



## MODIFICATION OF WIND-WAVE MODEL WAM AND ITS VERIFICATION AGAINST BUOY DATA IN THE INDIAN OCEAN

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**Abstract:** *In the present study, we made an attempt to improve the accuracy of wave prediction in the Indian Ocean. For this purpose, two versions of the wave model WAM have been used: WAM (Cycle 4) and the new version of WAM, called NewWAM. The three important source functions, namely, wind input, breaking dissipation, and nonlinear interaction were modified, and incorporated in the NewWAM model by fitting new coefficients into their respective parameterizations based on the physical processes related to seasonal changes and extreme weather events in the Indian Ocean. The model results thus obtained have been verified using the moored buoy data deployed in the Indian Ocean. Based on the error estimates extracted for the available 5 buoy data for the period 2005-2006, the accuracy of the NewWAM model has been discussed.*

**keywords:** *wind waves, NewWAM, WAM, wave model, source functions, buoy data, Indian Ocean, significant wave height*

### INTRODUCTION

The study of wind waves deserves major attention because waves play a very important role in areas such as offshore oil exploration and exploitation, fisheries, marine transportation and structural design. From a more fundamental point of view, wind waves play a crucial role in other important marine processes such as exchange of gases, heat and momentum between the atmosphere and the sea, interchange of energy with currents and interaction with the bottom. These are processes which affect the climate and marine lives. Waves grow from absorbing the energy and momentum of the forcing wind. Nonlinear wave-wave interaction transfers energy from the spectral peak region to both lower and higher frequencies and broadens the wave spectrum. During the growth process, waves also become steeper and eventually break with accompanying energy dissipation. The wind input, breaking dissipation, and nonlinear wave-wave interaction are the three most important source terms governing the dynamics of the surface gravity wave evolution (for example, Hasselmann et al., 1973; Phillips, 1977, 1985; Komen et al., 1994; Janssen, 2004).

The details of the source functions are still active research subjects, and one of the methods to gauge the performance of numerical or theoretical wave models is to compare the model results with fetch- or duration limited wave growth functions (e.g., Komen et al., 1984, 1994; Janssen et al., 1994; Janssen, 2004; Ardhuin et al., 2007; Romero and Melville, 2010). Because the duration limited condition occurs rarely in the natural environment, by far the fetch-limited growth research contributes the most to the benchmark database. Over the last few decades, many fetch growth functions have been proposed (e.g., reviews by Hasselmann

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et al., 1973; Donelan et al., 1985; Kahma and Calkoen, 1994; Young, 1999). The experimental data suggest that the fetch-limited growth can be expressed in power-law functions, but the coefficients of the power-law functions vary with the atmospheric stability condition and the range of wind fetch coverage in the data. Along with the time required for the calculations, the issue of their accuracy is the most important, since the discrepancies between the results obtained with different models appear to be too great (Tolman et al., 2002, Cavaleri et al., 2007). Numerical modelling is therefore one of the most important tools for the estimation of wave parameters, not only for practical operational use, but also for further conceptual and engineering studies.

All the present-day numerical models for wind waves are based on the solution of the evolution equation for a two-dimensional wave spectrum. From mathematical point of view, a wind wave field has a stochastic feature, and its properties should be governed by a proper statistical ensemble. Therefore, the best way to describe the phenomenon lies in the domain of statistical characteristics. The most significant of them is the two-dimensional spatial wave energy spectrum,  $S(\mathbf{k}, \mathbf{x}, t) \equiv S$ , spread in the space,  $\mathbf{x}$ , and time,  $t$ . The space-time evolution of this characteristics is described by the so called transport equation written in the spectral representation (Komen et al., 1984):

$$\frac{\partial S}{\partial t} + C_{gx} \frac{\partial S}{\partial x} + C_{gy} \frac{\partial S}{\partial y} = F \equiv NL + IN - DIS \quad (1)$$

