

## Comparative study on spreading function for directional wave spectra

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**Abstract-** The planning and design of all coastal and offshore installations call for an information on wave directionality. This can be accurately obtained through the knowledge of the directional wave spectrum which is commonly given as a product of one-dimensional wave energy  $S(f)$  and the directional spreading function  $D(f, \theta)$ . This paper reviews various spreading functions proposed in the past for estimating the directional wave energy and presents their application to the Indian wave condition. It is found that the directional spreading function  $\text{sech}^2(\beta\theta)$  estimates energy distribution reasonably well for our monsoon wave conditions.

### INTRODUCTION

The directional properties of ocean waves are of increasing interest to engineers involved in the design of offshore and coastal structures. Proper estimation of the directional spreading found to reduce design wave loads on structures when compared to that from unidirectional waves with the same height and period. (MALHOTRA and PENZEIN, 1970; MUNK, 1966; LONGUET-HIGGINS, 1962; BORGMAN, 1962; DEO, 1989; DEO and NARASIMHAN, 1988).

Directional wave energy spectra  $S(f, \theta)$  is simply expressed as the product of one-directional spectra  $S(f)$  and its angular spread function  $D(f, \theta)$ . As a primary step, it is important to visualise the spreading function 's' by parameterising wind input to wave and non-linear wave interaction terms. It is assumed that the sensitivity of 's' which governs the shape of the directional spectra of a given sea state, is proportional to  $U/c$  and  $f/f_p$  (where  $U$  is the wind speed;  $c$  is the wave celerity and  $f/f_p$  is the relative wave frequency).

In this paper a brief discussion on few important spreading functions proposed in the past has been made. A comparative study of directional spectra computed from theoretical spreading functions and the measured directional wave spectra has been made. Finally, a suitable spreading function which describes the measured directional spectra off Karwar, West Coast of India has been evaluated.

### DIRECTIONAL WAVE SPECTRUM

#### Theory

Sea surface elevation can be best represented by visualising the energy contribution to the spectrum from waves having different frequencies and directions. Pitch-roll buoy measures heave and 2 horizontally perpendicular slope signals, allowing estimation of  $S(f)$ ,  $A_1, B_1$  and  $A_2, B_2$  only through cross spectral calculations (Appendix). The directional information is contained in these harmonic coefficient. These are first converted into harmonic amplitude  $m_1$  and then mean direction  $\theta_0(f)$ , the spread  $\sigma_0(f)$  to compute 's' parameter. The standard method of analysis is described by LONGUET-HIGGINS *et al.* (1963) and CARTWRIGHT (1963). It requires the 2-D spectrum  $S(f, \theta)$  to be expressed

