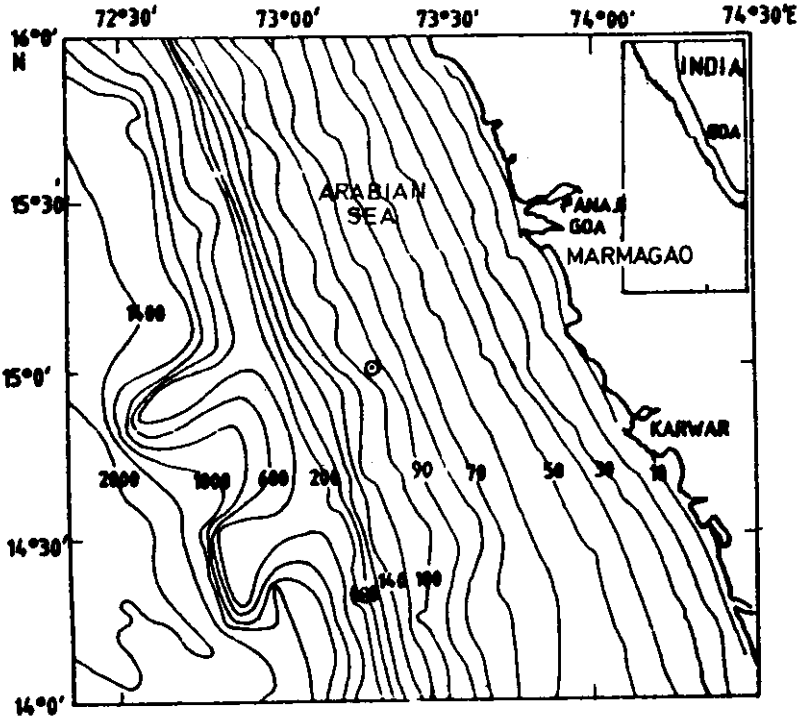


Fig. 1 — Location of the mooring and hydrographic stations across the shelf off the west coast of India (depth contours are in m)

(8-14 days), they provide an opportunity to examine the M_2 tidal fields over the shelf off Goa. The May and November moorings represent two extremes of stratification in this region.

The vertical sections of σ_t across the shelf (Fig. 2) show that the density structure during May is typical of a well stratified water column with the upward tilt of isopycnals near the coast. On the contrary, the σ_t structure during November lacks the upward tilt and higher gradients, indicating a weak stratification of the water column. If σ_t was uniform over the water column, the tidal currents would have been barotropic and would have resulted from the surface gravity waves alone. But the well-defined vertical stratification observed in this region is expected to support the propagation of internal waves at tidal periods. The result is that the tidal currents in this region are influenced by both barotropic and baroclinic modes. The theory on internal tides (baroclinic tides) suggests an increase in internal tidal energy with increasing stratification [2]. Some observations [7,8,12] report an increase in the amplitudes of the internal tides with increasing stratification at various locations over the continental shelves. It is

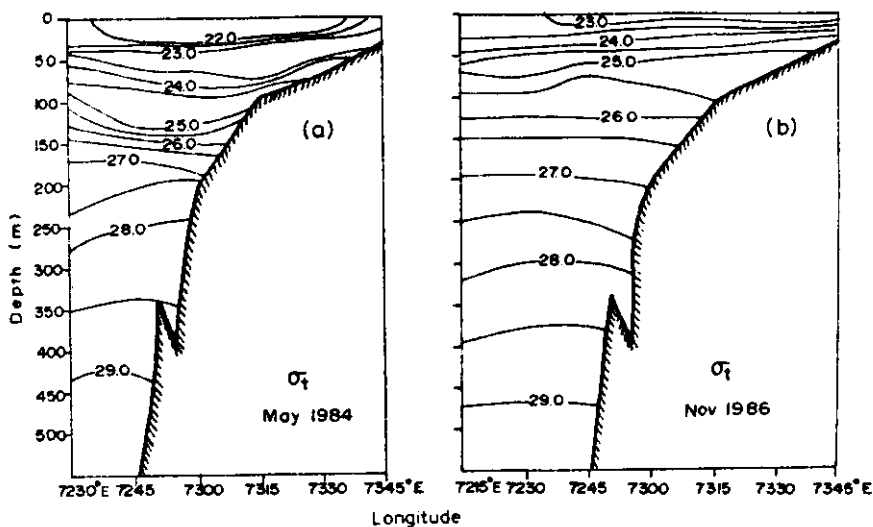


Fig. 2 — σ_t cross-section over the shelf and adjacent area off the west coast of India [hydrographic stations taken on (a) May 1984 and (b) November 1986]

unknown whether the variable stratification modifies the internal tide amplitude by affecting the process of generation, propagation, or dissipation, but it seems likely that all three may be important.