The LHS is the full time-derivative of the spectrum, and the RHS is the so called source function (“forcing”),  $F$ . Vector  $(C_{gx}, C_{gy})$  is the group velocity corresponding to a wave component with wave vector  $\mathbf{k}$ , which is defined by the ratio

$$\mathbf{C}_g = \frac{\partial \sigma(k)}{\partial k} \frac{\mathbf{k}}{k} = (C_{gx}, C_{gy})$$

Dependence of frequency on the wave vector  $\sigma(\mathbf{k})$  is given by the expression,  $\sigma = \sqrt{gk}$  known as the dispersion relation for the case of deep water, considered below. The LHS of equation (1) is responsible for “mathematical” part of the model, which is not discussed here, whilst the physical essence of the model is held by the source function,  $F$ . At present, it is widely recognized that  $F$  can be written as a sum of three terms – three parts of the united evolution mechanism for wind waves:

- (i) The rate of conservative non-linear energy transfer through a wave spectrum,  $NL$  (non-linear term)
- (ii) The rate of energy transfer from wind to waves,  $IN$  (input term)
- (iii) The rate of wave energy loss due to numerous dissipative processes,  $DIS$  (dissipation term).

The different mathematical expressions (parameterizations) used for the source functions define physical specification or processes of each model. The widely used third generation

models, WAM (The WAMDI group 1988) and WAWEWATCH (WWIII) (Tolman and Chalikov, 1996), are examples for such kind of models. These models are used to solve global tasks of the wave forecast in deep water. SWAN (Booij, 1999) is also a third generation model for the case of finite depth water, and it is used to solve regional tasks.

In this paper, we present the model results of improved version of WAM; two versions of WAM model has been setup for the Indian Ocean - one is WAM (Cycle 4) and the other new version of WAM (NewWAM). Three source functions have been modified in the new version of WAM. The model results obtained are verified using the moored buoy data from the Indian Ocean.

## METHODOLOGY

### Wave models used

#### *WAM (Cycle 4)*

The WAM-model (Cycle 4) is a third generation wave model which solves the wave transport equation explicitly without any presumptions on the shape of the wave spectrum. It represents the physics of the wave evolution in accordance with our knowledge today for the full set of degrees of freedom of a 2d wave spectrum. The model runs for any given regional or global grid with a prescribed topographic dataset. The grid resolution can be arbitrary in space and time. The propagation can be done on a latitudinal – longitudinal or on a Cartesian grid. The model outputs are the significant wave height, mean wave direction and frequency, the swell height and mean direction, wind stress fields corrected by including the wave induced stress and the drag coefficient at each grid point at chosen output times and also the 2D wave spectrum at chosen grid points and output times. The model runs for deep and shallow water and includes depth and current refraction.

#### *NewWAM*

In NewWAM three source functions are modified as follows:

##### *Optimized source function*

i) The wind-wave energy exchange term,  $In$ , is written in the commonly used representation as follows:

$$In = \beta(\sigma, \theta, u_*) \sigma S(\sigma, \theta) \quad . \quad (2)$$

In our version, the wave growth increment  $\beta(\sigma, \theta, u_*)$  has modified as

$$\beta = \max \left\{ -b_L, \left[ 0.04 \left( \frac{u_* \sigma}{g} \right)^2 + 0.00544 \frac{u_* \sigma}{g} + 0.000055 \right] \cos(\theta - \theta_w) - 0.00031 \right\}, \quad (3)$$

Where,  $\sigma$  and  $\theta$  are the frequency and propagation angle, respectively of a wave component;  $u_*$  is the friction velocity,  $g$  is the acceleration due to gravity and  $\theta_w$  is the local wind direction. Note that the condition of existence, a negative value of  $\beta$ , is included into parameterization (3), and the limiting magnitude of which is equal to

$$b_L = 0.000005. \quad (4)$$

The sense of negative value of  $\beta$  indicates to the fact that waves overtaking the local wind must supply their energy to the wind. Transition from the wind  $W_{10}$ , given at the standard height,  $z = 10\text{m}$ , to the friction velocity,  $u_*$ , in the optimized source function is realized by means of a special block for the dynamic boundary layer (DBL) whose complete description is presented in Polnikov (2005). However, in the present simulations, we did not switch on the block of DBL, instead we used  $u_*$  value calculated within the “body” of WAM model (Cycle 4).

