

APPROXIMATE EQUATIONS AT BREAKING FOR NEARSHORE WAVE TRANSFORMATION COEFFICIENTS

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ABSTRACT

Based on small amplitude wave theory, approximate equations are evaluated for determining the coefficients of shoaling, refraction, bottom friction, bottom percolation and viscous dissipation at breaking. The results obtained by these equations agree within the reasonable range with the results based on numerical integration. These equations will be useful for field engineers for easy application.

INTRODUCTION

Coastal engineers are often provided with wave characteristics at deep water or at some intermediate depth. The surfzone dynamics and the longshore sediment transport rates are evaluated based on the wave characteristics at breaking. When the deepwater wave propagates into shallow water, it is subjected to the effect of wave shoaling. If the wave crest is oblique to bottom contours, the difference in celerity along the wave crest causes the wave to bend, called wave refraction. The roughness of the seabed and the adjoining turbulent boundary layer retards the wave motion, termed as bottom friction. If the seabed is permeable, the percolation of water into the seabed would further retard the wave motion. The viscosity of the interior body of the water causes the wave energy to dissipate, termed as viscous dissipation. In the present study, approximate equations based on small amplitude wave theory are evaluated for determining the coefficients of shoaling, refraction, bottom friction, bottom percolation and viscous dissipation at wave breaking location.

Nearshore Wave Transformation Coefficients

Fig.1 shows the volume of water enclosed over the depth between the two adjacent orthogonals and two vertical sections perpendicular to these orthogonals. Assuming that no energy propagates across the orthogonals,

$$b_2 E_2 = b_1 E_1 - E_{1,2} \quad (1)$$

where, b_1 = distance between orthogonals at section (1), b_2 = distance between orthogonals at section (2), E_1 = energy flux at section 1, E_2 = energy flux at section 2, $E_{1,2}$ = loss of energy between sections 1 and 2.

The loss of energy would partly be due to bottom friction, bottom percolation, viscous dissipation etc. and if they are assumed to be negligible Eq.(1) reduces to,

$$b_1 E_1 = b_2 E_2 \quad (2)$$

Shoaling

Waves are assumed approaching parallel to the straight

and parallel bottom contours. Hence, no refraction is encountered, i.e., $b_1 = b_2$ and Eq.(2) becomes,

$$E_{11} = E_{12} \quad (3)$$

wave energy flux is given by (Svendsen and Jonsson, 1976),

$$E_1 = \frac{\rho g H^2}{16} c (1 + G) \quad (4)$$

Where,

- G = $2kh / \sinh 2kh$
- ρ = Density of seawater
- g = Acceleration due to gravity
- H = Wave height
- c = Wave celerity
- k = Wave number = $2\pi/L$
- h = Water depth
- L = Wave length.

considering the section 1 at deep water and section 2 at desired depth h and introducing Eq.(4) in Eq.(3) and rearranging,

$$K_s = \left(\frac{1}{2} \frac{1}{n} \frac{C_0}{C} \right)^{0.5} \quad (5)$$

where, K_s = shoaling coefficient, $n = 0.5 (1 + (2kh / \sinh 2kh))$, C_0 = wave celerity at deep water. Eq.(5) is simplified for wave breaking location using the small amplitude wave theory.

Refraction

Using Eq.(4) in Eq.(2),

$$b_1 H_1^2 C_1 (1 + G_1) = b_2 H_2^2 C_2 (1 + G_2) \quad (6)$$

considering the section 1 at deep water and section 2 at desired depth h and introducing Eq.(5) in Eq.(6),

$$\frac{H}{H_0} = K_s \left(\frac{b_0}{b} \right)^{0.5} \quad (7)$$

where, K_s = refraction coefficient = $\left(\frac{b_0}{b} \right)^{0.5}$ and b_0 and b are the distance between orthogonals at deepwater and at required depth respectively.

For straight and parallel contours, the orthogonals would be parallel and the horizontal distance is constant between two adjacent orthogonals. Therefore,

$$\begin{aligned} \frac{b_0}{\cos \alpha_0} &= \frac{b}{\cos \alpha} \\ \frac{b_0}{b} &= \frac{\cos \alpha_0}{\cos \alpha} \\ K_s &= \left(\frac{b_0}{b} \right)^{0.5} = \left(\frac{\cos \alpha_0}{\cos \alpha} \right)^{0.5} \end{aligned} \quad (8)$$

where, α_0 and α are the angle of the wave crest with respect to the coast at deepwater and at required depth respectively. Eq.(8) is simplified for wave breaking location using small amplitude wave theory.

