

ESTIMATION OF DIRECTIONAL WAVE SPREADING

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ABSTRACT

Directional properties of ocean waves are of great economic interest. The knowledge of wave directionality is important for the design of maritime structures and offshore operations. Two main aspects are considered for this study for the data collected over 2 years. Firstly, the characteristics of observed frequency spectra were studied and compared with general JONSWAP and PM spectrum is compared with theoretical two dimensional spectra using various spectral spreadings. Study indicates that for monsoon condition, the JONSWAP spectra is suitable as the sea state was developing and young. Directional spreading function with $\text{sech}^2(\theta/\theta_0)$ distribution found more suitable to the present data since the data seems to have less influence of swell and contains purely locally generated wind waves.

1. INTRODUCTION

The directionality of wind waves requires that they be addressed in probabilistic terms. Initial study on wave spectra was on the concept that the energy is distributed uni-directionally. But, only recently the concept of directional energy distribution was introduced to represent the real sea state reasonably well. The directionality of waves can be studied statistically as short term spreading of wave energy around mean direction as well as change in the mean direction over a long term. On the other hand it can be spectrally analysed examining the nature of energy distribution. In offshore engineering problems such as fluid loading, fatigue behaviour, dynamics of moored or anchored vessels, the influence of wave directionality has assumed growing importance in recent past (Malhotra & Penzein 1970, Deo 1988, Deo & Narasimhan 1989). The study also has bearing on shoaling, reflection, refraction and diffraction studies and wave generation and decay process (Borgman, 1962 and Munk et al 1966).

2. THEORY

A 2-dimensional directional spectrum $S(f, \theta)$ may be adopted to describe the real sea state in the continuum of frequency and direction regardless of wave numbers. Physically, $S(f, \theta) d\theta df$ represents the contribution to the variance due to waves with frequencies between f and $f + df$ and directions between θ and $\theta + d\theta$ (Sarpakaya and Isaacson, 1984). The variance of the sea surface elevation is given by 2-dimensional spectrum and can be expressed as the product of non-directional frequency spectrum $S(f)$ and its angular distribution $D(f, \theta)$ at each frequency (Borgman, 1969).

$$S(f, \theta) = S(f) \cdot D(f, \theta) \quad (2.1)$$

where $D(f, \theta)$ a parameter such that

$$\int_{-\pi}^{\pi} D(f, \theta) d\theta = 1 \text{ at each frequency} \quad (2.2)$$

3. DATA COLLECTION AND ANALYSIS

Different directional wave measurement techniques have been proposed and adopted based on direct and remote sensing methods. Panicker (1972) has summarised the usefulness and functional/operational aspects of these methods discussing the analytical approaches of each technique. For the present study, most common type of pitch-roll buoy measurement was carried out. Figure 1 is the location map of the data collection site. The location is 8 km away from the coast and the water is 16 m. Waves prevailing at the site has mean period of 8 sec approximately.

Figure 2 indicates the WAVEC system with the mooring details used for the above location. Buoy allows simultaneous measurement of heave and two orthogonal surface slopes at a single point. The procedure of FFT adopted yields spectral estimates at a resolution of 0.005 Hz and cut-off of a 0.65 Hz with sampling interval of 0.78 sec and it enables the computation of first four Fourier coefficients of the series to represent the directional spectra. Cross spectral analysis for the time series obtained is used to computed frequency spectra, mean direction and beam width at each frequency (Bhat et al, 1979). A typical 3-D spectral view is as shown in Figure 3 using Equation 2.1.

Longuet-Higgins et al (1963) suggested the convolution of distribution and considered it as sum of normalised angular harmonics at each frequency.

$$D(f, \theta) = \frac{1}{\pi} + \left\{ \frac{1}{2} + \sum_{n=1}^2 (A_n(f) \cos(n\theta) + B_n(f) \sin(n\theta)) \right\} \quad (3.1)$$

$$m_1^2 = A_1^2 + B_1^2 \quad (3.2)$$

$$\theta_0(f) = \arctan(B_1/A_1) \quad (3.3)$$

$$\sigma_0(f) = (2(1 - m_2))^{1/2} \quad (3.4)$$

$$s_1 = m_1 / (1 - m_2) \quad (3.5)$$

4. THEORETICAL SPREADING FUNCTIONS

Basically two classes of analysis are available. First

method involves the reconstruction of the energy distribution and second involves identification of distribution characteristics at such frequency. The first method directly computes harmonic component from buoy signal (Longuet-Higgins, 1963) where as second method parametrically models the behaviour of the energy distribution based on a prior assumed shape (Borgman, 1969).

Most commonly used functions are:

4.1 PNJ Distribution

The cosine square spreading function, defined by Pierson et al (1957), given as

$$D(f, \theta) = (2/\pi) \cos^2(\theta - \theta_m) \quad (4.1)$$

where

θ_m = mean wave direction.

This offers a simple formulation and is useful for design purposes.