ii) The term, *Dis* is represented by the original theoretically substantiated parameterization of the form

$$Dis(\sigma, \theta, S, W) = \tilde{\gamma}(\sigma, \theta, W) \frac{\sigma^6}{g^2} S^2(\sigma, \theta) \quad (5)$$

Where, the non-dimensional function  $\tilde{\gamma}(\sigma, \theta, W)$  is specified as

$$\tilde{\gamma}(\sigma, \theta, W) = c(\sigma, \theta, \sigma_p) \max[0.00005, \beta(\sigma, \theta, u_*)]. \quad (6)$$

Here the value of  $\beta(\sigma, \theta, u_*)$  is defined by formula (3), while the non-dimensional function  $c(\sigma, \theta, \sigma_p)$ , describing details of dissipation processes in the vicinity of peak frequency of the spectrum,  $\sigma_p$ , in our simulations is specified as,

$$c(\sigma, \theta, \sigma_p) = 45 \max[0, (\sigma - 0.5\sigma_p) / \sigma] T(\sigma, \theta, \sigma_p) \quad (7)$$

with the angular function of the following form

$$T(\sigma, \theta, \sigma_p) = \left\{ 1 + 4 \frac{\sigma}{\sigma_p} \sin^2 \left( \frac{\theta - \theta_w}{2} \right) \right\} \max[1, 1 - \cos(\theta - \theta_w)]. \quad (8)$$

It may be noted that in formula (6) for the parameterization of *Dis*, the condition of non-zero level of energy loss by waves is involved, for the frequencies of the order or less than the peak frequency,  $\sigma_p$  or lower, which reflects the fact of the existence of background dissipation processes. This element of parameterization of *Dis* is introduced for the first time in the model (Polnikov, 2005).

iii) For representation of the term *Nl* in our source function we used the Fast version of the Discrete Interaction Approximation (Fast DIA - FDIA), which has already provided high efficiency (Polnikov, 2002). In short, the computational efficiency is achieved by avoiding complicated procedure of interpolation of the present spectra over the calculation grid of the wave components  $\sigma$  and  $\theta$ , which is incorporated into the original version of DIA used in WAM and WWIII (Tolman and Chalikov, 2002; SWAMP, 1985). Numerical realization of FDIA method is given in Polnikov (2005).

The calculation of frequency-angle grid  $\{\sigma, \theta\}$  is given in representation typical for modern wave models

$$\sigma(i) = \sigma_0 \cdot e^{i-1} \quad (0 \leq i \leq I) \quad (9)$$

$$\theta(j) = -\pi + j \cdot \Delta\theta \quad (0 \leq j \leq J) \quad (10)$$

Where,  $\sigma_0$  is the lowest frequency,  $e$  is the exponential increment of the frequency grid,  $I$  is the number of frequencies considered,  $\Delta\theta = 2\pi / J$  is the angular resolution in radians and  $J$

is the number of directions considered. Further, we used the values of these parameters typical of the WAM model version (Cycle 4):

$$\sigma_0 = 2\pi \cdot 0,05 \text{ p/c}, \quad e = 1,1, \quad I = 30, \quad J = 24, \quad \Delta\theta = \pi/12 \quad . \quad (11)$$

In such a case, the optimal configuration of the four interacting waves in the version with FDIA is given by the following ratios:

a) for frequencies

$$\sigma_1 = \sigma e^3, \quad \sigma_2 = \sigma e^4, \quad \sigma_3 = \sigma e^5, \quad (12a)$$

b) for angles

$$\theta_1 = \theta + 2 \Delta\theta, \quad \theta_2 = \theta + 2 \Delta\theta, \quad \theta_3 = \theta + 3 \Delta\theta \quad (12b)$$

