

TRACTIVE FORCES

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Derivation of horizontal and vertical components of tide-producing forces.

Let B be a point of unit mass, E the centre of the earth and M the centre of the moon. The force of attraction at B is $A = \frac{xM}{\rho^2}$ acting along the direction BM (Fig. 1).

From Fig. 1, the horizontal and vertical components of this force can be written as

$$A_n = \frac{xM}{\rho^2} \sin \gamma' \quad ; \quad A_v = \frac{xM}{\rho^2} \cos \gamma'$$

The centrifugal force acts along EM. The components of the centrifugal force can be written as

$$z_n = \frac{xM}{a^2} \sin \gamma \quad \text{and} \quad z_v = \frac{xM}{a^2} \cos \gamma$$

$$\cos \gamma' = \frac{a \cos \gamma - R}{\rho} \quad \sin \gamma' = \frac{a \sin \gamma}{\rho}$$

$$\rho = a \left(1 + \frac{R^2}{a^2} - \frac{2R}{a} \cos \gamma \right)^{\frac{1}{2}}$$

Using binomial expansion,

$$\frac{1}{\rho^3} = \frac{1}{a^3} \left(1 + \frac{3R}{a} \cos \gamma \right) \text{ neglecting the terms of higher order.}$$

The horizontal and vertical components become

$$K_h = A_h - Z_h = \frac{3}{2} \frac{x M R}{a^3} \sin 2\gamma \quad \dots (1)$$

$$K_v = A_v - Z_v = 3 \frac{x M R}{a^3} \left(\cos^2 \gamma - \frac{1}{3} \right) \quad \dots (2)$$

(1) and (2) represent the expressions for horizontal and vertical tide-producing forces. If E is the mass of the earth, the gravity for a mass element μ on the earth's surface is $\frac{x \mu E}{R^2}$

$$\begin{aligned} \frac{\text{Maximum tide-producing forces}}{\text{force of gravity}} &= 2 \frac{M R^3}{E a^3} \\ &= 2 \frac{1}{81.5} \frac{1}{60^3} \frac{1}{9 \text{ million}} \end{aligned}$$

The vertical component of tide-producing forces acts in the direction of gravity and they are extremely small, when compared to gravity. The horizontal components, though small, are unopposed. They cause the generation of tides. These forces are called tractive forces.

Tractive forces and their distribution over the globe

Now look at the equations (1) and (2) when $\gamma = 0$ i.e. at the points Z (zenith) and N (Nadir). $K_h = 0$ and $K_v = 3xMR/a^3$ when $\gamma = 90^\circ$, $K_h = 0$ and $k_v = xMR/a^3$

At two points N and Z (Fig.1) and along the longitude circle of 90° , the tractive forces are zero. Along 45° circle from these, tractive forces are maximum.

Generation of tides:

Theoretically, the tractive forces, produce two bulges forming an ellipsoid with the semi-major axis directed towards the moon. However, in practice, this ellipsoid is not formed, because of the rotation of the earth. The two bulges, in order to maintain the same position relative to the moon, would have to travel around the world at the same rate (but in opposite direction) as the Earth rotates with respect to the moon.

Note that along the equator, the linear velocity for a particle on the surface of the earth is 449 m s^{-1} . Assuming a depth of 5.5 km., the long wave speed is approximately 230 m s^{-1} . According to the dynamic theory of tides, the disturbance produced at a given place due to tide-producing forces is assumed to propagate zonally. So, high tide at a given location along the equator will be retarded by 6 hours and 12 minutes after the moon's passage overhead. The above estimate is only

approximate. Firstly, the ocean depth is less than 5.5 km in many places. Secondly, the propagation of tides is not entirely zonal, there is a meridional component as well.

Suggested Reading

1. Defant, A. (1961). *Volume II of Physical Oceanography*, Pergamon Press, 598 pp.
2. Doodson, A.T. and Warburg, H.D. (1941). *Admiralty Manual of Tides*. Her Majesty's Stationery Office, London, 270 pp.
3. The Open University Course Team (1989). *Waves, Tides and Shallow-water Processes*. Pergamon Press, 187 pp.