

## Response of surface buoy moorings in steady and wave flows

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**Abstract** - A numerical model has been developed to evaluate the dynamics of surface buoy mooring systems under wave and current loading. Method of solution proposed by Berteaux and Chhabra for static analysis has been modified to account for dynamic analysis in the frequency domain considering the loads on the mooring configuration through the spectral approach. System tension response and variation of tension in the mooring line at various depths have been evaluated for deep water as well as for shallow water moorings having three different damping ratios equal to 0.1, 0.2 & 0.3. In all cases considered, as the damping ratio increases the mean square tension response decreases. Tension response spectra of deep water mooring seem to show a narrow band behaviour for all the three damping ratios considered. Where as in the case of shallow water mooring tension response of the system which is narrow band for  $\xi = 0.1$  found to change to a broad band one for higher values of  $\xi = 0.2$  &  $0.3$ .

### INTRODUCTION

Mooring system is an integral part of a floating off-shore or coastal structure, a navigational buoy, a S.P.M or a deep sea met-ocean data buoy system. Mooring lines are defined as flexible mechanical members that connect and attach the floating structure to its anchoring points. Even for a simple moored system which consists of one float, one line and one anchor and possibly some ancillary equipment, the analytical solutions are surprisingly numerous and complex. As in other engineering problems, the validity of their use depends on the degree of correspondence between the real condition and stated assumptions. In this respect mooring lines have a number of weighing factors like length, stretch, weight to drag ratio etc, that must be taken into account while selecting a method of solution.

Surface buoy moorings play a very important role in the environmental data collection programme. Surface buoys can be surface following (disks or skoffs) or surface decoupled (spar) buoys. Surface following buoys have the advantage of a large buoyancy to drag ratio. However the heave and roll motions of these buoys impart severe dynamic stresses to the line, reduces the efficiency of the radio transmission and may severely limit the threshold of scientific measurements made from the moving platform. When mooring cables are used to support oceanographic sensors, the quality of data may be affected by the energy transmitted down the line from surface waves. The wave excitation at the buoy may cause dynamic response of mooring cables and sensors attached to them. Degree of success in any environmental data collection programme depends very much on the integrity of the mooring system. Proper evaluation of environmental loading which a moored system experiences during its operational period should be made while designing the mooring system.

### THE MODEL

The numerical model developed for the analysis of single point surface buoy mooring considers the dynamic effects due to waves in addition to static loads from steady currents. The static analysis procedure proposed for a single point surface buoy

mooring by BERTEAUX and CHHABRA (1973) has been expanded to account for the dynamic effects due to waves by the spectral approach. Main assumptions made are :

- \* Mooring line is considered as a slender structure with negligible moments and shear forces.

- \* The force considered in the cable is only the tension, which is acting tangential to the cable direction.

- \* The instruments are rigid and do not stretch under tension.

- \* Flow normal to the cable is independent of flow tangential to the cable, so that velocity can be resolved into components one normal and the other tangential to the cable.

- \* The current velocity is assumed to be constant over the length of the segment.

- \* In spectral approach the sea state is assumed to be zero mean Gaussian . -

- \* The whole structure is idealised as a single degree Maxwell model with motion restricted to longitudinal axis i.e, heave motion only.

- \* The spring constant for the whole structure is approximated as

$$1/k = 1/k_1 + 1/k_2 + 1/k_3 + \dots \text{etc.}$$

Where  $k_1, k_2, k_3$  etc are spring constants for different mooring line segments and  $K$  is the overall stiffness for the whole mooring line.

- \* Dynamic force in the mooring line due to horizontal movement of the buoy under the action of the fluctuating wind and waves is assumed to be negligible compared with that due to vertical motion of the buoy.

- \* Stress due to strumming is neglected.

- \* The damping ratio is considered as a parameter for studying the effect on the dynamic behaviour of the system.