4.2 SWOP Distribution

Based on the Stereo Wave Observation Project (SWOP) Cote et al (1960), described the directional spreading function as

$$D(f, \theta) = (1/\pi) (1 + A \cos(2\theta) + B \cos(4\theta)) \quad (4.2)$$

for $\theta < \pi/2$

= 0 otherwise

where

$$A = 0.5 + 0.32 \exp(-0.5 (U^2 \pi f / g)^{**4})$$

$$B = 0.32 \exp(-0.5 (U^2 \pi f / g)^{**4})$$

U = Wind speed at 10m height above sea

g = Gravitational acceleration 9.81 m/s^2

f = Frequency Hz

4.3 Donelan Formulation

Assuming that the energy distribution in the direction transverse to the main direction behaves like sech^2 Donelan et al (1985) proposed the following function:

$$D(f, \theta) = (\beta/2) \cdot \text{sech}^2(\beta(\theta - \theta_m)) \quad (4.3)$$

where

$$\beta = 2.61 (f/f_p)^{1.3} \quad \text{for } 0.56 < (f/f_p) < 0.95$$

$$= 2.28 (f/f_p)^{1.3} \quad \text{for } 0.95 < (f/f_p) < 1.60$$

$$= 1.24 \quad \text{otherwise}$$

Figure 4, 5 and 6 indicate behaviour of functions against θ for different f/f_p values and Figure 7 gives the relationship between β and f/f_p .

5. RESULTS AND DISCUSSION

Monsoon period wave records were analysed for the directional wave parameters and results were tabulated in Table 1. It is observed that during the short period of data collection significant wave height, wind direction and speed were almost constant. The mean wave period T_z and peak period T_p remained close to 8.5 secs and 12 secs respectively. Frequency spectra of heave time series was obtained and plotted in Figure 8. This indicates the sea waves had dominant single peak. Since spectra belong to monsoon period, it is compared with JONSWAP spectra. Similarly, PM spectra is also drawn to compare the goodness of the estimation. JONSWAP spectra seems to estimate the sea state more conformly in the low frequency part and at peak, on the other hand PM spectra finds a larger discrepancy. The deviation on the tail part of the spectra indicates both spectra do not fit the observed spectra.

Figures 9, 10, and 11 are the polar plots for record No.165. These are obtained from product of the wave frequency spectrum and directional spread function. The measured polar spectra (Fig.9) is predominantly unimodal and tend to be more or less symmetrical about the peak. The slight difference between wind direction and direction of wave propagation may be due to relaxation time of the directional response of the waves. It is interesting to study the directional spread of energy contained region of the spectra, the peak value and width at half height of the 2-D spectra. The parameter s_3 is defined as that width which matches the half height, half width $\theta/2$ of the test model (Donelan et al, 1985). Figure 9d is the plot of $\lg(s_3)$ v/s $\lg(f/f_p)$. The Figure indicates s_3 values as lower for the sech^2 distribution (Eq.4.3) when compared to that of other functional variations (Eqs.4.1 & 4.2). The shape of the distribution is mainly controlled by β parameter which depends on (f/f_p) ratio and since monsoon data has energy centered around the peak, it seems it follows the real sea state more reasonably. This justifies that the waves of the present record are not sensitive to distant fetch but are influenced by the local wind.

6. CONCLUSIONS

Commonly used directional spreading functions described in this paper are tested for behaviour against monsoon condition. Uncommon sech^2 Donelan formulation seems to show better representation of energy spread. The monsoon data follows the JONSWAP spectra reasonably well. However, it is suggested that a more comprehensive data base has to be chosen for the purpose of arriving at a specific conclusion.

ACKNOWLEDGEMENT

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APPENDIX

The coefficient of the first two harmonics can be estimated from the co and quadrature spectra from the heave and 2 slope signals are:

$$A_1 = \int_0^{2\pi} D(f, \theta) \cos(\theta) d\theta = Q_{xy} / (k C_{zz}) \quad (A1)$$

$$B_1 = \int_0^{2\pi} D(f, \theta) \sin(\theta) d\theta = Q_{yx} / (k C_{zz}) \quad (A2)$$

$$A_2 = \int_0^{2\pi} D(f, \theta) \cos(2\theta) d\theta = C_{xx} - C_{yy} / (k^2 C_{zz}) \quad (A3)$$

$$B_2 = \int_0^{2\pi} D(f, \theta) \sin(2\theta) d\theta = 2C_{xy} / (k^2 C_{zz}) \quad (A4)$$

where C and Q are the co and quadrature spectra respectively, x and y indicating the x & y component of the sea surface slope and z indicating heave. The wave number is defined by k which is computed from data as

$$k = \sqrt{\frac{(C_{xx} + C_{yy})}{C_{zz}}} \quad (A5)$$

The mean wave direction $\theta_0(f)$, directional width $\sigma_0(f)$ are obtained from above four Fourier coefficients.

