# Preliminary results of an acoustic tomography experiment (ATE-93) in the eastern Arabian Sea

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An acoustic transmission experiment was conducted in the eastern Arabian Sea by deploying two transceiver systems on deep sea moorings separated by 270 km. The experiment lasted for a duration of ten days (2-12 May 1993). The acoustic transmissions took place at hourly intervals in both forward and reciprocal directions. The reception time of TR2 was selected to match the transmission time of TR1 at 00 minutes of every hour, and vice versa at 30 minutes of each hour. The transmitted signal consisted of 511 digit phase coded shift register sequences. From the measured acoustic multipath arrivals, significant peaks were identified. This information was utilised to derive the layer-wise sound speed perturbation field following generalized inverse SVD. Results from preliminary analysis showed that inversion with six energetic modes and eight eigen rays can adequately reconstruct the perturbation in six pre-set layers.

## 1. INTRODUCTION

During the summer of 1993, an acoustic transmission experiment (ATE-93) was conducted in the eastern Arabian Sea by deploying two tomographic transceiver systems, separated over 270 km (Figure 1), for a duration of ten days (2-12 May). The major objectives of the experiment were 1) to map the time evolution of the sound speed anomaly field (in vertical) from travel time perturbations using the tomographic technique and 2) to investigate the general characteristics of acoustic propagation. In this paper we present the preliminary results obtained from the experiment and show that the generalized inverse technique adequately reconstructs the layer averaged sound speed structure.

#### 2. EXPERIMENTAL CONFIGURATION

During cruise 83 of ORV Saga Kanya, two transceivers TR1 and TR2 were moored at depths of 1535 m and 1685 m respectively above the sound channel axis (Figure 1). The experimental site along 12.5° N lat. was so chosen that its bathometry was near uniform (~ 4100m). The transceivers had a central frequency of 400 Hz with 100 Hz bandwidth. The acoustic transmission took place every hour while reception every half hour for TR1 and vice versa for TR2. The transmitted signal consisted of 511 digit phase coded shift register sequences. The experiment started on Julian day 122 (2 May 1993) at 2000 hrs (Indian Standard Time) when TR1 transmitted and TR2 received. In order to keep a track of the mooring drift, tilt meters were attached near transceivers on both the moorings. The computed drifts were of the order of 100 m over the experimental duration. While

the transmissions were going on a CTD survey was also carried out covering a total number of 16 stations (Figure 1). The sound speed profiles at TR1 and TR2 broadly resembled one another with the SOFAR channel axis situated at about 1800 m.

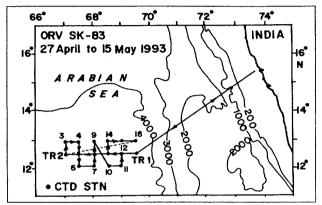


Figure. 1. Location map of transceivers and CTD stations

# 3. SIGNAL PROCESSING

As the source used was peakpower limited, a phase coded signal was transmitted and suitably processed at the receiver to increase the signal to noise ratio. The transmitted signal consisted of 3, 17, 9 and 12 repetitions of a phase modulated code of 5.11 s duration lasting for 15.33, 86.87, 45.99 and 61,32 s respectively.

The received signal is preamplified using an in-built automatic gain controller (agc), complex demodulated, low-pass filtered, sampled and coherently averaged [1]. These demodulates consist of in-phase and out-phase components of the coherently averaged receptions.

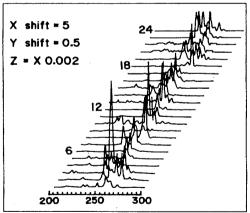


Figure 2. Waterfall plot of TR1 - DAY 4 reception - No. of sequences 16

The signals thus obtained were cross-correlated with the replica of the source signal (pulse compression) during the post processing to generate amplitude peaks corresponding to multipath arrivals. Figure 2 shows the typical waterfall plot of the intensity versus travel time generated using 24 processed transmissions.