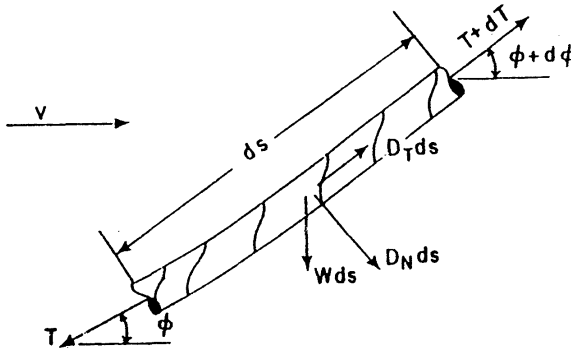


Fig. 1. Uniformly loaded cable

### Equilibrium equations

Mooring line is considered as a uniformly loaded cable (Fig. 1). Resolving the forces normal and tangential to the cable element the cable differential equations can be written as

$$d\phi = (N/T) ds \quad (1)$$

$$dT = P ds \quad (2)$$

where,

$P$  = sum of the forces parallel to segment 'n',  $\phi$  = angle made by the cable with flow direction,  $T$  = tension in the cable element of length  $ds$ ,  $dT$  = change of tension over the cable element,  $N$  = sum of the forces normal to segment 'n'

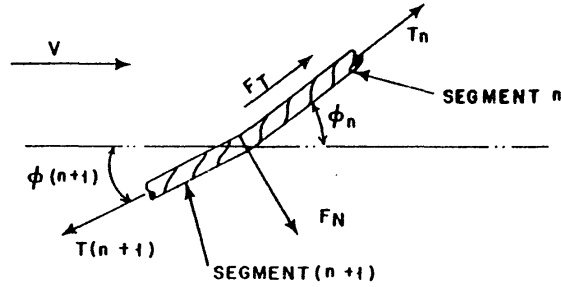


Fig. 2. Discretely loaded cable

When discrete forces are applied to the mooring line (Fig. 2) different equilibrium conditions are to be considered. In such cases the conditions of equilibrium are

$$\phi_{(n+1)} = \tan^{-1} \frac{(T_n + F_t) \sin \phi_n - F_n \cos \phi_n}{(T_n + F_t) \cos \phi_n + F_n \sin \phi_n} \quad (3)$$

$$T_{(n+1)} = \frac{(T_n + F_t) \cos \phi_n + F_n \sin \phi_n}{\cos \phi_{(n+1)}} \quad (4)$$

where,

$F_n$  = normal component of discrete force,  $F_t$  = tangential component of discrete force,  $T_n$  = tension in segment 'n' and  $\phi_n$  = inclination of segment 'n'

The equilibrium conditions at the surface buoy are,

$$T \sin \phi = B_B - W_B$$

$$T \cos \phi = D_{AB} + D_{WB}$$

where,

$T$  = tension at the buoy,  $\phi$  = inclination of the line with respect to horizontal,  $B_B$  = buoyancy of the buoy,  $W_B$  = weight of the buoy,  $D_{AB}$  = drag force due to air and  $D_{WB}$  = drag force due to water.

In a realistic study of mooring line statics, elastic response of different materials of the mooring line should also be included. Detailed description on the above has been given by BERTEAUX and CHHABRA (1973).

#### Computation procedure

Numerical integration of the cable differential equations are made using finite element method. Certain initial conditions are assumed prior to the start of the computation process. The results obtained after the first iteration are compared with known

physical constraints and the computation errors arrived at are used to correct the assumptions. The iteration process is then repeated until the errors are minimised.

### Static analysis

In the static analysis static displacements of the mooring line configuration and variation of mooring line tension at various depths due to wave and current loading are evaluated. In order to numerically integrate the differential equations, the entire cable profile is divided into several segments of finite length. The changes of tension and inclination over the length of the segment 'n' can be determined by

$$\delta T_n = P_n \quad (5)$$

$$\delta \phi_n = -N_n/T_n \quad (6)$$

Where,

$P_n$  = resultant of forces parallel to the segment 'n',  $N_n$  = resultant of forces perpendicular to segment 'n' and  $T_n$  = tension in the segment 'n'