TABLE I: Directional Wave Parameters

Date	Time	Record	Hs	Tz	Tp	θ_0	U	Uo	Lo	L	Cp	Uc	Uc/Cp
29-06-89	1900	164	2.61	6.5	10.0	257.37	10.30	270	156.0	110.6	11.06	-2.26	.204
	2130	165	2.62	6.7	11.1	251.70	11.00	270	192.6	128.3	11.55	-3.23	.274
30-06-89	0030	166	2.46	6.5	10.0	251.70	11.84	242	156.0	110.6	11.06	-3.71	.335
	0330	167	2.91	6.6	10.0	254.53	9.78	255	156.0	110.6	11.00	-2.68	.240
	0630	168	2.47	6.5	11.1	251.72	12.36	270	192.6	128.3	11.56	-3.87	.335
	0930	169	2.60	6.7	10.0	254.53	11.33	273	156.0	110.6	11.06	-3.01	.270
	1030	170	2.61	6.6	10.0	253.13	11.84	254	172.0	119.5	11.84	-3.44	.300
	1400	171	2.64	6.8	10.0	250.30	9.78	264	156.0	110.6	11.06	-3.29	.290
	1530	172	2.62	6.6	10.0	254.53	10.81	273	156.0	110.6	11.06	-2.88	.260
	1630	173	2.46	6.4	11.1	251.72	9.72	260	192.6	128.3	11.56	-3.07	.270
	1830	174	2.53	6.7	10.5	251.72	9.27	250	172.0	119.5	11.38	-2.91	.250
	2130	175	2.68	6.7	11.7	251.72	10.30	261	213.55	134.9	11.53	-3.23	.280