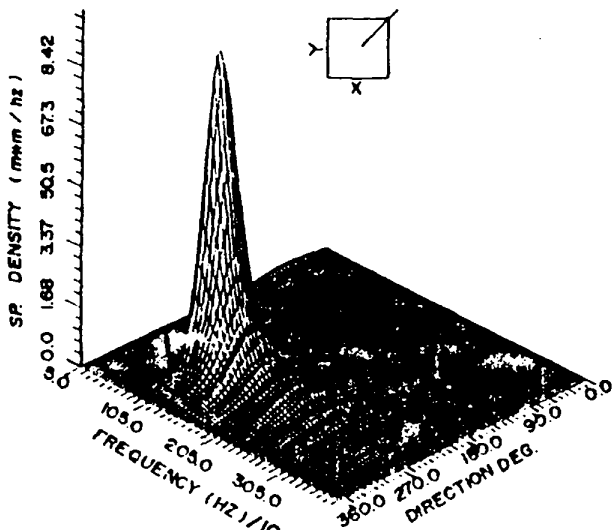


Fig. 1

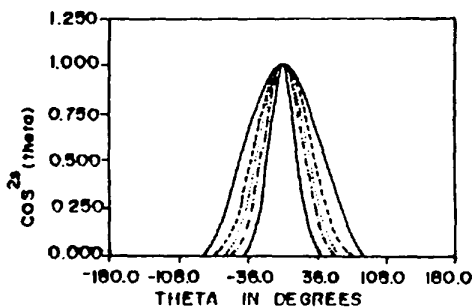


Fig. 2

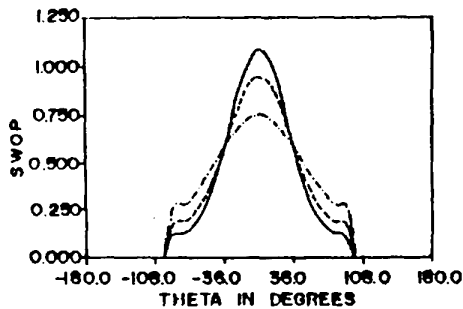


Fig. 3

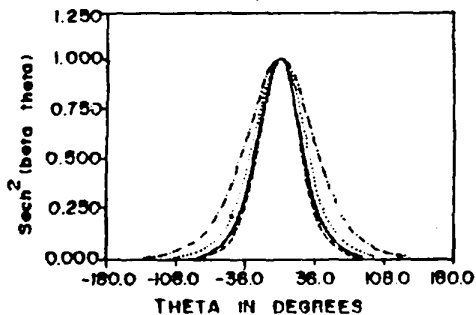


Fig. 4a

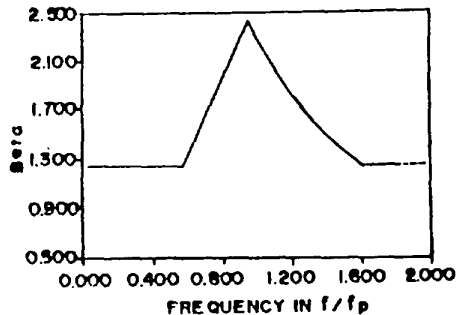


Fig. 4b

Fig.1. Directional spectra, Fig. 2. Directional distribution  $\cos^2(\theta)$  spreading, Fig.3. Directional distribution SWOP spreading, Fig.4a. Directional distribution  $\text{sech}^2(\beta\theta)$  spreading and Fig.4b. Variation of  $\beta$  with frequency ( $f/f_p$ )

as the product of its non-directional frequency spectrum  $S(f)$ , and its angular distribution  $D(f, \theta)$ , at each frequency  $f$  (Fig. 1).

$$S(f, \theta) = S(f) * D(f, \theta) \quad (2.1)$$

$$\text{where } \int_{-\pi}^{\pi} D(f, \theta) = 1. \quad (2.2)$$

The distribution is considered as sum of normalised angular harmonics at each frequency.

$$D(f, \theta) = 1/\pi ( 1/2 + \sum ( A_n(f) \cos(n\theta) + B_n(f) \sin (n\theta) ) \quad (2.3)$$

$$m_1^2 = A_1^2 + B_1^2 \quad (2.4)$$

$$\theta_o(f) = \arctan (B_1/A_1) \quad (2.5)$$

$$\sigma_o(f) = (2(1-m_1))^{1/2} \quad (2.6)$$

$$s_1 = m_1 / (1-m_1) \quad (2.7)$$

### Theoretical spreading functions

Basically two techniques of analysis are available. First method involves the reconstruction of the energy distribution and second involves the identification of distribution characteristics at each frequency. The first method although involves direct computation of harmonic components (Eqn. 2.1) from the buoy signals, convolution of the distribution suggested by LONGUET-HIGGINS *et al.* (1963) arbitrarily broadens the estimations. Second method involves parametric models which are based on a priori assumed shape of the directional energy distribution. Here, the number of independent shape parameters chosen should be less than four since only four Fourier coefficients are available (KUIK *et al.*, 1988).

There are quite a few directional spreading functions available for practical application.

### PNJ distribution

PIERSON (1955) proposed a cosine square type spreading function which is defined as

$$D(\theta) = (2/\pi) * \cos^{2s}(\theta - \theta_m) \quad \text{for } (\theta - \theta_m) < \pi/2$$

$$D(\theta) = 0 \quad \text{otherwise} \quad (2.8)$$

Figure 2 is the plot of variation of  $\cos^{2s}$  function against  $\theta$  where smaller 's' is used for near sea state condition and larger 's' is for swell. This offers a simple formulation and for this simplicity it has been accepted by many off-shore engineers (BORGMAN 1968).