Tension in the segment 'n+1' and angle of segment 'n+1' are given by

$$T_{n+1} = T_n + \delta T_n \quad (7)$$

$$\phi_{n+1} = \phi_n + \delta \phi_n \quad (8)$$

The elastic characteristics of the segment 'n+1' and the tension  $T_{n+1}$  are then used to compute the stretched length of the segment. This stretched length and the angle  $\phi_{(n+1)}$  are used to compute the forces in segment 'n+1', the change of tension and inclination resulting from the application of these forces and the location of end point of the segment. At places where discrete forces are applied on the mooring line equations (3) and (4) are used to compute the tension and inclination of the following segment. This procedure is repeated for all segments. At the end of the first computation the depth reached by the last segment will most probably not coincide with the actual water depth. The tension initially assumed in the first segment must then be corrected and the computation procedure is resumed. A new depth error and new correction will result from the second computation. The process is repeated until the depth error lies in the prescribed limits.

### Dynamic analysis

In the dynamic analysis amplification in the mooring line tension due to heave response of the buoy at various depths under random wave loading have been evaluated. Pierson-Moskowitz spectra was used for representing the sea state. From the vertical velocity and acceleration spectra the load spectra is computed by linearising the drag term (BORGMAN, 1969).

For a single degree freedom structure, the equation of motion is,

$$M d^2x/dt^2 + C dx/dt + K X = F(t) \quad (9)$$

Since the forcing function  $F(t)$  can be written as sum of infinite random forces

$$M d^2x/dt^2 + C dx/dt + K X = F_0 e^{i\omega t} \quad (10)$$

where,  $M$  and  $K$  are the mass and stiffness of the system and  $C$  the damping force.

$$M d^2x/dt^2 + 2 \sqrt{MK} \xi dx/dt + K X = F_0 e^{i\omega t} \quad (11)$$

where,  $\xi = C/C_c$  and  $C_c$  is the critical damping.

The solution of equation (10) is  $X = X_0 e^{(i\omega t - \theta)}$

where  $\theta$  is the phase difference between input and output quantities.

$$X = i\omega X_0 e^{i(\omega t - \theta)} = i\omega X \quad (12)$$

$$\text{and } X = -\omega^2 e^{i(\omega t - \theta)} = -\omega^2 X \quad (13)$$

$$ie, M (-iw^2X) + C (iwX) + K X = F(t) \quad (14)$$

and 
$$X = \frac{F(t)}{K - Mw^2 + iCw} \quad (15)$$

$$X(t) = H(iw) F(t) \text{ where } H(iw) = \frac{1}{K - Mw^2 + iCw} \text{ is known}$$

as the complex transfer function or receptance function.

The tension response spectra is then computed as, 
$$S_{XX}(w) = H(iw)^2 S_{FF}(w) \quad (17)$$