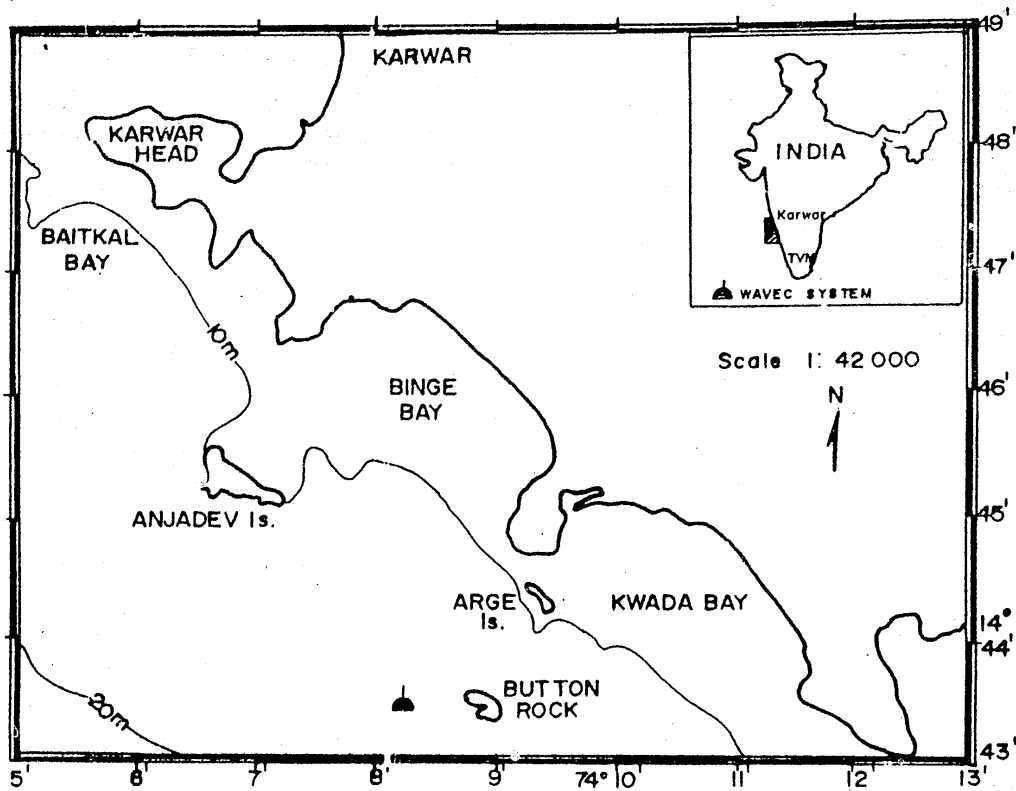


Fig.1 Location map

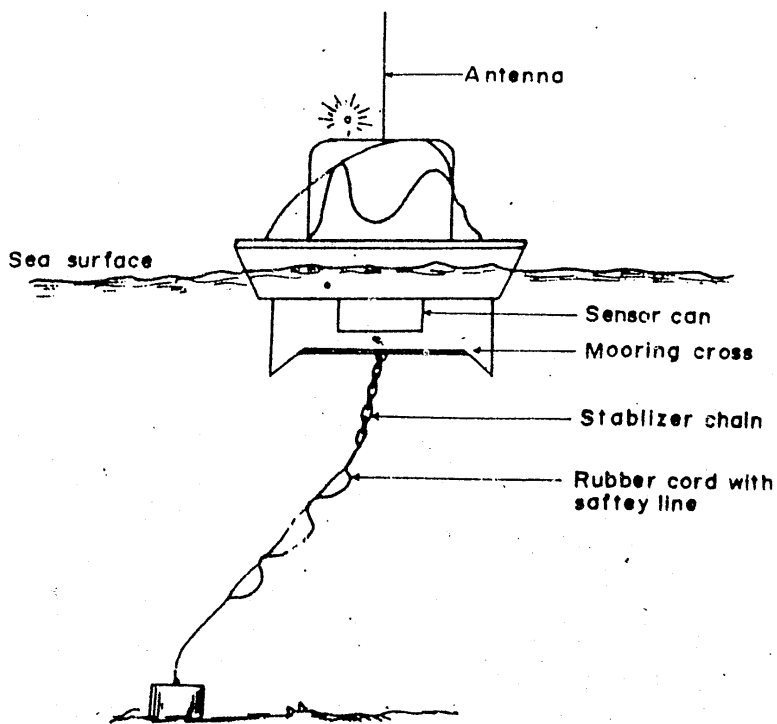


Fig.2 Wavec buoy with Mooring

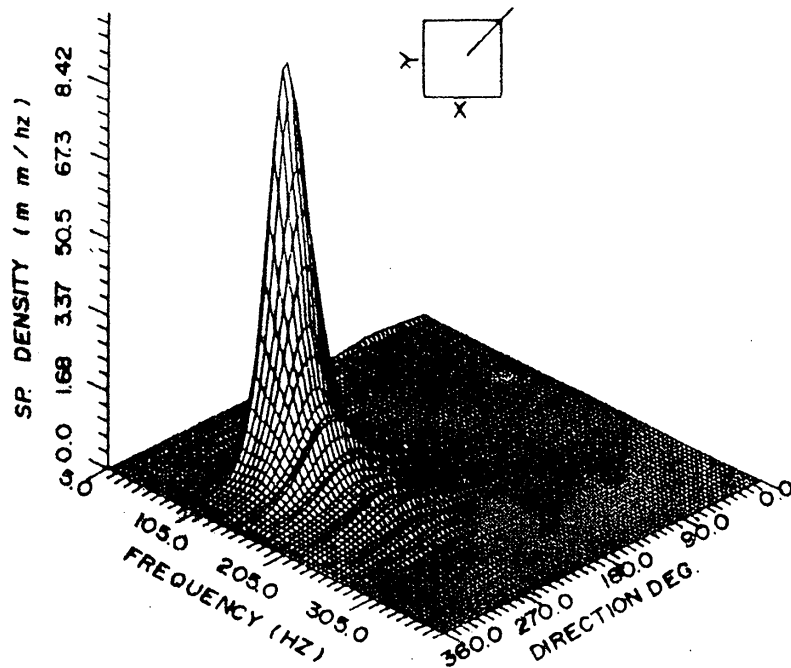


Fig. 3 Directional spectra

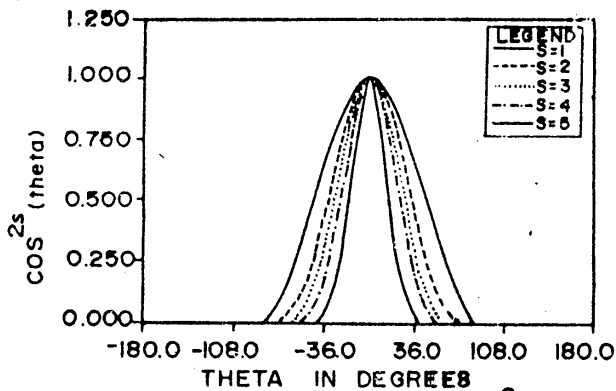


Fig. 4 Directional distribution, COS^2 (theta) spreading.

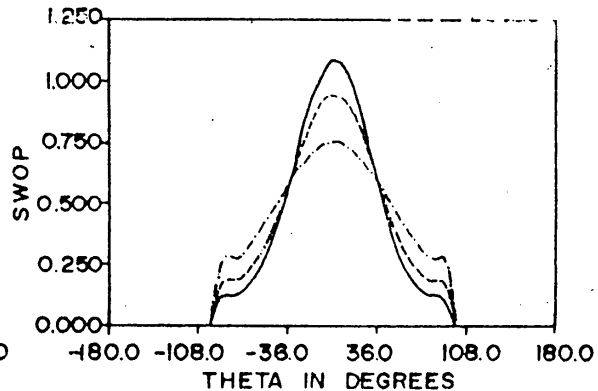


Fig. 5 Directional distribution, SWOP spreading.