In this paper, we discuss the properties of M_2 tides on the western continental shelf of India, observed at a station off Goa, using three sets of mooring data collected during three different months. The vertical structure of the tidal currents has been examined here. The vertical structure of the M_2 tidal current is consistent with the interpretation of the first baroclinic mode. Evidence presented here suggests that the strength of the stratification over the continental shelf and the adjacent slope may be one of the factors controlling the amplitude of M_2 internal tide. Shenoi and Antony [14] have earlier presented preliminary results on the analysis of current records on the western shelf of India. The spectra of the current records suggest that the semi-diurnal and diurnal bands contain a significant portion of the energy. Here, we confine our attention to the M_2 constituent of the tidal currents.

DATA AND METHODS

The location of the mooring and the major features of bottom topography are shown in Fig. 1. A general description of the mooring was given by Shenoi and Antony [14]. Five current meters were attached to each mooring deployed in a total water depth of 92 m. The data obtained from some of the current meters were

Table 1 — Details of Current Meter Moorings

Moorings	Depth of current meter (m)	Duration of acceptable records(h)
11-19 May 1984	17	188
	27	—
	42	156
	62	—
	82	204
17-27 March 1986	15	240
	25	—
	40	240
	60	240
	80	240
29 Oct.-11 Nov. 1986	10	312
	25	312
	40	276
	60	—
	80	312

unreliable either because of failure of the instrument or because of the obstructions created by fouling animals on the rotor. The depth of the instruments and the data from the current meters are summarized in Table 1.

The tidal analysis reported here is based on the least-square fit method [6]. Computer algorithms developed by Foreman [5] were used for this purpose. As the series are too short to resolve S_2 from M_2 , the S_2 constituent was inferred from the amplitude ratios and phase differences determined from the analysis of tidal data [4] from a nearby tide gauge (at Marmagao 15°25'N, 73°48'E). The phase of each constituent is represented as the Greenwich phase. This is defined as the difference between the astronomical argument for Greenwich and the phase of the observed constituent signal. The time series was thus referred to GMT before subjecting it to analysis.

RESULTS

The tidal ellipses at M_2 frequencies show the amplitude and phase to be quite variable in time and depth (Fig.3, Table 2). The sense of rotation is clockwise for all the ellipses, except the one during March 1986 at 15 m depth. The variations

in tidal ellipses with depth point to the strong possibility of baroclinic tide here. The ellipses show large variations from one season to another. The semi-major axis of M₂ ellipses varied from 15.3 cm/s at the top current meter during May to 3.7 cm/s at the top current meter during November. The ellipses also do not show a consistent depth dependency in any of the data sets. The middle ellipse (at 42 m) in the May mooring was the weakest (4.1 cm/s) with stronger ellipses at the top and bottom current meters. From the orientation of the ellipses, it appears that the M₂ tidal currents at middle and the bottom current meters have opposite flow directions compared to the top current meter. The other two data sets (March and November) do not show such marked differences in their orientations. The ellipses were oriented from 12.6° to 32.8° during March and from 14.3° to 55.8° during November. Similarly, their semi-major axis also do not show large variations (4.5 cm/s to 7.8 cm/s during March and 3.7 cm/s to 7.3 cm/s during November) compared to the ellipses obtained during May (4.1 cm/s to 15.3 cm/s).

Since it is possible that the horizontal tidal currents represented by the ellipses in Fig. 3 have included the effects of both barotropic and baroclinic tides, we had separated them to study their properties. The nature of barotropic tidal ellipses is constant phase and amplitude with depth, except near the bottom where friction