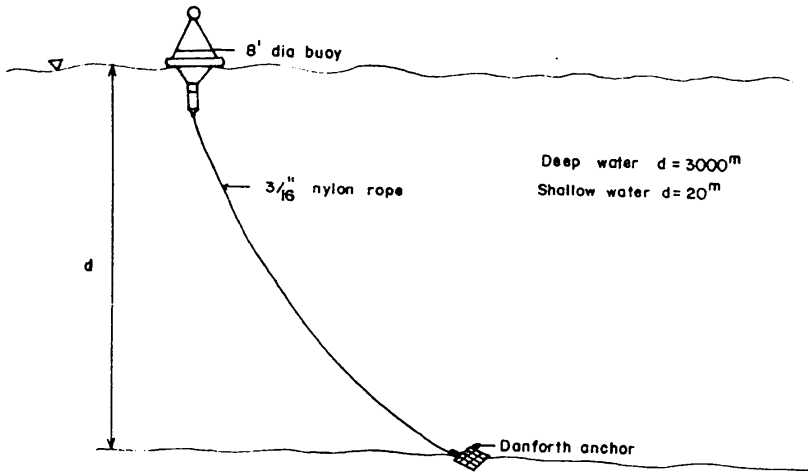


Fig. 3. Surface buoy mooring

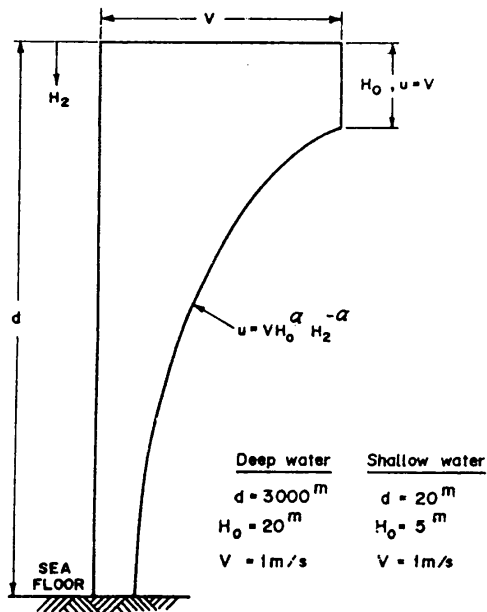


Fig. 4. Current profile

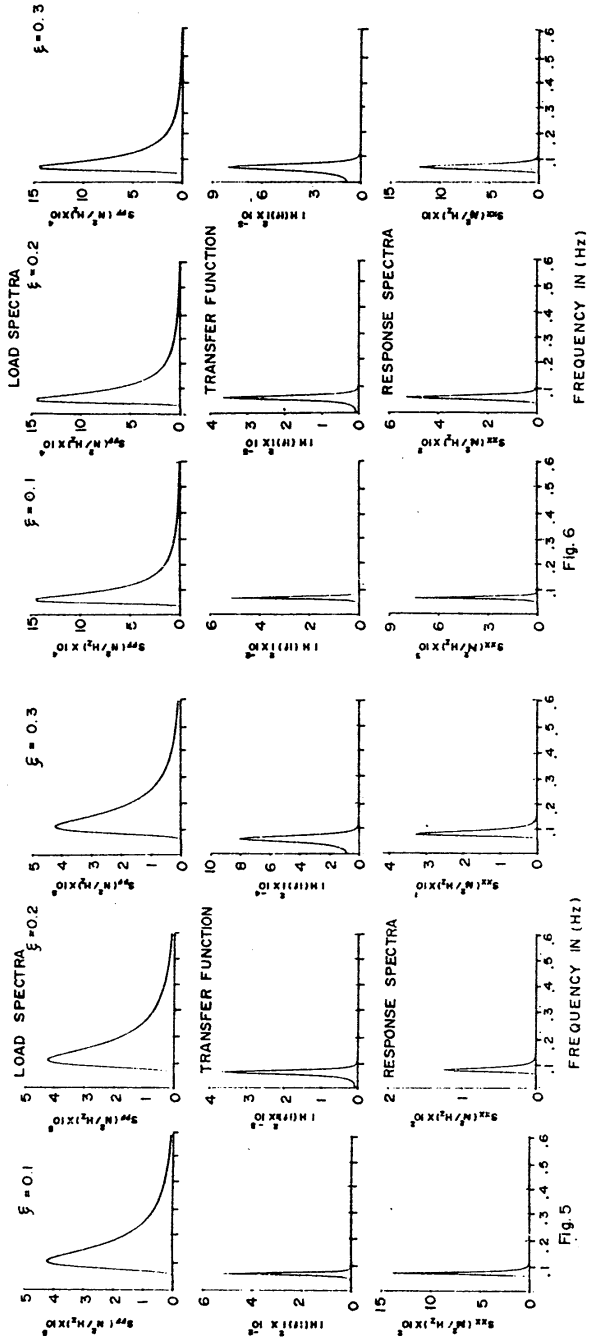
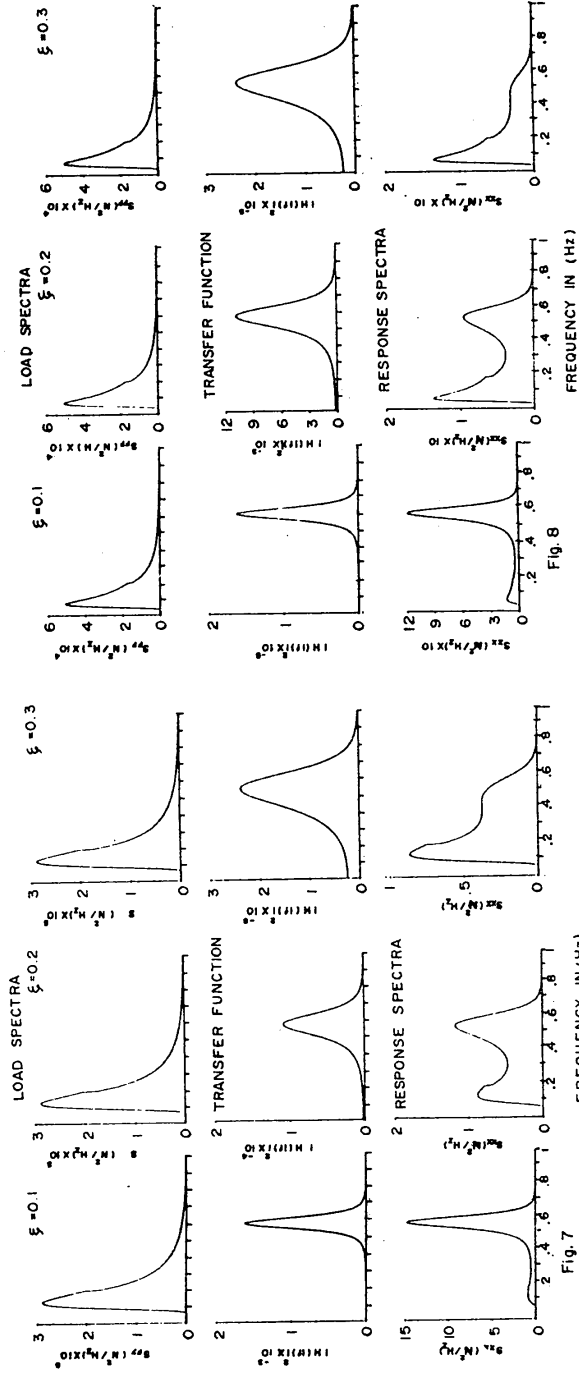