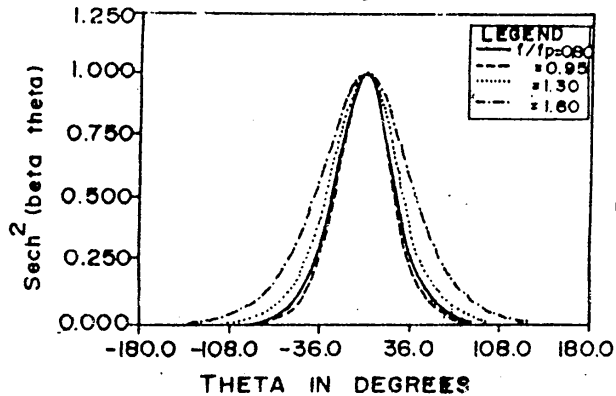


Fig. 6 Directional distribution, Sech^2 (beta theta) spreading.

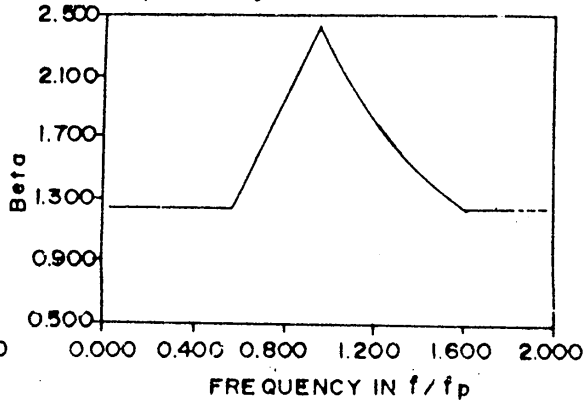


Fig. 7 Variation of Beta with frequency (f/fp)

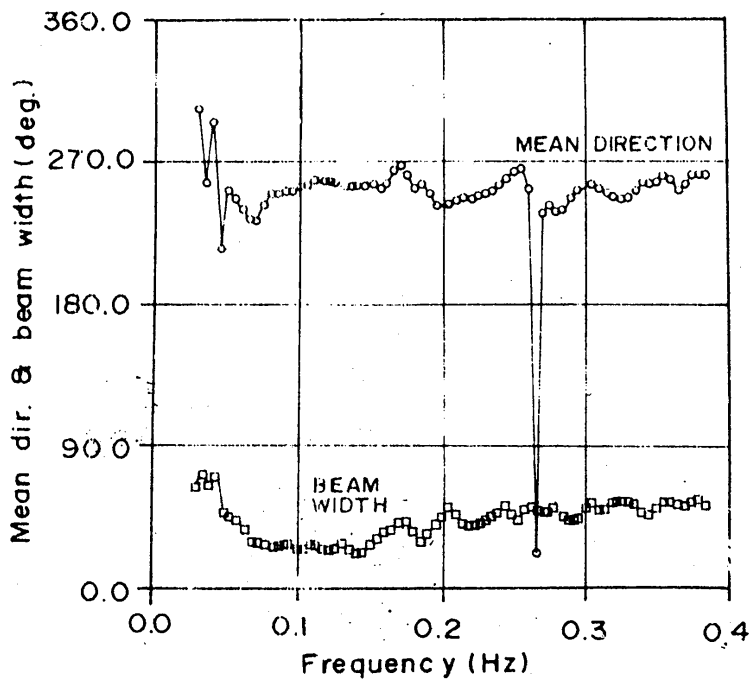
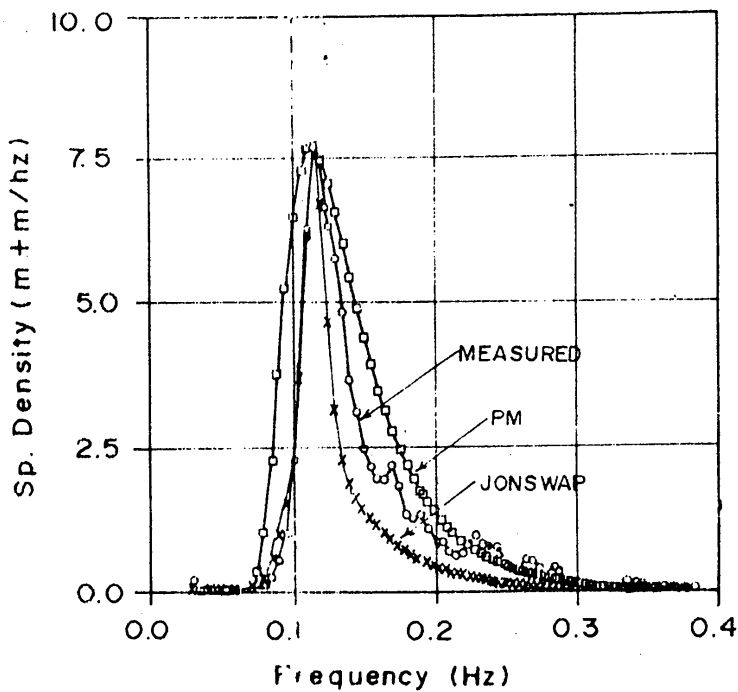


Fig.8 Measured REC.165 $H_s=2.62m$ $T_p=8.7s$ wind speed = 12.11 m/s, direction = 270N with JONSWAP & PM spectra and mean direction & beam width.