### 4. ACOUSTIC RAY TRACING

The temperature and salinities from the historical hydrographic data in the study region were range averaged to construct a climatological sound speed profile using Chen and Millero [2]. this profile was used for acoustic ray tracing (Figure 3) using a range-independent program [3] to obtain the predicted arrivals. Ray tracing was carried out assuming a flat bottom 4100 m for emergence angles between +12° and -12° at 0.1° intervals, keeping the source at 1535 m (TR1) and the receiver at 1685 m (TR2) for the exact range of 270.92 km. Having obtained the predicted arrivals, the next step is to match the measured travel times with the predicted ones to obtain the ray travel time perturbations for use in the inverse analysis. Towards this, we chose the measured ray travel time at TR2 for Day 1, hour 1 reception.

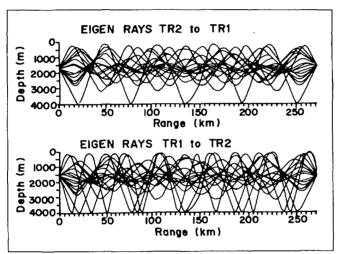


Figure 3. Eigen rays from TR1 to TR2 and vice versa for a range of 270.92 km

# 5. TRAVEL TIME INVERSION

In order to perform inversion of travel time data, each measured arrival must be associated with a particular ray path corresponding to the predicted arrival. This identification is achieved by comparing the simulated and predicted arrivals in time domain (Figure 4).

The inversion procedure used here is based on singular value decomposition (SVD). Following Chester et al. [4] the perturbation in travel time could be written as

$$\delta T_i = -\int \frac{\delta C(x,t)}{C_0^2(x,t_o)} ds \tag{1}$$

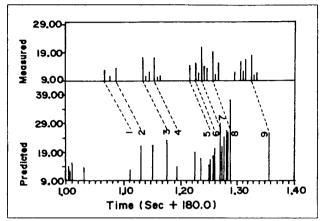


Figure 4. Measured and predicted arrival pattern

The model equation could be rewritten as

$$\left[\frac{-R_{ij}}{C_o^2}\right]\delta C_j = \delta T_i \tag{2}$$

After parameterising the model, the above equation can be expressed in matrix-vector notation as

$$\delta T = A \delta C \tag{3}$$

where  $\delta T$  is the travel time differences between the measurements and those obtained by the ray model;

 $A = -R_{ij}/C_o^2$  where  $R_{ij}$  is the path length of ray i in layer j and  $C_o$  is the reference sound speed. The travel time perturbation is related to sound speed perturbation through matrix A.

In the following analysis, the construction of a generalized inverse operator using SVD and employing the eigen value technique has been considered [5]. According to Jackson [6], the natural inverse operator  $A_{\rm g}^{-1}$  always exists and is given by

$$A_g^{-1} = V \Gamma^{-1} U^T \tag{4}$$

where matrices V and U are obtained by the following coupled eigen value problem

$$AA^TU_j = U_j (5)$$

$$A^T A V_i = V_i \tag{6}$$

In the above equations  $U_j$  and  $V_j$ , the column vectors of matrices U and V, are eigen values of matrix A arranged in decreasing order. The eigen vectors corresponding to the largest eigen value indicate that large scale factors can best be determined. The selection of number of eigne values is decided by the closeness ratio approach. The solution of equation (4) is given by

$$\delta C = [V\Gamma^{-1}U^T]\delta T \tag{7}$$

Calculations of the model resolution matrix and data resolution matrix are needed for the assessment of the resulting sound speed model.

# 5.1 Model Resolution

The model resolution of generalized inverse (5) is given by  $R = A_p^{-1} A = V_p V_p^T$  here, p indicates the number of factors used in SVD which is less than or equal to rank of the matrix A. The model parameters will be perfectly resolved if  $V_p$  spans the complete space of the model parameter i.e.  $V_p V_p^T = I$ .