For known values of  $\sigma$  and  $\theta$ , for which the calculating loop is arranged. In this loop the expression for the term  $Nl[S(\sigma, \theta)]$  is calculated by known formulas

$$NL(\sigma, \theta) = I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma, \theta) \quad (13a)$$

$$NL(\sigma_3, \theta_3) = I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma, \theta) \quad (13b)$$

$$NL(\sigma_1, \theta_1) = -I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma, \theta) \quad (13c)$$

$$NL(\sigma_2, \theta_2) = -I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma, \theta) \quad (13d)$$

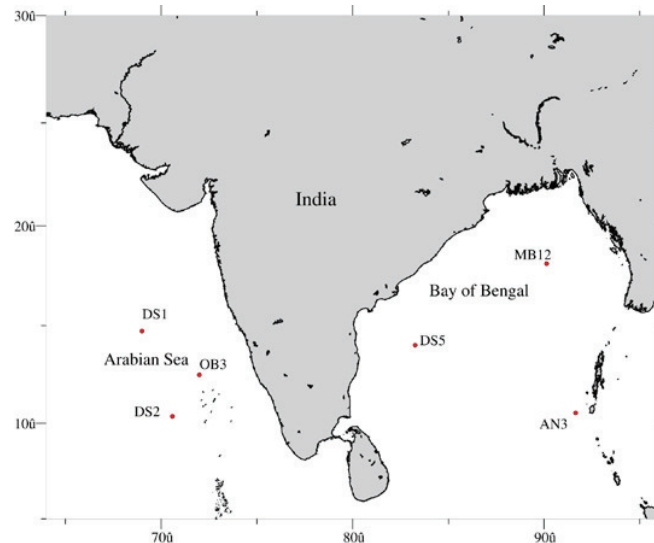
Where

$$I(\sigma_1, \theta_1, \sigma_2, \theta_2, \sigma_3, \theta_3, \sigma, \theta) = \\ = C \sigma^{11} \left[ S_1 S_2 (S_3 + (\sigma_3 / \sigma)^4 S_1) - S_3 S \left( (\sigma_2 / \sigma)^4 S_1 + (\sigma_1 / \sigma)^4 S_2 \right) \right]. \quad (14)$$

In equations 13 and 14, short notations are used;  $S_i = S(\sigma_i, \theta_i)$ ; index 4 is omitted. Values  $(\sigma_i, \theta_i)$  for indices  $i = 1,2,3$  are given by the relations (eqn. 12). The adjusting constant  $C$  has the value 12000, for optimized version of FDIA in the source function given above.

The bathymetry of the study area is prepared using the ETOPO2 (Earth Topography 2 minute) obtained from the National Geophysical Data Centre, USA, with a spatial resolution of  $1.5^\circ \times 1.5^\circ$  in the model. Most of the wave models could simulate the average wave parameters to an agreeable extent, but the reliable simulation of the swell wave components remains as a major limiting factor. The sensitivity analysis of the wave model in which we extended the southern boundary of Indian Ocean from  $5^\circ\text{N}$  till  $60^\circ\text{S}$  proved that the inclusion of south Indian Ocean in the model domain significantly improved the model performance. Hence we considered the model domain bounded by the latitudes  $60^\circ\text{S}$  to  $30^\circ\text{N}$  and longitudes  $10^\circ\text{E}$  to  $144^\circ\text{E}$  for the present study. In the pre-processing, the European Centre for Medium-Range Weather Forecast (ECMWF) winds has been used as the input for the models with a resolution of  $1.5^\circ \times 1.5^\circ$  at every 6h interval. Wave growth was computed using the source terms to account for wind input, non-linear wave-wave interactions and white-capping. Model simulations have

been made for the years 2005 and 2006 and validated with the available buoy measurements (Fig. 1).