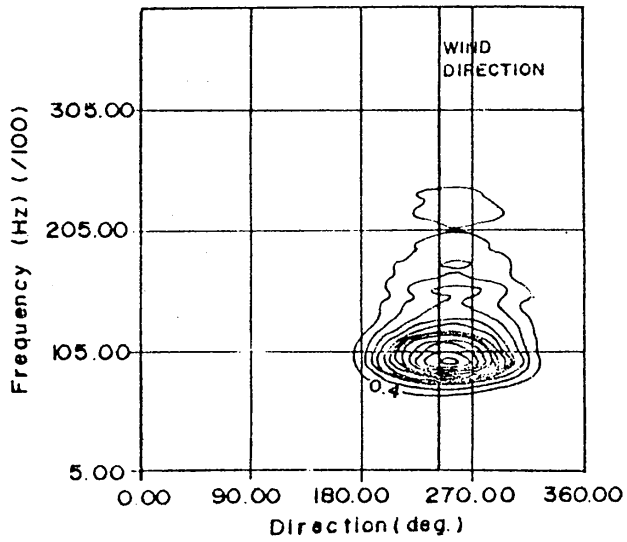


Fig.9 a Polar plot (PNJ Dist measured)

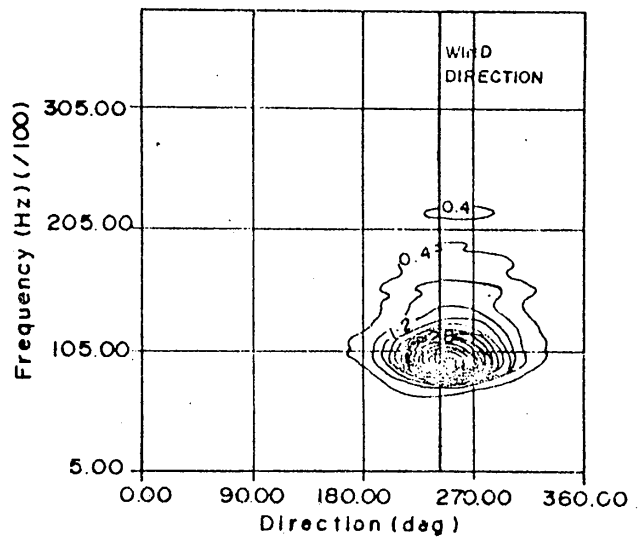


Fig.9 b Polar plot(SWOP Dist measured)

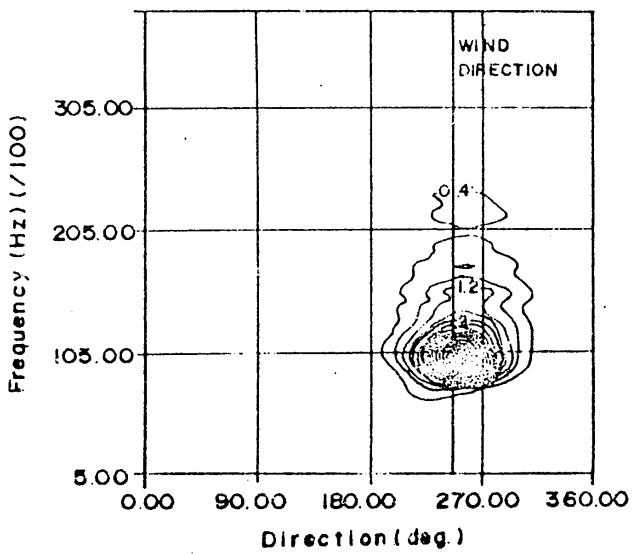


Fig.9 c Polar polar (DONELAN Dist measured)

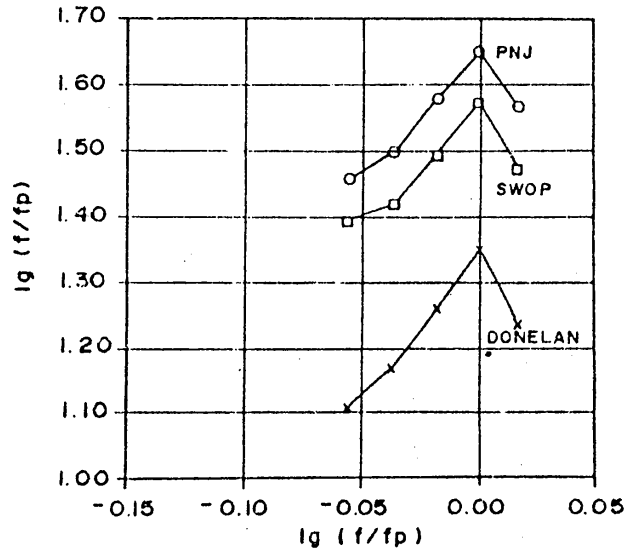


Fig.9 d Parameter s_3 against $\lg(f/f_p)$
 PNJ, SWOP and DONELAN
 models F varies from 7.8s to 9.2s