Dissipative Forces

Considering the dissipative forces are due to bottom friction, bottom percolation and viscous dissipation, the mean energy flux between the orthogonals O_1 and O_2 in Fig.2 is given by,

$$E_t D_t = \frac{\rho g H^2}{16} C (1 + G) D_t \quad (9)$$

where, D_t = elementary length of wave front between the orthogonals.

Taking the ratio between Eq.(9) and the energy flux at deep water,

$$H/H_0 = K_s K_r K_{dis} \quad (10)$$

where,

$$K_{dis} = K_r K_p K_v \quad (11)$$

and referring to Fig.2,

$$K_{dis} = \left(\frac{E_t D_t}{E_{fr} D_{fr}} \right)^{0.5} \quad (12)$$

K_s , K_r , K_r , K_p , K_v are the shoaling, refraction, bottom friction, bottom percolation and viscous dissipation coefficients.

If the bed is deep and horizontal K_s and K_r are equal to 1, if the orthogonals are perpendicular to straight and parallel contours, K_r is equal to 1 and if dissipation is neglected $K_{dis} = 1$.

Bottom Friction

In Fig.2, considering the sections 1 and 2, spaced by infinitesimal distance D_s , the difference equation (1) transforms to a differential equation,

$$\left(\frac{D}{D_s} \right) E_t D_t = -E_d D_t \quad (13)$$

Squaring Eq.(12) and differentiating with reference to s ,

$$\frac{d K_{dis}^2}{dt} = \frac{C E_{dis} \beta}{E_{fr}} \quad (14)$$

Where, E_{dis} is the total energy dissipated due to bottom friction, bottom percolation and viscous dissipation.

Considering the dissipation is only due to bottom friction, Jonsson (1966) gives an expression for E_{dis} ,

$$E_{dis} = E_{fr} = \left(\frac{2}{3} \pi \right) \rho f_b a_m^3 \quad (15)$$

where, E_{fr} = energy loss due to bottom friction, f_b = friction parameter (Jonsson, 1966, Kamphuis, 1975, Lambrakos, 1982), a_m = horizontal particle amplitude at bed.

Using Eq.(15) in Eq.(14) and replacing K_{dis} with K_r since the dissipation is only due to friction, and assuming that the starting point is in deep water (Skovgaard, 1975),

$$\frac{d K_r}{dt} = - \left(\frac{8}{3} \frac{1}{Th} \frac{G}{1+G} \right) a_m f_b K_r \quad (16)$$

$$\frac{d K_r}{dt} = - \left(\frac{8}{3} \frac{dc}{L dh} \right) a_m f_b K_r \quad (17)$$

where,

$$\frac{dc}{dh} = \frac{c}{h} \frac{G}{1+G} \quad (18)$$

The non-linear first order differential equation (16) is solved and simplified for wave breaking location.

Bottom Percolation

Reid and Kajiura (1957) give the mean energy loss due to bottom percolation per second per unit area of the bed (E_p) as,

$$E_p = \frac{\pi}{2} \rho g \frac{K_D}{L_0} \frac{H^2}{\sinh 2 kh} \quad (19)$$

where, K_D = permeability coefficient (Oza, 1969). Loss due to permeability is usually considered only if the depth of permeable layer is more than 30 percent of the wave length (Svendsen and Jonsson, 1976).

Referring to Fig.1 and considering the Eq.(14) and assuming that the dissipation is only due to bottom percolation, the percolation coefficient can be written as,

$$K_p = \left(\frac{E_f D_f}{E_{fs} D_{fs}} \right)^{0.5} \quad (20)$$

Squaring Eq.(20) and differentiating with respect to s,

$$\frac{dK_p^2}{dt} = \frac{C E_f \beta}{E_{fs}} \quad (21)$$

where, K_p = the percolation coefficient.

$$E_{fs} = \left(\frac{\rho g H_0^2}{8} \right) \left(\frac{C_0}{2} \right) \text{ in deep water} \quad (22)$$

Assuming that the starting point is in deep water, using Eqs.(19) and (22) in Eq.(21), differentiating and simplifying using small amplitude wave theory approximations,

$$\frac{dK_p}{dt} = -K_p \frac{C}{C_0} \frac{1}{h} \frac{G}{1+G} K_p \quad (23)$$

using Eq.(18) in Eq.(23),

$$\frac{dK_p}{dt} = -K_p \frac{1}{C_0} \frac{dc}{dh} K_p \quad (24)$$

The non-linear first order differential equation (24) is solved and simplified for wave breaking location.

