

Variation of wave directional spread parameters along the Indian coast

V. Sanil Kumar

Ocean Engineering Division

National Institute of Oceanography, Goa - 403 004, India

Tel: 0091 832 2450327 : Fax: 0091 832 2450604 Email: sanil@nio.org

Abstract

Directional spreading of wave energy is popularly modeled with the help of the Cosine Power model and it mainly depends on the spreading parameter. This paper describes the variation of the spreading parameter estimated based on the wave data collected at four locations along the East as well as West side of the Indian coast. The directional spreading parameter was correlated with other characteristic wave parameters, like non-linearity parameter, directional width. Working empirical relationships between them had been established. The mean spreading angle was found to be slightly lower than the directional width with a correlation coefficient of 0.8. The maximum spreading parameter, s , underwent a sudden reduction in its value with the increase in the value of directional width. The average value of the maximum spreading parameter for the locations studied was found to be 23. The study shows that the spreading parameter can be related to significant wave height, mean period and water depth through the non-linearity parameter and can be estimated with an average correlation coefficient of 0.7 for the Indian coast and with higher correlation coefficient of 0.9 for the high waves ($H_s > 1.5$ m).

Keywords: Directional waves, Spreading function, non-linearity parameter, directional width, mean spreading angle

1. Introduction

One of the major wave characteristics is the directional distribution of wave energy at a given location. Short-term directional wave characteristics can be conveniently studied through a directional wave spectrum, which represents distribution of wave energies over various wave frequencies and directions. Most widely practiced technique for directional data collection involves use of the floating buoys. The data analysis methods for such systems are based on cross-spectral analyses of the three signals obtained from the buoys. In the analysis it is assumed that the sea is made up of non-interacting waves and also that over the frequency range of interest; the buoy follows the slope of sea surface perfectly. In the literature a number of methods are available to estimate directional spectrum from the measurements made by a moored buoy [1-6]. The simplest among them considers representation of the directional spectrum as a product of unidirectional spectrum and a directional spreading function. The directional spreading function can be modelled using a variety of parametric models [1,7-9]. No single model, however, is universally accepted due to either the idealization involved in its formulation or site-specific nature associated with it [10]. Ewans [11] found that the angular distribution is bimodal at frequencies greater than the spectral peak frequency. Hwang and Wang [12] found that unimodal distribution exists in a narrow wave number range near the spectral peak. For wave components shorter than the dominant wave length, bimodality is a robust feature of the directional distribution [13, 14]. Non-linear wave-wave interaction is the mechanism that generates bimodal feature [15].

Cosine power-2s model, proposed by Longuet-Higgins *et al.* [1], is popular and common, owing perhaps to its ease of applications. This model is unimodal and involves a sensitive parameter 's' called the spreading parameter and has varying values as per the method of its derivation. Cartwright [16] showed that 's' can be related either to the first order Fourier coefficients a_0 , a_1 and b_1 , or second order ones, viz., a_2 and b_2 . Accordingly two estimates of 's' viz., 's₁' and 's₂' result. Results of Mitsuyasu *et al.* [17], Cartwright [16], Hasselman *et al.* [18], Tucker [19], Wang *et al.* [20] and Wang [21] show that the values of 's₁' and 's₂' are different. The difference may be due to the bimodality, the noise in the system and the limitation of the resolution of the pitch-roll buoys [16, 19]. The effect of noise on 's₁' and 's₂' by numerical simulation was studied by Ewing and Laing [22] who concluded that 's₁' is more sensitive to noise than 's₂'. Though it can be calculated using alternative formulae, many researchers have assumed a constant value of 's'. However in actual, magnitude of 's' varies with that of significant wave height, its frequency, etc. and a uniform recommendation in this regard is lacking. Kumar *et al.* [23] related spreading parameter obtained from first order Fourier coefficients with significant wave height, mean wave period and water depth based on the south-west monsoon data collected at 23 m water depth along the west coast of India. Deo *et al.* [24] found that the spreading parameter 's' was highly correlated with significant wave height and mean period using the neural network technique. The information of directional spreading parameters based on data covering all seasons in a year is not available.

In the light of the above discussion, objectives of the present study was: i) to estimate the directional spreading parameter near peak frequency at different locations using the data covering all seasons in a year, ii) study the interrelationship among the various parameters involved, viz., mean spreading angle, directional width, significant wave height, mean period and non-linearity parameter and iii) arrive at empirical equations for the spread parameter at peak frequency.