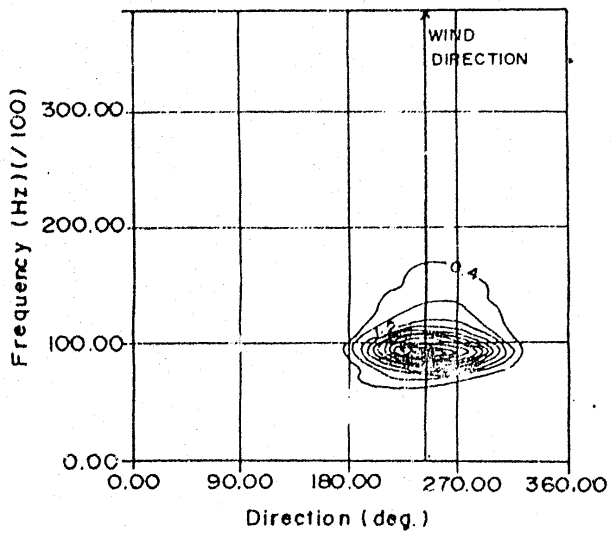


Fig.10a. Polar plot (PNJ DIST. JONSWAP)

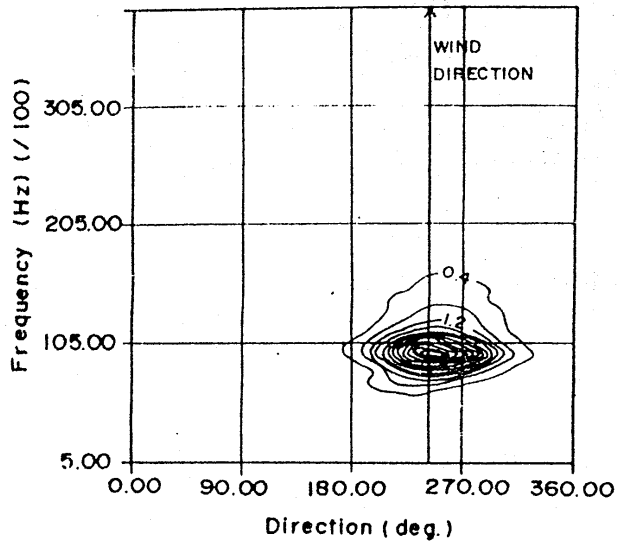


Fig.10b. Polar plot (SWOP DIST. JONSWAP)

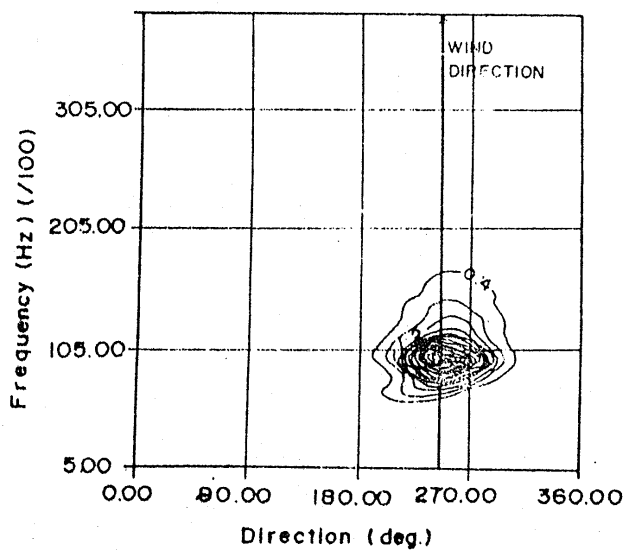


Fig.10c. Polar plot (DONELAN DIST. JONSWAP)

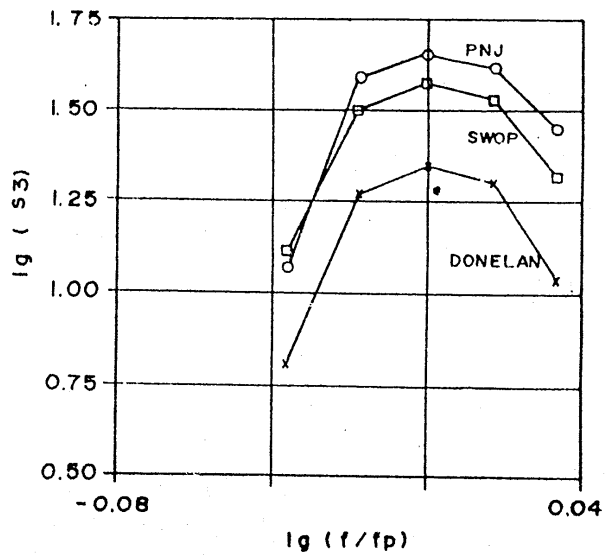


Fig.10d. Parameter s_3 against $\lg(f/f_p)$
PNJ, SWOP AND DONELAN
models F varies from 8.0s to 9.45s.