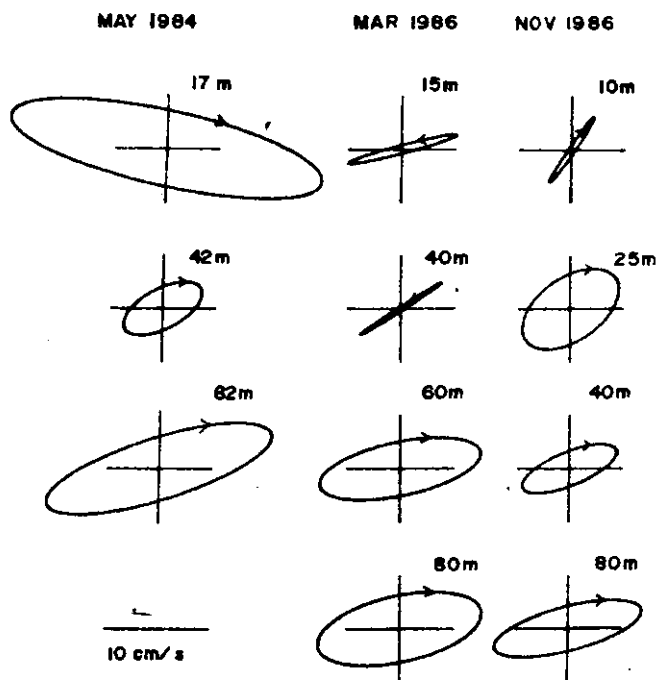


Fig. 3 — M₂ tidal current ellipses during May 1984, March 1986 and November 1986 [depths of instruments are also indicated]

Table 2 — M_2 Ellipse Parameters
 [A negative sign for the minor axis indicates the clockwise rotation of the tidal ellipse]

Depth (m)	Semi-major axis (cm/s)	Semi-minor axis (cm/s)	Inclination (deg)	Phase (deg)
<i>May 1984</i>				
17	15.30	-3.53	168.6	332.7
42	4.10	-1.88	27.4	119.7
82	11.20	-2.89	17.5	41.0
<i>March 1986</i>				
15	5.22	0.39	15.0	17.6
40	4.49	0.19	32.8	68.2
60	7.66	-2.52	11.9	0.6
80	7.80	-3.12	12.6	85.9
<i>November 1986</i>				
10	3.71	-0.29	55.8	78.5
25	5.21	-2.98	34.1	101.9
40	4.84	-1.59	21.8	67.6
80	7.27	-2.08	14.3	61.9

reduces the amplitude. On the other hand, a baroclinic tide would have low horizontal and vertical coherence and would produce an ellipse that is anomalous in size and phase with the barotropic tidal ellipses because of the characteristic lack of coherence between barotropic and baroclinic tides. The top and bottom ellipses during May mooring are anomalously large and they can be expected to contain a large portion of the baroclinic tidal component.

Barotropic tidal ellipses — The barotropic tidal ellipses were calculated based on the assumption that the simple average of the measured current from all the current meters available in the water column represent the barotropic currents, and the deviation from this average represents the baroclinic currents. Data from three current meters were averaged for May mooring and data from 4 current meters each were averaged for March and November moorings. The resulting time series were subjected to the tidal analysis program to generate the barotropic ellipse parameters. The barotropic ellipses are given in Fig. 4 and the ellipse parameters in Table 3. The M_2 amplitude was highest (7 cm/s) during May and lowest (4.3 cm/s) during November. The greater amplitude of M_2 barotropic tide during May is possibly due to the spring-neap cycle. It may be noted that the May observations

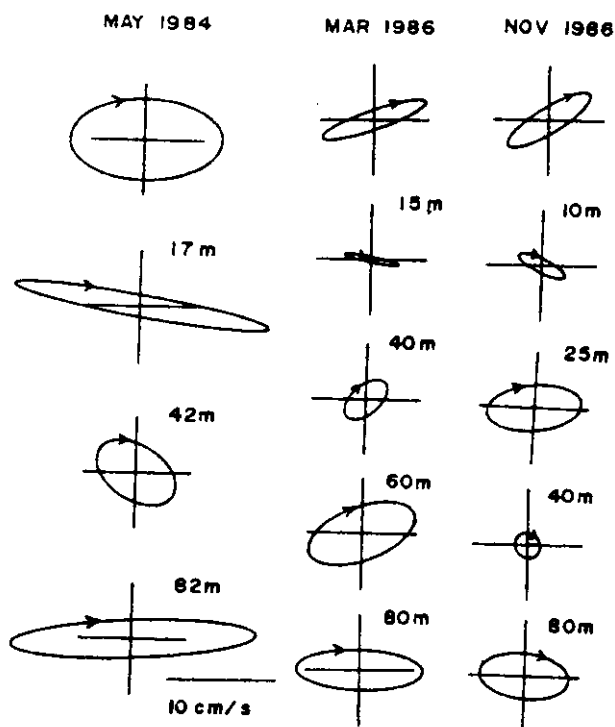


Fig. 4 — M₂ barotropic and baroclinic tidal current ellipses during May 1984, March 1986 and November 1986 [depths of the instruments are also indicated]