2. Materials and methods

The data collected by National Institute of Oceanography, Goa using Datawell directional waverider buoy [25] at 4 locations (Table 1) along the Indian coast in the past were used for the analysis. At all locations the data were recorded for 20 minutes duration at every 3 hr. interval for a period of one year. Data were sampled at a frequency of 1.28 Hz and fast Fourier transform of 8 series, each consisting of 256 data points were added to give spectra with 16 degrees of freedom. The high frequency cut off was set at 0.6 Hz and the resolution was 0.005 Hz. The significant wave height (H_s) and the average wave period (T_{02}) were obtained from the spectral analysis. The low rate of data at location 4 is due to drifting of the deployed buoy from its moored location due to cutting of the mooring line by fishermen. At all locations the internal battery of the buoy was changed once and the buoy was retrieved for cleaning the biofouling growth on the outer surface. Hence 100% data could not be collected. Also the collected time series was subjected to error checks as mentioned in Kumar [26] and the records, which were found suitable, only were included in the analysis. Due to low

sample size, any bias or special behaviour in results of location 4 was not observed. The data analysis was carried out by using the technique proposed by Kuik *et al.* [6] wherein the characteristic parameters of directional spreading function at each frequency were obtained directly from Fourier coefficients a_0 , a_1 , b_1 , a_2 and b_2 without any assumption of model form. Fourier coefficients were estimated from auto, co- and quadrature spectra of the collected buoy signals. Cartwright [16] showed that spreading parameter 's' can be related either to the first order Fourier coefficients a_0 , a_1 and b_1 , as s_1 or second order ones, a_2 and b_2 as s_2 as given below:

$$s_1 = \frac{r_1}{1-r_1} \quad \text{and} \quad s_2 = \frac{1+3r_2+(1+14r_2+r_2^2)^{1/2}}{2(1-r_2)} \quad (1)$$

$$\text{where } r_1 = \frac{\sqrt{a_1^2 + b_1^2}}{a_0} \quad \text{and} \quad r_2 = \frac{\sqrt{a_2^2 + b_2^2}}{a_0} \quad (2)$$

$a_0 = 1$ for the given definitions of a_1 , b_1 , a_2 and b_2 .

For fairly steady wind condition and mild sea states, Mitsuyasu *et al.* [17] showed that maximum spreading parameter corresponding to the peak frequency of wave spectrum namely, s_U , can be determined from the non-dimensional parameter of wave age, which is the ratio of wave speed to wind speed as below.

$$s_U = 11.5 (c_m/U)^{2.5} \quad (3)$$

In the above equation c_m is wave speed associated with peak frequency (f_p) and U is measured wind speed at 10 m height above mean sea level.

Wang [21] showed that value of spreading parameter at peak frequency namely, s_L , can be related to the wave length (L_p) associated with peak frequency (f_p) of the spectrum (determined from the linear dispersion relation), and the significant wave height (H_s) as under:

$$s_L = 0.2 (H_s/L_p)^{-1.28} \quad (4)$$

Kumar *et al.* [23] related the spreading parameter obtained from first order Fourier coefficients with wave non-linearity parameter, F_C based on the data collected at location 1 during southwest monsoon (June to September) as follows.

$$s_3 = 5.28 F_C^{0.59} (f/f_p)^b \quad \text{with } b=5 \text{ for } f \leq f_p \text{ and } b=-2.5 \text{ for } f > f_p \quad (5)$$

where F_C is related to significant wave height (H_s), wave period (T_{02}) and water depth (d) as given below [27]. The lower and upper limits of the frequency considered were 0.025 and 0.6 Hz. These were based on the data measuring system.

$$F_C = \sqrt{\frac{H_s}{d}} \left(\frac{g T_{02}^2}{d} \right)^{0.2} \sqrt{\frac{g}{d}} \quad (6)$$

and g is acceleration due to gravity.

The directional spreading parameters obtained from the measured data are directional width (σ) as given by Kuik *et al.* [6] and mean spreading angle (θ_k) and long crestedness parameter (τ) as per Goda *et al.* [28].

$$\sigma = \sqrt{2(1-r_1)} \quad (7)$$

$$\theta_k = \arctan \left\{ \frac{\sqrt{0.5 b_1^2 (1 + a_2) - a_1 b_1 b_2 + 0.5 a_1^2 (1 - a_2)}}{a_1^2 + b_1^2} \right\} \quad (8)$$

$$\tau = \left\{ \frac{1 - \sqrt{a_2^2 + b_2^2}}{1 + \sqrt{a_2^2 + b_2^2}} \right\}^{1/2} \quad (9)$$