Viscous Dissipation

Based on the work done by the laminar shear stresses on the small particle velocity gradients (Batchelor, 1967), the approximate expression for the mean energy loss due to direct viscous dissipation in the interior body of the water per second per unit area (E_v) is given by,

$$E_v = 2 \pi^2 \rho g \gamma \left(\frac{H}{L} \right)^2 \quad (25)$$

where, γ = kinematic viscosity of the sea water

Considering the Eq.(12) and assuming that the dissipation is only due to viscous dissipation, the viscous dissipation coefficient is written as,

$$K_v = \left(\frac{E_f D_f}{E_{fs} D_{fs}} \right)^{0.5} \quad (26)$$

Squaring Eq.(26) and differentiating with respect to s,

$$\frac{dK_v^2}{dt} = \frac{C E_v \beta}{E_{fs}} \quad (27)$$

where K_v = viscous dissipation coefficient.

Assuming that the starting point is in deep water, using Eqs.(25) and (20) in Eq.(27), differentiating and simplifying by applying small amplitude wave theory relationships,

$$\frac{dK_v}{dt} = -\frac{4 \gamma \pi}{L} \frac{2 kh}{h} \frac{K_v}{1+G} \quad (28)$$

$$\frac{dK_v}{dt} = -4 \gamma \pi \frac{dc}{dh} \frac{\sinh 2 kh}{C^2 T} K_v \quad (29)$$

The non-linear first order differential equation (29) is solved and simplified at wave breaking location.

RESULTS

Approximate Equations at Breaking

At wave breaking the small amplitude wave theory approximations are,

$$n = 1 \quad (30)$$

$$C_0 = \frac{g T}{2 \pi} \quad (31)$$

$$C_b = (g h_b)^{0.5} \quad (32)$$

$$h_b = 1.28 H_b \text{ (Mc Cowan, 1981)} \quad (33)$$

Shoaling

Using Eqs.(30), (31), (32) and (33) in Eq.(5) and assuming H_b is approximately equal to H_0 in this case,

$$K_s = 0.4697 \frac{\sqrt{T}}{H_0^{0.75}} \quad (34)$$

Refraction

By Snell's law,

$$\frac{C_1}{\sin \alpha_1} = \frac{C_2}{\sin \alpha_2} \quad (35)$$

Assuming the two points, one at deep water and the other at breaking,

$$\frac{C_0}{\sin \alpha_0} = \frac{C_b}{\sin \alpha_b} \quad (36)$$

$$\alpha_b = \sin^{-1} \left(2.27 \frac{C_b}{C_0} \sin \alpha_0 \right) \quad (37)$$

Using Eqs.(31), (32) and (33) in Eq.(37)

$$\alpha_b = \sin^{-1} \left(2.27 \frac{\sqrt{H_0}}{T} \sin \alpha_0 \right) \quad (38)$$

using Eq.(38) in Eq (9)

$$K_r = \left[\frac{\cos \alpha_0}{\cos A} \right]^{0.5} \quad (39)$$

where, $A = \alpha_b$, given by Eq.(38)

Bottom Friction

Writing the differential Eq.(17) in finite difference form for incremental time Δt ,

$$\frac{K_{f,t+\Delta t} - K_{f,t}}{\Delta t} = -\frac{8}{3L} \frac{dc}{dh} \alpha_{in} f_0 K_{f,t} \quad (40)$$

slope of the sea bed (s) can be written as,

$$s = \frac{dh}{c \Delta t} \quad (41)$$

Using Eq.(41) in Eq.(40) and rearranging,

$$K_{f,t+\Delta t} = \frac{K_{f,t}}{\left(1 + \frac{8}{3L} \frac{dc}{cs} \alpha_{in} f_0\right)} \quad (42)$$

Eq.(42) in its general form can be used to estimate the bottom friction coefficient for any depth. Considering initial time t is in deep water and (t + Δt) is just before breaking,

$$K_{f,t} = 1 \text{ (in deep water)} \quad (43)$$

$$\text{and } (\alpha_{in})_b = \frac{H_b}{2kh_b} \quad (44)$$

using Eqs.(31),(32),(33), (43) and (44) in Eq.(42) the friction coefficient just before breaking is given by

$$(K_f)_b = \frac{1}{\left(1 + \frac{8}{3L} \frac{dc}{c_b s} \frac{H_b}{2kh_b} f_0\right)} \quad (45)$$