Fig. 5

Fig. 6

Figs. 5 & 6. Load spectra, transfer function & response spectra for deep water mooring when  $H_s = 2m$  and  $T_p = 8$  and 15 seconds respectively.



Figs. 7 & 8. Load spectra, transfer function & response spectra for shallow water mooring when  $H_s = 2m$  and  $T_p = 8$  and 15 seconds respectively.

Mean square tension response and the design tension in the mooring line at the buoy connection are then obtained for different damping ratios considered. This value is added to the static tension and by the iteration procedure discussed earlier tension at various levels are evaluated.

#### *Test mooring and environmental conditions*

Two mooring configurations, one for deep water (3000m) and one for shallow water (20m) have been tested (Fig. 3). Mooring line is a nylon rope of 16 mm diameter having a rated breaking strength 4.7 tons. Diameter of the surface buoy is 2.45m. Damping ratios 0.1, 0.2 and 0.3 have been used for studying the effect of damping on dynamic behaviour of the tension in the mooring system. Current profile used has a surface current velocity of 1m/sec and is constant upto a depth of 20m in the case of deep water and 5m for shallow water (Fig. 4). Pierson-Moskowitz spectra considering a significant wave height of 2m and peak periods 8sec and 15sec have been adopted for modelling the sea states.

## RESULTS AND DISCUSSION

#### *Deep water mooring*

The excursion of the mooring line and tension at various depths are given in Table 1. Tension values along the length of the mooring line under static loading conditions found to increase when subjected to dynamic loads due to waves. Maximum value of tension found to occur at the surface buoy level and decrease gradually down to anchor point.