# 5.2 Data Resolution

The data resolution matrix is given by  $N = A A_g^{-1} = U_p U_p^T$ 

The data are perfectly resolved if  $U_p$  spans the complete space of data. This usually tells the convergence of rays in the given domain. The SVD provides a simple framework for determining how well the inverted model parameters fit the data and how closely the model parameter estimates the true values.

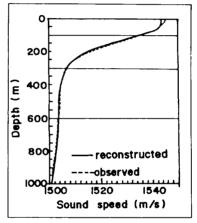


Figure 5. Reconstructed profile

The larger eigen values determine the large scale features of the problem while smaller eigen values correspond to small scale or higher frequency features.

Table 1. Ray Identifier

	Launch angle θ	Total turning + N	Upper, lower turnings (p, -p)	Total travel time T (s)	
1	8.10	9	(5,-4)		
2	6.87	8	(4,-4)	181.2587	
3	5.39	8	(4,-4)	181.2166	
4	4.30	9	(5,-4)	181.2865	
5	2.13	13 11 (6,-5)		181.3595	
6	-1.56	12	(6,-6)	181.3763	
7	-3.14	11	(6,-5)	181.3494	
8	-5.30 9		(5,-4)	181.2357	

# 6. RESULTS

For a number of cases using different layer depths, inversions were carried out with cight eigen rays (Table 1). From these numerical experiments, it was found that the inversion carried out using six layers with layer depths 100-300, 300-600, 600-1000, 1000-1500, 1500-2000, 2000-3500 give the best result (Table 2), especially in the upper 1000 m depth (Figure 5). Note that the upper 100 m and the bottom 600 m, i.e. 3500-4100 m, is not accounted in the computations as the identified rays do not scan these depth layers. The departures in the lower layers are the artefacts of over sampling of the same layers by rays leading to over determinancy.

Table 2. Layer-wise model and reconstructed sound speed perturbations

Laye	r dep	oth (m)	Sound speed perturbation (m.s <sup>-1</sup> ) obtained				
			from CTD	through inversion			
100	_	300	+0.818	+0.714			
300	-	600	+0.864	+0.156			
600	-	1000	+0.906	+0.920			
1000	_	1500	+0.805	+ 1.054			
1500	_	2000	-0.401	+0.760			
2000	_	3500	-0.387	+1.462			

						.110	.237	.483	.486	.775	1.000
						.135	.380	.390	.906	.907	. 927
.000	.004	.024	.024	.151	1.00	.144	.342	.344	.395	.397	.398
.004	.065	.425	.425	.969	1.00	.146	.170	.173	.244	-410	.907
.060	.469	.882	.910	.976	1.00	.132	.162	.225	.512	.614	.801
.253	.454	.527	.891	.975	1.00	101.	.140	.489	.490	.928	1.000
.158	.376	.377	.977	.991	1.00	.116	.221	.419	.462	.463	.464
.524	.632	.765	.770	.938	1.00	-115	.250	.478	.505	-506	.507

Figure 6a. Model resolution

Figure 6b. Data resolution

For the assessment of the validity of the layering of the model used for inversion, it is essential to look at model and data resolution matrices.

Model parameter resolution is an indication of how perfectly and independently each model parameter is determined. The diagonal elements of the model parameter resolution matrix (Figure 6a) give a clue as to how well the individual model parameters are resolved. A value of unity indicates a perfectly resolved parameter whereas a smaller value means inadequate resolution. It can be seen that eight eigen rays and six layers with six

eigen modes lead to a perfect resolution with unity diagonal elements.

The data resolution matrix is an indication of the information density of the data kernel, i.e. it indicates which data contribute independent information to the solution. The diagonal elements of the data resolution matrix (Figure 6b) show unity in the case of boxes 1 and 6, indicating that the contribution of information by steep and near axial rays (8.1° and -1.56° launch angle) are exclusive. The low values observed in boxes 3, 7 and 8 indicate poor information resolution associated with 3° to 5° rays which scan nearly the same region of the water column.

Analysis of data from all 480 transmissions is in progress which will lead to the reconstruction of the time evolution of the sound speed anomaly in the vertical (2-D) using a box model.

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