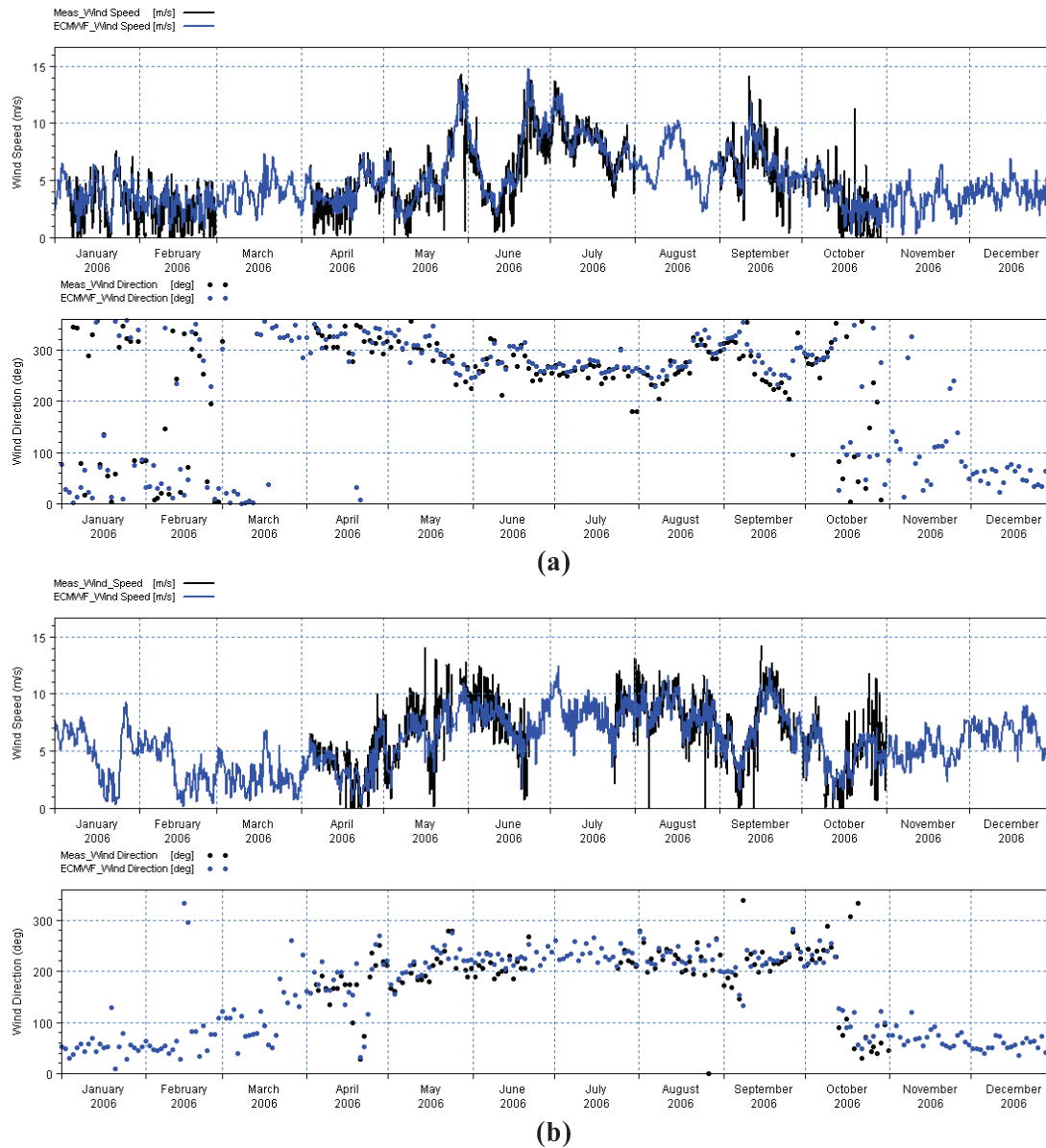
**Figure 1. Location Map with buoys in the Indian Ocean**

The WAM model was constructed on the basis of scientific knowledge, numerics and computational facility of 20 years ago. Therefore, they suffer from processes to be included in the source function terms and mathematical formulation. This necessitates the construction of a modern model of new generation. It is primarily related to the modification of source function  $F$  (Polnikov, 2005).