were made during a spring tidal cycle, where the full moon occurred on 15 May, i.e. right in the middle of the observation period. The barotropic tides on the northwest shelf of Australia also have a pronounced spring-neap cycle [8]. An interesting feature to be noticed in the barotropic ellipses is the consistency in their orientations during all the three moorings. The M₂ ellipse inclinations are within 19° - 33° anticlockwise from the east-west axis. The orientation of the barotropic tidal ellipses defines the horizontal direction in which the barotropic tide is propagating, but with an ambiguity of 180°. From the ellipse orientations, it appears that the barotropic tidal currents (M₂) are directed perpendicular to the bathymetric orientations of the shelf. As the barotropic tide is expected to propagate towards the shelf from the open ocean, it may be concluded that the M₂ barotropic tidal currents in this region are directed onshore.

Baroclinic tides — Time series of baroclinic currents were calculated at each level by subtracting the barotropic (mean) tidal currents from the observed series. The resulting time series were subjected to the tidal analysis to obtain the ellipse

Table 3 — M_2 Barotropic and Baroclinic Ellipse Parameters
 [A negative sign for the minor axis indicates the clockwise rotation of the tidal ellipse]

Depth (m)	Semi-major axis (cm/s)	Semi-minor axis (cm/s)	Inclination (deg)	Phase (deg)
		May 1984		
		<i>Barotropic</i>		
	7.00	-3.66	20.4	114.1
		<i>Baroclinic</i>		
17	11.81	-1.01	171.3	350.0
42	4.00	-2.48	149.2	115.3
82	11.26	-1.70	3.3	7.1
		March 1986		
	5.02	-0.86	19.3	40.5
		<i>Baroclinic</i>		
15	2.56	-0.27	169.6	130.3
40	2.34	-1.30	40.8	147.9
60	5.15	-2.39	21.4	316.1
80	5.77	-1.81	178.8	310.3
		November 1986		
		<i>Barotropic</i>		
	4.35	-1.13	32.7	77.1
		<i>Baroclinic</i>		
10	2.31	-0.64	154.9	102.0
25	4.32	-2.05	7.5	194.3
40	1.19	-1.11	139.2	254.2
80	4.06	-2.07	175.4	225.1

parameters. The baroclinic ellipse parameters are given in Table 3 and the ellipses are shown in Fig. 4. The plotted M_2 baroclinic ellipses (Fig. 4) show a strong variability in the currents with depth and season. There is $\sim 180^\circ$ change in ellipse orientation from the near surface to near bottom ellipses during May mooring. This means that the baroclinic M_2 tidal currents have opposite directions at the current meters near surface and bottom. The major axis lengths have similar magnitudes (~ 11.5 cm/s) at both the current meters with a minimum (4 cm/s) at mid-depth (42 m). The March and November moorings also show large differen-

ces in ellipse orientations, but the ellipses become much smaller. The ellipses have minimum major axis lengths at 40 m for both March and November sets of moorings.

Further insight into the nature of tidal motions can be gained from the linear internal wave dynamics. To interpret the observed tidal motions, we have computed the normal modes of the internal waves using hydrographic data. The modes associated with the buoyancy frequency profile can be calculated [11] by solving the vertical structure equation for the vertical velocity *W*:

$$(\omega^2 - f^2) \frac{\delta}{\delta z} \left(\bar{\rho} \frac{\delta W}{\delta z} \right) - \bar{\rho} (\omega^2 - N^2) k^2 W = 0 \quad \dots(1)$$