### SWOP distribution

Based on the Stereo Wave Observation Project (SWOP), COTE *et al.* (1960) described the function as

$$D(f, \theta) = 1/\pi [ 1 + a \cos (2\theta) + b \cos (4\theta) ] \quad \text{for } \theta < \pi/2 \quad (2.9)$$

$$= 0 \quad \text{otherwise.}$$

where

$$a = 0.5 + 0.82 \exp [-0.5 (U * 2\pi f / g)^{**4}]$$

$$b = 0.32 \exp[-0.5 (U * 2\pi f / g)^{**4}]$$

U = wind speed at 10 m above the sea

g = gravitational acceleration 9.81 m/s<sup>2</sup>

$f$  = frequency (Hz)

Figure 3 is the functional variation of Equation 2.9 for various frequency ratios ( $f/f_p$ ) against  $\theta$ , dropping the constant .

*SECH<sup>2</sup>( $\theta$ ) distribution*

DONELAN *et al.* (1985) verifying the appropriateness of the  $\cos^2(\theta)$  spreading function, found that it consistently errs by overestimating the wave energy at peak in the direction of wave propagation. He proposed an alternative function on the assumption that the distribution of wave energy in the direction transverse to the main direction is proportional  $\text{sech}^2(\theta\beta)$ . Here  $\beta$  is to determine the energy spread.

$$D(f,\theta) = (\beta/2) * \text{sech}^2\{\beta(\theta-\theta_m)\} \tag{2.10}$$

$$\beta = 2.61 (f/f_p)^{1.3} \quad \text{for } 0.56 < f/f_p < 0.95$$

$$= 2.28 (f/f_p)^{-1.3} \quad \text{for } 0.95 < f/f_p < 1.60$$

$$= 1.24 \quad \text{otherwise}$$

Figure 4a indicates for various  $f/f_p$  values behaviour of  $\text{sech}^2(\beta\theta)$  as against  $\theta$ . Values of  $\beta$  are taken as indicated in Equation 2.10 and Figure 4b gives functional variation of  $\beta$  with respect to frequency ratio.

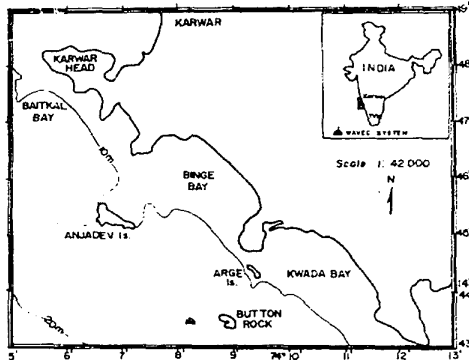


Fig. 5

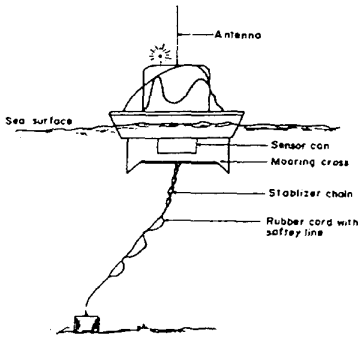


Fig. 6

Fig. 5. Location map, and  
Fig. 6. WAVEC buoy with mooring

## LOCATION, DATA COLLECTION AND ANALYSIS

The Datawell WAVEC buoy was deployed at a water depth of approximately 16m and about 8km away from the Karwar coast in the Arabian Sea (Fig. 5). Buoy has a diameter of 2.50m. A cylindrical can 0.50m diameter and 0.70m height, contains sensors, radio transmitter, other electronics and batteries. It protrudes through the bottom centre of buoy to reduce current induced tilt of the buoy and to protect the main sensor from freezing. To prevent capsizing, a foam body is placed on the top of the hull. This foam body contains radar reflector. The main sensor is a Hippy-120A which measures the heave, pitch and roll motions of the buoy in a coordinate system relative to the buoy hull. The heave is determined by double integration of vertical acceleration of the platform. Pitch and roll angles are determined by measuring the fluxes of two oscillating magnetic fields through a pick-up coil on the platform. These fields are generated by two orthogonal coil systems in a fixed position relative to the buoy hull. A three-dimensional fluxgate meter is used to measure the orientation of the hull fluxgate to magnetic north. Figure 6 shows the details of the buoy and mooring system.