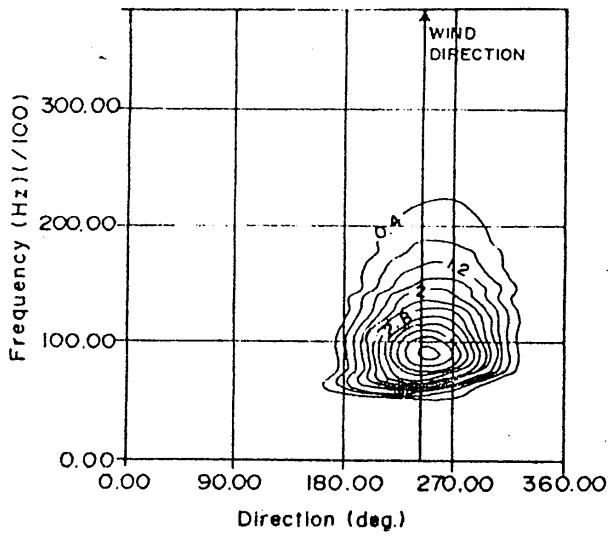


Fig. II a. Polar plot (PNJ DIST. PM)

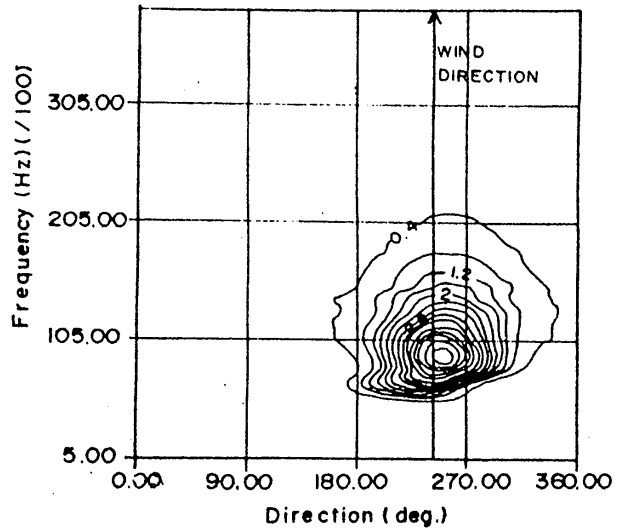


Fig. II b. Polar plot (SWOW DIST. PM)

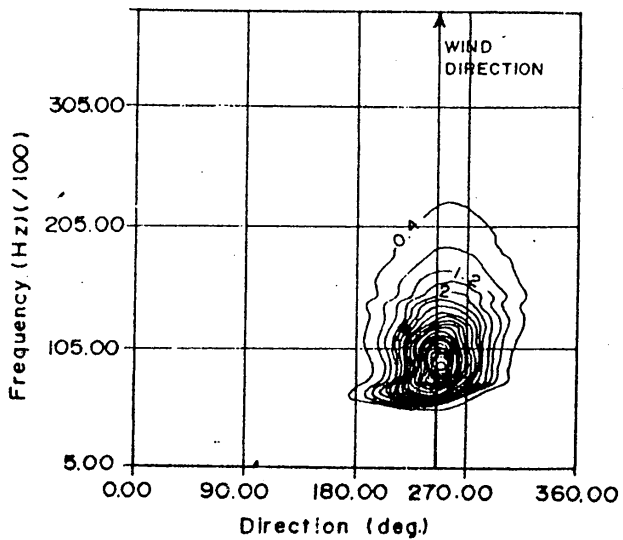


Fig. II c. Polar plot (DONELAN DIST. PM)

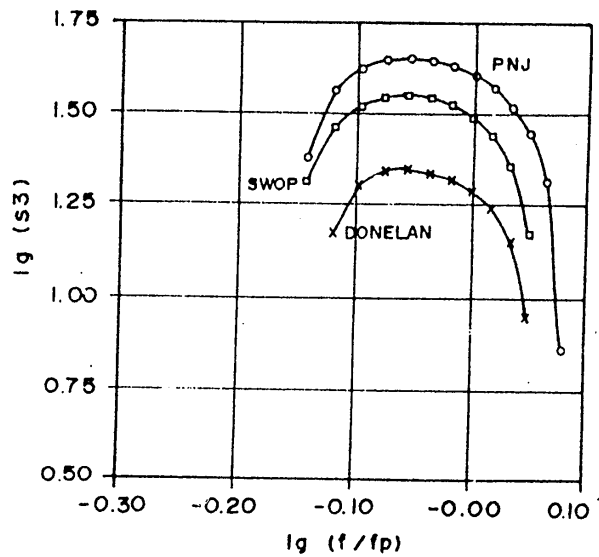


Fig. II d. Parameter s_3 against $\lg(f/f_p)$
PNJ, SWOP AND DONELAN
models F varies from 6.2s to 10.7s.