where $\bar{\rho}$ is the mean density, $N^2 = (g/\bar{\rho})(\delta\bar{\rho}/\delta z)$ is the square of the Brunt Vaisala frequency; k^2 , the eigenvalue (the square of the horizontal wave number); f , the Coriolis parameter; and ω , the frequency of the wave. Strictly speaking, the above equation is valid for a flat bottom. However, it has been shown that where the bottom slope is gentle, the horizontal wavelength is locally the same as would be predicted for a flat bottom [15]. The hydrographic data collected near the mooring during the current measurements were used to compute the buoyancy frequency profiles. Eq. (1) was solved subject to the boundary conditions $W=0$ at surface and bottom. A NAG Fortran library routine (subroutine D02kef) available to solve the Sturm-Liouville equation was used to obtain the eigenvalue k^2 and eigenfunction *W*. As the horizontal velocities were proportional to the *z* derivative of the vertical velocity, the eigenfunctions were differentiated to obtain the eigenfunctions for horizontal velocity components. Finally, all the eigenfunctions were normalized, so that the horizontal velocity is unity at the surface for each mode. The horizontal wavelengths, the horizontal phase speed, $C_p = \omega/k$, and the horizontal group speed, $C_g = \left[C_p \frac{k^2}{\omega^2 - f^2} \right]^{-1}$, are then found from the horizontal wave number *k* [12].

Figure 5 shows the vertical structure of the horizontal velocity for the first three modes for *M₂* tidal frequency. The buoyancy frequency profiles were computed from the hydrographic data collected on 13 May 1984 and 3 November 1986. There is no buoyancy frequency profile available for the March mooring.

It can be seen that the observed baroclinic tidal ellipses at different depths for May and November moorings are consistent with the first mode structure. Features such as the vertical position of the minimum in horizontal velocity and 180° shift in their orientation across that minimum compare favourably with the observations. The current data for *M₂* ellipse show minimum magnitudes around 40 m. They also depict opposite flows on either side of the minimum. The first mode structure also predicts a zero crossing in horizontal velocity to be at 42 m of water depth for May mooring and at 30 m of water depth for November mooring. It should be remembered when making these comparisons that the hydrographic

data with which the dynamical modes are calculated are a snapshot in time, whereas the tidal current ellipses represent an average over 8 - 14 days.

The horizontal phase speed, group speed, wave number and wave length predicted for M_2 baroclinic modes for the May and November moorings are listed in Table 4. These were calculated according to Eq. (1) using the eigenvalues found by assuming a flat bottom locally. The group speed for the first baroclinic mode

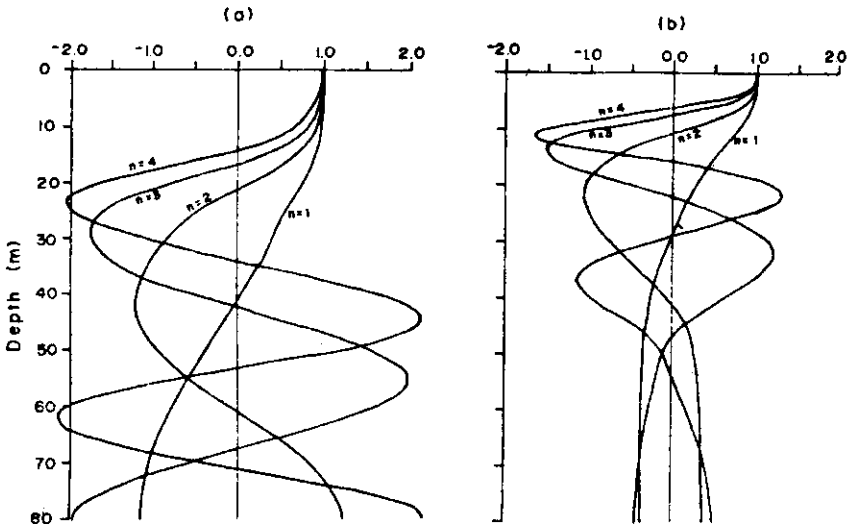


Fig. 5 — The first three baroclinic dynamical mode horizontal velocity structures calculated for M_2 tide during (a) May 1984 and (b) November 1986

Table 4 — Horizontal Wavelength (L), Phase Speed (C_p) and Group Speed (C_g) for M_2 Tide

Mode no.	L (km)	C_p (cm/s)	C_g (cm/s)
<i>May 1984</i>			
1	23.74	53.09	49.12
2	12.00	26.84	24.89
3	7.77	17.38	16.11
<i>Nov. 1986</i>			
1	20.06	44.86	41.61
2	9.40	21.02	19.50
3	5.65	12.64	11.76

is about 0.45 m/s, which means that energy takes approximately two and a half days to cross the shelf having a width of about 100 km. The horizontal wavelength predicted over the shelf in this case is about 22 km.