Table 1. Directional wave parameters

Date & Time	Rec.	Sig. Wave Hs (m)	Tz (s)	Tp (s)	$\theta$ (deg)	U (m/s)	Wind dir.	U <sub>c</sub> /C <sub>p</sub>
29-06-1989 1900	164	2.61	6.5	10.0	257.4	10.3	270	0.204
2130	165	2.62	6.7	11.1	251.7	11.0	270	0.274
30-06-1989 0030	166	2.46	6.5	10.0	251.7	11.8	242	0.335
0330	167	2.91	6.6	10.0	254.5	9.8	255	0.240
0630	168	2.47	6.5	11.1	251.7	12.4	270	0.335
0930	169	2.60	6.7	10.0	254.5	11.3	273	0.270
1030	170	2.61	6.6	10.0	253.1	11.8	254	0.300
1400	171	2.64	6.8	10.0	250.3	9.8	264	0.290
1530	172	2.62	6.6	10.0	254.5	10.8	273	0.260
1630	173	2.65	6.4	11.1	251.7	9.7	260	0.270
1830	174	2.53	6.7	10.5	251.7	9.3	230	0.250
2130	175	2.68	6.7	11.7	251.7	10.3	261	0.280

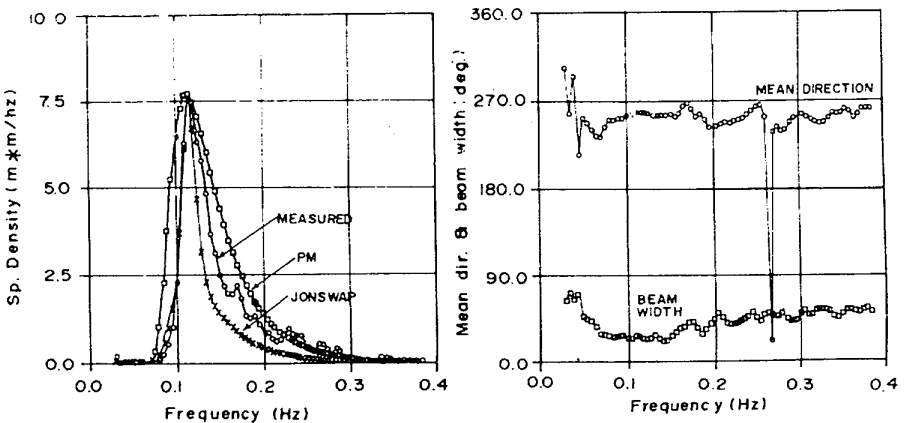


Fig. 7. Measured Rec. No.165 ( $H_s=2.62m$ ,  $T_p=8.7s$  wind speed 12.11m/s Direction - 270°N with JONSWAP & PM SPECTRA and Mean direction and Beam width.

