

# SHALLOW WATER TIDES

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In shallow waters, long waves propagate at a speed approximately equal to  $\sqrt{gh}$ , the crests tend to move faster than the troughs resulting in an asymmetry of the tidal wave. In estuaries, as the tidal wave propagates upstream, the trough of the wave becomes flatter. The duration of ebb increases, while the duration of flood decreases. The propagation of tides in an estuary (or other semi-enclosed bodies) can be studied with the help of data from a series of tide gauges from the mouth to the upstream end, where tidal oscillations cease. In India, this type of observations are made in Hoogly estuary and Gulf of Kutch.

## Modelling of tidal propagation in estuaries

Numerical models have been widely used to compute tides and tidal currents in estuaries since last two decades. A brief description of a model developed for French estuaries is given below. Note that tides are barotropic and they can be modelled by using vertically integrated shallow water equations.

Consider a rectangular cartesian co-ordinate system in which the direction  $x$  is taken positive towards the head of the estuary. Vertically integrated shallow water equations take the following form. The equations (1) and (2) are the momentum equations in  $x$  and  $y$  directions and equation (3) is the continuity equation.

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - fV + g \frac{\partial \zeta}{\partial x} + \frac{gU (U^2 + V^2)^{\frac{1}{2}}}{C^2 H} + \frac{\tau_{sx}}{\rho H} + \epsilon \Delta U = 0 \quad \dots (1)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} - fU + g \frac{\partial \zeta}{\partial y} + \frac{gV (U^2 + V^2)^{\frac{1}{2}}}{C^2 H} + \frac{\tau_{sy}}{\rho H} + \epsilon \Delta V = 0 \quad \dots (2)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial(HU)}{\partial x} + \frac{\partial(HV)}{\partial y} = 0 \quad \dots (3)$$

when  $u$ ,  $v$  are the velocity components in  $x$  and  $y$  directions,  $C$  is the coefficient of bottom friction,  $f$  the Coriolis parameter,  $g$  is the acceleration due to gravity.  $\zeta$  is the water surface elevation,  $\tau_{sx}$  and  $\tau_{sy}$  are the wind stress components.  $H$  is the total depth and  $\epsilon$  is the coefficient of pseudo viscosity.

On the seaward side of the domain, a periodic boundary condition is prescribed. For the open boundary near the upstream side, boundary condition is defined in terms of river discharges. For the closed boundaries, zero normal flux condition is imposed.

Note that the momentum equations contain non-linear terms such as  $u \frac{\partial u}{\partial x}$ ,  $v \frac{\partial u}{\partial y}$  etc. The bottom friction term is quadratic. There is also non-linearities in the continuity equation. The above system cannot be solved analytically. However, they can be solved by numerical methods, normally by a finite difference technique. A detailed discussion of the method is beyond the scope of this lecture. Only a brief analysis of various terms in the equations is attempted here.

In equations (1) and (2),  $g \frac{\partial y}{\partial x}$  and  $g \frac{\partial y}{\partial y}$  terms are greater than one order of magnitude when compared to  $u \frac{\partial u}{\partial x}$ ,  $v \frac{\partial u}{\partial y}$  terms.  $g \frac{\partial y}{\partial x}$  and  $g \frac{\partial y}{\partial y}$  contribute to the velocity components. During high tide or low tide, these terms are zero, the tidal currents are also zero during slack. Maximum currents are found during middle of flood and ebb.

## Application of the model

The domain is schematised by constructing a mesh with a grid size of about 0.25-1 km. (in the case of estuaries) and the coastline represented by a stair step boundary. The equation (1) to (3) can be written in finite difference form. The various schemes used for this purpose have been widely described in literature (for instance, Salomon, 1979). After constructing the mesh, the depth values are given at each grid point from the bathymetric chart. The model is run by specifying the necessary boundary conditions the seaward and landward sides. In order to have reliable results, it is necessary to calibrate the model. This is done by varying the coefficients of bottom friction in various parts of the domain, so that the difference between the computed and observed tides is minimum. Once the model reproduces the tides fairly accurately, the computed currents are also reliable, as the currents depend mainly on  $g \frac{\partial y}{\partial x}$ ,  $g \frac{\partial y}{\partial y}$  terms.

In estuaries, the results of tidal model are used to study the sedimentation in channels. In the continental shelf regions, the results of tidal model are used to prepare the charts of co-tidal and co-range lines. Co-tidal lines are lines of simultaneous occurrence of tides and co-range lines are lines joining equal tidal range. Amphidromic points are places around which the crest of the tidal wave rotates. At amphidromic points, the tidal range is zero and the tidal range increases away from these points. Amphidromic points are formed in certain places due to the combined effects of bottom topography and rotation of the earth. From the charts of co-range lines, amphidromic points can be identified.

### **Shallow water constituents**

The distortion produced in shallow water for the tidal wave is due to the generation of overtides and compound tides. The overtides produced are the multiples of the frequency of the principal tides. The overtides usually taken into account are those originating from  $M_2$  and  $S_2$ . They are  $M_4$ ,  $M_6$ ,  $M_8$ ,  $S_4$ ,  $S_6$  and  $S_8$ , the subscript denotes the number of oscillations per day. The principal astronomical constituents from which the compound tides are derived are  $M_2$ ,  $S_2$ ,  $N_2$ ,  $K_1$  and  $O_1$ . For instance,  $MS_4$  has a speed equal to sum of speeds of  $M_2$  and  $S_2$ .  $2MS_6$  has a speed twice that of  $M_2$  plus the speed of  $S_2$ .

### **Suggested Readings**

1. Doodson, A.T. and Warburg, H.D. (1941). *Admiralty Manual of Tides*. Her Majesty's Stationery Office, London, 270 pp.
2. Unnikrishnan, A.S. (1988). *Link node model for the simulation of tides in the Gironde estuary, France*. *Indian Journal of Marine Sciences*, Vol. 17, pp. 9-13.