Regions of generation of internal tides — Baroclinic/internal tides are believed to be generated by the interaction of barotropic tides with bottom topography and/or the density stratified water column, and, modified by the influence of the earth's rotation. The most favourable regions for the generation of internal tides are where the slope of the bottom equals the slope of the internal-wave characteristics [2]. For continental shelf regions, the generation process is generally strongest at or near the shelf break, where the generating force is largest [1]. The slope of the internal-wave characteristics is defined as

$$c^2 = \left[\frac{\omega^2 - f^2}{N^2 - \omega^2} \right] \quad \dots(2)$$

The bottom slope (α) and the slope of the characteristics (c) near the bottom over the shelf and adjoining region are given in Table 5. The slopes of the M₂ characteristics were computed from the hydrographic data collected at stations across (Fig. 1) the shelf and slope region prior to laying of the mooring in May and November. It is seen from Table 5 that during May 1984, the bottom slope switches from being supercritical ($\alpha > c$) to subcritical ($\alpha < c$) in the shelf break region (between 73° N and 72° 45'N). Thus, the shelf break region could be identified as a probable region for the generation of M₂ internal tides. During November, stratification over the shelf and slope region differs from the May stratification (Fig. 2). During November, the bottom slope appears to be subcritical all over the shelf and slope region, suggesting no well defined region of generation. At the shelf break (200 m), the bottom slope is 1.25×10^{-2} and c is 1.61×10^{-2} . The steepest slope is at about 800 m depth (3.27×10^{-2}), but this is the region where the characteristics are also steeper (5.22×10^{-2}). Thus, it is quite

Table 5 — Bottom Slopes (α) and Slopes of the M₂ Internal Tide Characteristic (c) during May 1984 and November 1986

Location	α	c	
		May 1984	Nov. 1986
73°30'	1.09×10^{-3}	6.68×10^{-4}	3.95×10^{-3}
73°15'	2.46×10^{-3}	1.08×10^{-3}	4.40×10^{-2}
73°00'	1.25×10^{-2}	8.73×10^{-3}	1.61×10^{-2}
72°45'	3.27×10^{-2}	7.97×10^{-2}	5.22×10^{-2}
72°30'	2.13×10^{-2}	1.22×10^{-1}	4.84×10^{-2}

possible that generation takes place over a very large section of the bottom unlike from a narrow region during May.

DISCUSSION

In this paper, we have discussed the M_2 tidal currents on the shelf off the west coast of India. The observations show that the tidal currents in this region are constituted by both barotropic and baroclinic currents. The M_2 tidal currents vary from 3.7 cm/s to 15.3 cm/s. The barotropic M_2 currents are directed onshore with a magnitude of 4.0 - 7.0 cm/s. As observed elsewhere, over the continental shelves [9,12], the internal tidal currents here also have similar or greater magnitudes (1.2-11.8 cm/s) than barotropic tidal currents. In the northern hemisphere, the currents due to internal waves/tides are expected to rotate in clockwise direction. From Table 3 it appears that all the baroclinic ellipses follow this rule as they rotate clockwise. For a two-dimensional internal wave [8], the absolute value of the ratio of the minor axis to the major axis equals the ratio f/ω . For the M_2 tide in this region, the value of f/ω is 0.27. The values of minor axis to major axis ratios for M_2 tide are approximately 0.09, 0.10 and 0.48 for May, March and November moorings. The greater deviations of these ratios from the f/ω ratio imply a three-dimensional structure for the internal waves in this region.

As in other continental shelves, the amplitude of the internal tide shows great temporal variability, the larger internal tidal currents being associated with a stronger stratification. The deep stratification over the slope is relatively constant during May and November moorings, but changes in the upper 300 m over the slope (Fig. 2) may be responsible for the variability in the internal tides over the shelf, as suggested by Baines [2].

The baroclinic ellipses were compared with the first baroclinic mode calculated from actual N^2 profile by assuming a flat bottom. The M_2 currents calculated separately for each current meter have a vertical structure consistent with the first baroclinic mode. In general, on the shelves, the baroclinic tidal energy is observed to be concentrated in the lowest mode [3]. The first M_2 mode has a horizontal wavelength of approximately 22 km. During May mooring, the shelf switches from being supercritical to subcritical in the shelf break region, suggesting the break region as the probable source region for M_2 internal tides. During November, the shelf and slope regions are subcritical with respect to the internal waves of M_2 frequency and the generation of internal tides may take place over a vast region of the slope. The May and November σ_t structures being representative of two different seasons which control the coastal currents along the west coast of India, it may be concluded that there is always sufficient stratification over the shelf and slope regions to generate the internal tides.

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