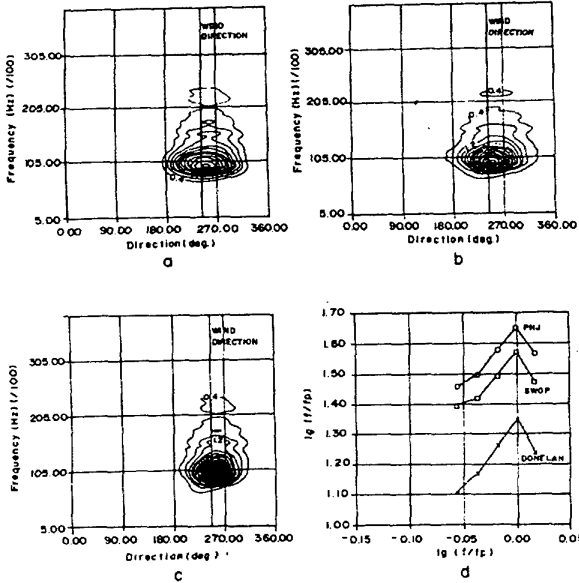


Fig. 8

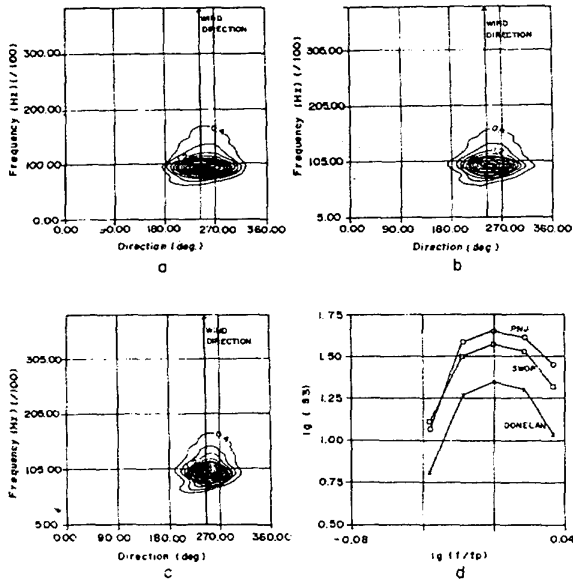


Fig. 9

Fig. 8a to 8c. Polar plots of PNJ, SWOP & DONELAN distribution (measured) respectively. Fig. 8d. Parameter  $s_3$  against  $\log_e(f/f_p)$  PNJ, SWOP & DONELAN models,  $T_p$  varies from 8 to 9.45 s. Fig. 9a to 9c. Polar plots of PNJ dist., JONSWAP, SWOP dist., JONSWAP & DONELAN dist. JONSWAP respectively and Fig. 9d. Parameter  $s_3$  against  $\log_e(f/f_p)$  PNJ, SWOP & DONELAN models,  $T_p$  varies from 8 to 9.45 s.

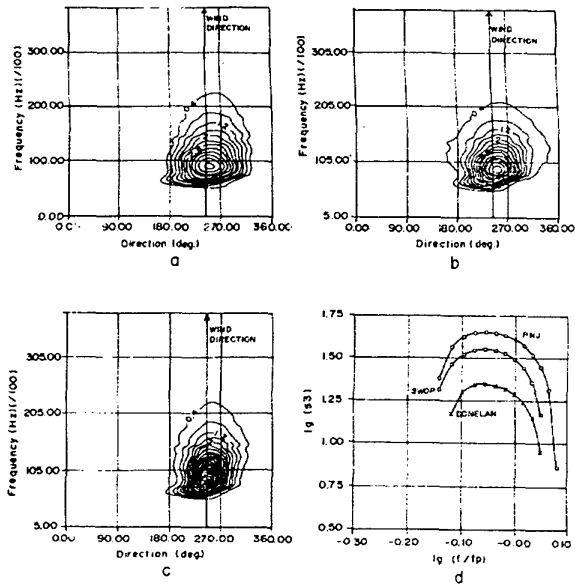


Fig. 10a to 10c. Polar plots of PNJ dist. PM, SWOP dist., and DONELAN dist, Fig. 10d. Parameter  $s_3$  against  $\log_e (f/f_p)$  PNJ, SWOP & DONELAN models,  $T_p$  varies from 8 to 9.45 s.

A 27 MHz radio link is used to send the data to a receiver which contains a microprocessor. This microprocessor performs real time analysis to determine directional wave characteristics. A desk-top computer is used to carry out on-line analysis and data storing. On-line analysis of six basic pitch-roll buoy signals is carried out for each record of 30 minutes to obtain information on the directional wave spectrum  $S(f, \theta)$ , along with mean direction, r.m.s. spread values of wave energy and 's' parameter of the distribution at each 0.005 Hz frequency. The timeseries is divided in to nine blocks of each 200s and FFT analysis is carried out with sampling interval of 1.28s and cut off frequency of 0.65 Hz ( BHAT *et al.*, 1989; KUIK *et al.*, 1988).

## RESULTS AND DISCUSSION

Monsoon period wave records are analysed for directional wave parameters and results are tabulated in Table 1. Present study consists of a discussion on the features of the frequency spectra and its comparison with JONSWAP and PM spectra. It also presents a discussion on characteristics of the directional spectra and modeling of the directional energy spread for a given sea state.

Many authors have discussed about the suitability of general spectra, its shape and location of peak for a given location (HASSELLMANN *et al.*, 1980.) Since the records analysed belong to monsoon period with developing sea state prevailing at the location, it is thought of comparing the spectral characteristics with JONSWAP and PM. Here we restrict our attention to only to the cases where there is no swell. In the general case of winds that are variable in speed and direction, a description of the parameters of spectrum in terms of local conditions would be valuable. Therefore,

it is thoughtful to relate spectral parameters to  $U_c/c_p$ ; where  $U_c$  is the component of 10m wind speed in the direction of the waves and  $c_p$  is the celerity of waves with peak frequency. However, wind intensity over the period of observation is found to be constant in speed and direction. Similarly value of  $H_s$  showed no much variation. The ratio of  $U_c/c_p$  also has the value in the range of 0.20 to 0.40. Since the data analysed consists of very small data set, it is not feasible to derive any conclusion. From the Figure 7 for typical non directional spectra it seems that the JONSWAP spectra compares well with the low frequency part than PM since the former is parameterised for developing seas. However, high frequency part of the observed spectra seems to be in no agreement with both of the empirical spectra.