Assuming contours are straight and parallel, and the wave crests are parallel to bottom contours,

$$K_{f,t} = 1 \quad \text{and at breaking using Eq.(31), (32) and (33) in Eq.(5)}$$

$$(K_s)_b = 1.2$$

hence,

$$H_b = H_0 \cdot K_f K_s = 1.2 H_0 \quad (46)$$

using Eq.(46) in Eq.(45),

$$(K_f)_b = \frac{1}{\left(1 + \frac{8}{12\pi} \frac{dc}{c_b} \frac{f_0}{1.28s}\right)} \quad (47)$$

According to the variations of f_0 with depth, near wave breaking for $k_N = 0.002$ the value of f_0 is 0.015 (Jonsson, 1966). Using this value in Eq.(47) and simplifying further,

$$(K_f)_b = \frac{1}{\left[1 + \frac{0.0006}{s} \left(\frac{T}{H_0^{0.5}} - 2.5\right)\right]} \quad (48)$$

For typical values of nearshore slope 1:200, T = 8s, $H_0 = 1$ m Eq. (48) gives $(K_f)_b = 0.6$ whereas the numerical integration of Eq. (17) gives $(K_f)_b = 0.7$ and the difference is found to be within reasonable limit.

Eq.(42) is an approximate solution from which the friction coefficient at any depth can be estimated assuming initial time (t) is in deep. Eq.(48) is the approximate solution at wave breaking.

Bottom Percolation

Writing the differential Eq.(24) in finite difference form for the incremental time Δt,

$$\frac{K_{p,t+\Delta t} - K_{p,t}}{\Delta t} = -\frac{K_{p,t}}{C_0} \frac{dc}{dh} K_{p,t} \quad (49)$$

using Eq.(19) and rearranging

$$K_{p,t+\Delta t} = \frac{K_{p,t}}{1 + \frac{K_{p,t}}{C_0} \frac{dc}{cs}} \quad (50)$$

Assuming the starting time t is in deep water ($K_p = 1$), time (t + Δt) is just before breaking, and using Eqs.(31), (32), (33), (43), (44) and (45) in Eq.(50)

$$(K_p)_b = \frac{1}{1 + \frac{0.26K_D}{sT} \left(\frac{T}{H_0^{0.5}} - 2.5\right)} \quad (51)$$

If the bed is assumed to consist of medium to fine sand and the permeability coefficient K_D is assumed as 0.0005 m/s.

$$(K_p)_b = \frac{1}{1 + \frac{0.00013}{sT} \left(\frac{T}{H_0^{0.5}} - 2.5\right)} \quad (52)$$

For values of nearshore slope = 1:50, T=8s, $H_0=1$ m, Eq.(52) gives $(K_p)_b = 0.9955$ where as for slope=1:500, T=8s, $H_0=1$ m and K_D for coarse sand bed =0.005m/s, it gives $(K_p)_b=0.6911$. It shows that though the effect of bottom percolation is small, in case of coarse sand and gravel bed, where the permeability is high, the value of percolation coefficient considerably increases.

Eq.(50) is an approximate solution from which the percolation coefficient at any depth can be estimated assuming initial time (t) is in deep water. Eq.(52) is the approximate solution at wave breaking.

Viscous Dissipation

Writing the first order differential Eq.(28) in finite difference form for the incremental time Δt,

$$\frac{K_{v,t+\Delta t} - K_{v,t}}{\Delta t} = -4\pi\gamma \frac{dc}{dh} \frac{\sinh 2Kh}{c^2 T} K_{v,t} \quad (53)$$

using Eq.(19) and rearranging,

$$K_{v,t+\Delta t} = \frac{K_{v,t}}{1 + 4\pi\gamma \frac{dc}{sT} \frac{\sinh 2Kh}{c^3 T}} \quad (54)$$

Considering the starting time t in deep water ($K_v = 1$), time (t + Δt) is just before breaking and using Eqs.(19), (33), (34), (35), (36), (37) and (38) in Eq.(56)

$$(K_v)_b = \frac{1}{1 + \frac{1.817}{s} \frac{\gamma}{T^2 H_0^{0.5}} \left(\frac{T}{H_0^{0.5}} - 2.5\right)} \quad (55)$$

substituting the kinematic viscosity of sea water as $\gamma = 0.00001945 \text{ m}^2/\text{s}$.

$$(K_v)_b = \frac{1}{1 + \frac{3.534 \times 10^{-6}}{s} \frac{1}{T^2 H_0^{0.5}} \left(\frac{T}{H_0^{0.5}} - 2.5 \right)} \quad (56)$$

Using the values of sea bed slope = 1:50, $T = 8\text{s}$, $H_0 = 1\text{m}$, in Eq.(47), it gives $(K_v)_b = 0.99998415$. For muddy nearshore water, when the bed slope = 1:500, $\gamma = 0.0001945 \text{ m}^2/\text{s}$, $T = 8\text{s}$, $H_0 = 1\text{m}$, it gives $(K_v)_b = 0.9850$. It indicates that the effect of viscous dissipation is negligibly small, but over the regions of mud banks, where the viscosity of the water increases considerably, the effect will be significant.