In the study carried out by Polnikov (2005), an optimal representation for source function with the criterion “accuracy-computational speed” was proposed, based on comparative analysis of results from model with new source terms and results of investigations conducted during the last 5 years with WAM (Cylce 4). On this basis, it was proposed that the parameterization of  $F$  could serve as a ground for construction of the NewWAM.

## **RESULTS AND DISCUSSION**

Selection of appropriate wind data with sufficient spatial and temporal resolutions over a large region is a major task which greatly determines the accuracy of the model wave parameters. The European Centre for Medium-Range Weather Forecasts 40-year reanalysis (ECMWF ERA-40) winds were used as input to the model. Accuracy of the wind input (ECMWF winds) was tested by comparing the ECMWF winds with the measured buoy winds at two locations (Fig. 2a & 2b): DS2 (Arabian Sea) and DS5 (Bay of Bengal). Correlation coefficients of 0.9 (bias = 0.38, rms=1.37) and 0.86 (bias = -0.14, rms=1.53) were obtained for the wind speeds at DS2 and DS5. Seasonal variabilities including monsoon conditions are clearly evident in the ECMWF winds as seen in the measurements. These give us the confidence to use ECMWF winds for the analysis and model simulations.



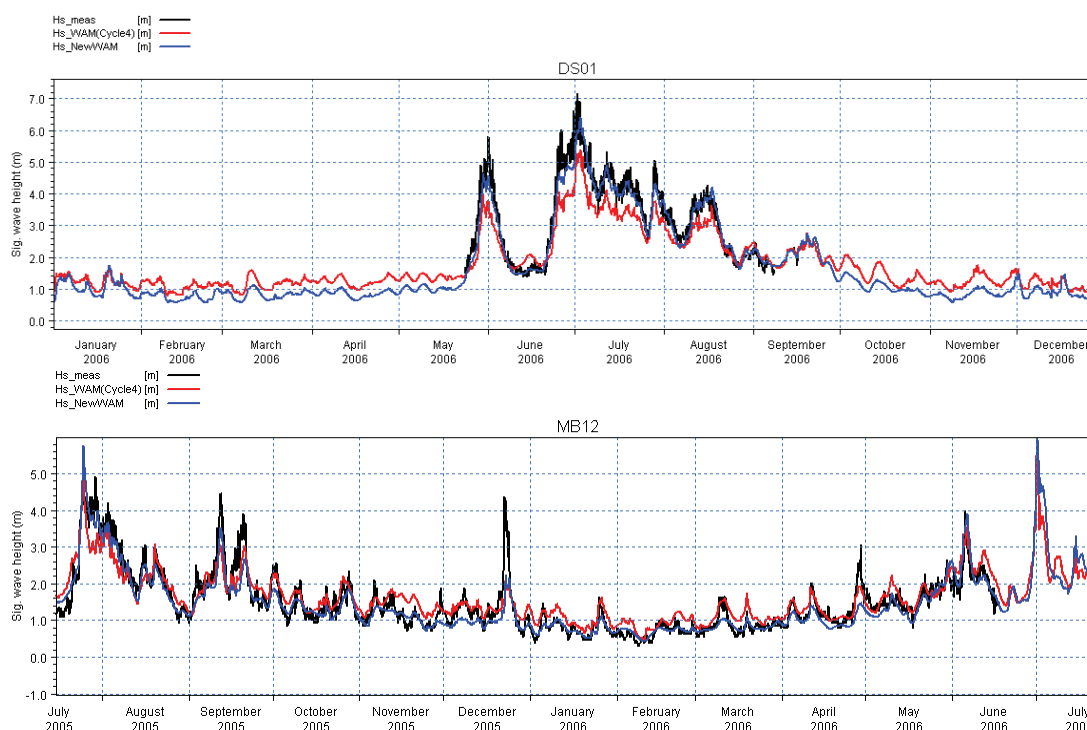
**Fig. 2: Comparison of time series of ECMWF winds (blue line) with buoy winds (black line) at two locations: (a) DS2 (Arabian Sea) and (b) DS5 (Bay of Bengal).**