Table 1. Deep water mooring

Seg No.	Slack length (m)	Depth (m)	Excursion (m)	Tension (kg)	Velocity (cm/sec)
Buoy	---	1.20	4458.20	614.06	100.00
1	168.00	220.48	4448.01	614.06	45.60
2	336.00	373.68	4289.26	598.84	28.90
3	504.00	522.00	4126.04	593.52	23.93
4	672.00	666.43	3959.44	588.32	21.01
5	840.00	807.42	3790.01	583.23	19.03
6	1008.00	945.21	3618.05	578.22	17.57
7	1176.00	1079.94	3443.76	573.32	16.44
8	1344.00	1211.71	3267.30	568.53	15.53
9	1512.00	1340.57	3088.77	563.82	14.78
10	1680.00	1466.54	2908.27	559.22	14.15
11	1848.00	1589.63	2725.87	554.71	13.61
12	2016.00	1709.85	2541.63	550.31	13.14
13	2184.00	1827.18	2355.60	546.00	12.72
14	2352.00	1941.62	2167.84	541.79	12.36
15	2520.00	2053.12	1978.39	537.69	12.03
16	2688.00	2161.68	1787.30	533.69	11.74
17	2856.00	2267.25	1594.60	529.80	11.47
18	3024.00	2369.81	1400.34	526.00	11.23
19	3192.00	2469.31	1204.55	522.32	11.01
20	3360.00	2565.73	1007.27	518.75	10.81
21	3528.00	2659.01	808.55	515.29	10.63
22	3696.00	2749.13	608.42	511.94	10.47
23	3864.00	2836.04	406.93	508.70	10.31
24	4032.00	2919.70	404.11	505.58	10.17
25	4200.00	3000.00	0.00	502.58	10.04

The mean square value of the tension response found to decrease as the damping ratio increases. Figs. 5 and 6 show the load spectra, transfer function and tension response spectra for wave spectra having  $H_s$  equal to 2m and peak period  $T_p$  equal to 8Sec and 15sec respectively. The response for  $T_p$  equal to 8sec is comparatively



lower compared to  $T_p$  equal to 15sec. Response is maximum at loading frequency which is close to natural frequency of the system. This is also true for the case  $T_p$  equal to 15sec. For both peak periods, 8sec and 15sec the response spectra is of narrow band in nature. As the value of damping ratio increases from 0.1 to 0.3 the response was found to decrease. However the tension response spectra seem to show a narrow band behaviour for all the values of damping ratios considered.

### *Shallow water mooring*

Table 2. Shallow water mooring

Seg No.	Slack length (m)	Depth (m)	Excursion (m)	Tension (kg)	Velocity (cm/sec)
Buoy	---	1.20	26.10	100.58	100.00
1	2.80	3.39	25.33	100.58	100.00
2	5.60	6.42	23.61	97.86	100.00
3	8.40	8.96	21.23	96.20	82.04
4	11.20	11.16	18.53	95.71	72.50
5	14.00	13.12	15.65	95.51	66.50
6	16.80	14.88	12.65	95.43	62.28
7	19.60	16.50	9.57	95.40	59.09
8	22.40	17.99	6.42	95.39	56.58
9	25.20	19.37	3.23	95.42	54.54
10	28.00	20.66	0.00	95.45	52.83

The excursion of the mooring line and the tension at various depths are given in Table 2. The tension response of the system for  $T_p$  equal to 8sec is comparatively lower compared to  $T_p$  equal to 15sec, and is maximum at natural frequency of the system for damping ratio equal to 0.1. In shallow water the tension response spectra which is narrow band in nature for damping ratio equal to 0.1 seem to change to broad band for higher values of damping ratios equal to 0.2 & 0.3 and maximum response shifts from system natural frequency to loading frequency. Figs. 7 and 8 show the load spectra, transfer function and the tension response spectra for peak periods equal to 8sec and 15sec.

## CONCLUSIONS

From the above case studies the following conclusions are drawn.

- i) In all cases considered, as the damping ratio increases the mean square response of tension shows a decreasing trend.
- ii) Response spectra is narrow banded for all damping ratios considered in the case of deep water mooring.
- iii) In the case of the shallow water mooring as the damping ratio increases the response spectra changes to a broad band one and the maximum response shifts from natural frequency of the system to the loading frequency.

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