In the second part of the study, the aspects of the energy dispersion are considered for frequency domain. In modelling the energy spread relationship between the wind intensity and direction is considered as an important factor.

Example of directional spectral contours defined in Equation 2.1 is given in Figures 8, 9 and 10 for the record no. 165. These are derived from the wave energy spectra (Fig. 7.) as the product of directional spread function with the non-directional spectral estimates. The polar spectra derived as the product of the measured non-directional spectra and the directional energy spreading function (Fig. 8), is predominantly unimodal and tend to be more or less symmetrical about the peak. The important feature is that the little difference between the wave propagation direction and wind direction near the peak. Another aspect is that the directional broadening of the polar curves with the increasing frequency above the peak. This may be attributed to the monsoon wave conditions where the locally generated wind waves normally tend to follow the wind direction. However, above slight difference may be due to the short relaxation time of the directional response of the waves. Hence, these waves are not very much sensitive to distant fetch but are influenced by the local wind. This justifies the earlier findings of DONELAN *et al.* (1985). Figure 8 a, b and c show that directional energy spectra obtained using the PNJ, SWOP and DONELAN distribution functions and the measured non-directional spectra. It can be seen that the 2-d contours are closely clustered for the  $\text{sech}^2(\theta)$  distribution function proposed by DONELAN compared to the other two and are more close on the lower frequency range. In practice, it is interesting to study about the directional spread in the energy contained region of the spectra. That is, the peak value and width at half height are of more interest than that of tail of the spread function. Figure 8 d is one such plot with  $\lg(s_3)$  vs  $\lg(f/f_p)$  where 's<sub>3</sub>' is the half height width of the polar plot. The figure indicates that 's<sub>3</sub>' values are very low for the  $\text{sech}^2(\theta)$  distribution when compared with the other functions. This is may be due to the fact that the data belongs to monsoon period and the distribution has  $\beta$  parameter which takes into effect of the frequency dependency of waves.

Figures 9 and 10 are similar plots obtained as a product of theoretical spectra viz., JONSWAP and PM spectra and the spreading functions. The results show obviously the JONSWAP spectra with  $\text{sech}^2(\theta)$  distribution seems to be in agreement with the 2-d spectra obtained from measured values (Fig. 8c).

## CONCLUSION

Monsoon wave records are analysed for the directional parameters. JONSWAP spectra seems to fit the observed spectra reasonably well at low frequency part compared to the PM spectra. Since the sea state is young and developing, it is not uncommon that observed spectra to have deviations from this empirical spectra. Few important directional spread functions are tested with measured data obtained off Karwar, west



coast of India. An uncommon spreading function  $\text{sech}^2(\theta)$  seems to describe the directional energy spread for the monsoon wave data. Since the number of sea states used here is small, the relationship between energy spread and U/c ratio could not be established. However, a long term data would give a better insight into the problem of modeling the directional energy dispersion of ocean waves.

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## APPENDIX

The coefficient of the first two harmonics can be estimated from the co- and quadrature spectra from the heave and 2 slope signals are :

$$A_1 = \int_0^{2\pi} D(f, \theta) \cos(\theta) d\theta = Q_{zx} / (k C_{zz}) \quad \dots (A1)$$

$$B_1 = \int_0^{2\pi} D(f, \theta) \sin(\theta) d\theta = Q_{zy} / (k C_{zz}) \quad \dots (A2)$$

$$A_2 = \int_0^{2\pi} D(f, \theta) \cos(2\theta) d\theta = C_{xx} - C_{yy} / (k^2 C_{zz}) \quad \dots (A3)$$

$$B_2 = \int_0^{2\pi} D(f, \theta) \sin(2\theta) d\theta = 2C_{xy} / (k^2 C_{zz}) \quad \dots (A4)$$

where C and Q are the co and quadrature spectra respectively , x and y indicating the x & y component of the sea surface slope and z indicating heave. The wave number is defined by k which is computed from data as

$$k = \sqrt{(C_{xx} + C_{yy}) / C_{zz}} \quad \dots (A5)$$

The mean wave direction  $\theta_0(f)$ , directional width  $\sigma_0(f)$  are obtained from above four Fourier coefficients.