Eq.(54) is an approximate solution from which the viscous dissipation coefficient at any depth can be estimated assuming that the initial time (t) is in deep water. Eq.(55) is the approximate solution at breaking.

CONCLUSIONS

Based on the small amplitude wave theory approximations, the following approximate equations are derived to estimate the nearshore wave transformation coefficients at wave breaking location.

i) Shoaling:

$$K_s = 0.4697 \frac{\sqrt{T}}{H_0^{0.25}}$$

ii) Refraction:

$$K_r = \left[\frac{\cos \alpha_0}{\cos \alpha_b} \right]^{0.5}$$

where,

$$\alpha_b = \sin^{-1} \left(2.27 \frac{\sqrt{H_0}}{T} \sin \alpha_0 \right)$$

iii) Bottom Friction:

$$(K_f)_b = \frac{1}{1 + \frac{0.0006}{s} \left(\frac{T}{H_0^{0.5}} - 2.5 \right)}$$

iv) Bottom Percolation:

$$(K_p)_b = \frac{1}{1 + \frac{0.00013}{sT} \left(\frac{T}{H_0^{0.5}} - 2.5 \right)}$$

v) Viscous Dissipation:

$$(K_w)_b = \frac{1}{1 + \frac{3.534 \times 10^{-6}}{s} \frac{1}{T^2 H_0^{0.5}} \left(\frac{T}{H_0^{0.5}} - 2.5 \right)}$$

The results obtained by the above equations agree within the reasonable range with the results based on numerical integration. These equations may be found handy for the field engineers for easy application.

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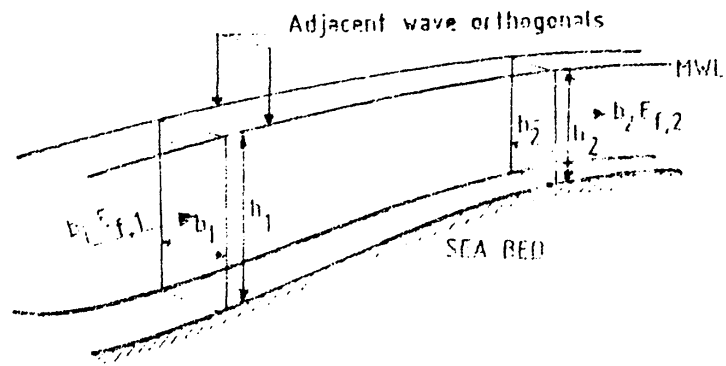


Fig. 1. Energy flux between sections through adjacent wave orthogonals

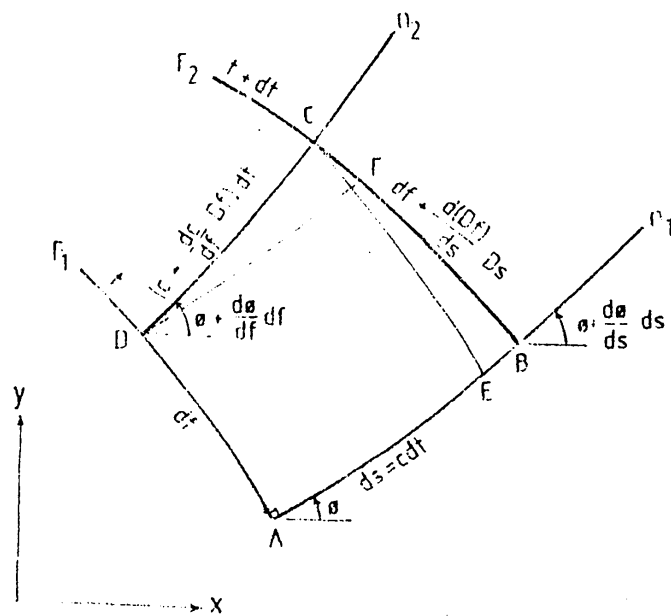


Fig. 2. System of wave orthogonals and fronts