Firstly, simulations were setup for the WAM (Cycle 4) for the Indian Ocean. Winds were extracted from ECMWF data server and converted in the format of WAM model. The model was run for the years 2005 and 2006. Similarly, the new version of WAM (NewWAM) was setup for the Indian Ocean with changed source functions. Various trial runs were carried out by varying four constants in the included physical parameterization of the input and the dissipation source terms. Thus, the merits of new source function were revealed by means of comparison the simulation results obtained for WAM and new version of WAM model. The results obtained from the two versions of the model were compared with the 5 moored buoy measurements available for the period 2005-2006 in the Indian Ocean. The statistical analysis

was carried out for WAM and the NewWAM, in order to estimate the best results obtained. The error estimates for the WAM (Cycle 4) and NewWAM are given in Table 1.

Year	Buoy ID	WAM (Cycle 4)				New WAM			
		Bias	RMS Error	Scatter Index	Corr Coefft	Bias	RMS Error	Scatter Index	Corr Coefft
2005	DS01	-0.05	0.47	0.28	0.96	-0.15	0.28	0.17	0.98
	DS02	0.19	0.47	0.34	0.91	0.02	0.42	0.30	0.91
	OB03	0.26	0.51	0.35	0.91	0.15	0.36	0.25	0.94
	MB12	0.08	0.4	0.28	0.9	-0.12	0.34	0.23	0.93
2006	MB12	0.18	0.29	0.25	0.92	-0.07	0.25	0.21	0.91
	AN03	0.07	0.3	0.17	0.94	-0.17	0.31	0.18	0.94

**Table 1. Error Estimates for WAM (Cycle 4) and the NewWAM**



**Fig. 3: Comparison of significant wave heights of Buoy (DS01 and MB12), WAM (Cycle 4) and NewWAM.**

Quantitatively, the characteristics of the differences in the model accuracies are obtained from routine statistical procedures. From the statistical analysis (Table 1) we see that there is good improvement in the wave prediction by NewWAM. Considering the wind and wave conditions prevailing over the Indian Ocean during different seasons and extreme weather conditions, we have varied the input and dissipation coefficients till we obtained accurate wave predictions. Also, the coefficients related to nonlinear and the background dissipation have been changed to check the difference in model outputs.



Comparison of significant wave heights  $H_s$ , obtained from the two models, is shown in Figure 3. The comparison shows that NewWAM reproduced the wave conditions of all seasons better than the WAM (Cycle 4). In fact, several runs that we have simulated with both the models, and their comparison with moored data buoy measurements at several locations proves that precisely the NEW model provides a more accurate presentation for extreme waves and high waves, as shown in the above typical figure (Fig. 3). In Fig. 3, though the data availability for buoy DS01 is very limited, it can be noted that the NewWAM reproduced the high waves of significant wave height above 7.0 m during the southwest monsoon season, very accurately than the WAM (Cycle 4). But, still there is underestimation of  $H_s$  during the high waves (needs to be improved). To get more accurate results, may be, we have to run the NewWAM with fine resolution winds. It may be noted from the Table 1 that the accuracy of wave prediction has been increased by 11 to 67 % for the different buoys considered in the present study. Taking into account the systematic accuracy improvement in the NEW model implemented for the Indian Ocean, and for all the buoys at a time, we may infer that, in our verification technique, this accuracy improvement is completely obtained only because of the changes made in the source functions, considering the specific oceanic and atmospheric processes related to the Indian Ocean.

## CONCLUSIONS

An attempt was made to improve the accuracy of wave predictions in the Indian Ocean by modifying the Original WAM (Cycle 4). The three important source functions wind input, breaking dissipation, and nonlinear interaction were modified in the wave model by fitting new coefficients into their respective parameterizations based on the physical processes related to seasonal changes in the Indian Ocean. Thus, the merits of new source function were revealed by means of comparison with the simulation results obtained for original and modified versions of model NewWAM with the moored buoy measurements in the Indian Ocean. From the error estimates extracted from the comparisons, it is found that the NewWAM improved the accuracy of the wave predictions in the deep waters of the Indian Ocean by 11 to 67%.

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