3. Results and discussions

3.1 Wave heights

The minimum, maximum and mean values of the H_S and H_{max} for locations 1 to 4 are given in Table 2. The values of H_{max} observed from each 20 minutes record show that they were approximately 1.65 times those of H_S values and that they co-vary with correlation coefficients of 0.99, 0.96 0.92 and 0.98 for locations 1 to 4 respectively [26]. The concept of statistical stationarity of wave height was originally proposed by Longuet-Higgins [29] and shown that the wave amplitudes in narrow banded spectrum will be Rayleigh distributed. The H_{max} values estimated based on Rayleigh distribution show that for the locations considered in the present study, the H_{max} value was 1.65 times H_S . An earlier study by Rao and Baba [30] on the one-week data collected off location 1 in 80 m water depth had indicated the ratio between H_{max} and H_S as 1.75.

The probability distributions of significant wave height (H_S) in different ranges are shown in Tables 3 to 6 for different locations. At all the locations the waves were predominantly between 0.5 and 1 m with a cumulative probability of 0.40, 0.59, 0.55 and 0.38 at locations 1, 2, 3 and 4 respectively. The significant wave heights

were generally low with values less than 2.5 m for all shallow water locations (Tables 4 to 6) with water depths around 15 m. At location 1, the wave heights were relatively high during June to September. Highest H_S of 5.69 m was observed in June during the passage of a storm. At location 2, wave heights were relatively high during the northeast monsoon period (October to January) and at location 3, the wave heights were relatively high during May to November. Highest H_S of 3.29 m was recorded in November during the passage of a storm close to the measurement location. At location 4, the wave heights were high during southwest monsoon period (June to September).

3.2 Spreading angle and directional width

Values, viz., the spreading angle, θ_k and directional width, σ provide a measure of the energy spread around the mean direction of wave propagation. While the directional width was calculated using the first order Fourier coefficients, the mean spreading angle was determined using first as well as second order Fourier Coefficients. The mean spreading angle generally varies from 0 to $\pi/2$ and will be zero for unidirectional waves. The present study revealed that the values of σ were larger than those of θ_k at all the four locations (Figure 1). Goda *et al.* [28], Benoit [31] and Besnard and Benoit [32] had earlier observed similar differences for these parameters. The exception to this is a very small range of high width, which might be due to the effect of consideration of second order Fourier Coefficients in deriving θ_k . The average value of σ was 22°, 20°, 17° and 17° for locations 1 to 4 and the corresponding value of θ_k was 16°, 16°, 13° and 13°. The low value of θ_k and σ was due to the fact that the waves were recorded close to

the coast at shallow water, where the wave directional spreading will be narrow due to refraction. The average correlation coefficient (r) between θ_k and σ was 0.8. The distributions of directional width parameter with H_S for all locations are given in Tables 3 to 6 and it shows that values of directional width predominantly varied between 10 and 40°. As the wave height increased the value of directional width reduced.

3.3 Spreading parameter and directional width

The value of the spreading parameter ' s_1 ' corresponding to the peak of the wave energy spectrum is called the maximum spreading parameter ' s_1 '. As expected, directional width, σ , increases, with reduction in the maximum spreading parameter ' s_1 '. The spreading parameter ' s_1 ' is related to the directional width as given below.

$$s_{1 \max} = 2 \sigma^{-2} - 1 \quad (10)$$

The average values of spreading parameter ' s_1 ' estimated were 16, 20, 28 and 29 for locations 1 to 4. Goda [33] has recommended maximum spreading parameter value of 10 for wind waves, 25 for swell with short decay and 75 for swell with long decay distance for deep water waves. When waves propagate into shallow water, the directional spreading is narrowed owing to the wave refraction effect and the equivalent value of maximum spreading parameter increases accordingly.

3.4 Spreading and other wave parameters

The average value of long crestedness parameter was 0.28, 0.28, 0.22 and 0.22 for locations 1 to 4. It generally varies from 0 to 1 and will be zero for unidirectional waves. As expected the spreading parameter 's₁' was decreasing at all locations for higher values of long crestedness parameter. The Kurtosis of the directional distribution at peak frequency was found to increase with increase in spreading parameter 's₁'.

Considering H_S, T₀₂ and d data of locations 1 to 4, a multiple regression was carried out and the expression for maximum spreading parameter was derived as given below with a multiple regression coefficient of 0.8.

$$S_{4 \max} = 46.25 H_S^{0.107} T_{02}^{0.963} d^{-0.854} \quad (11)$$

Figure 2 shows the variation of spreading parameter at peak frequency estimated based on above equation (11) and the one estimated from the earlier derived equation (5) based on non-linearity parameter F_C. It shows that the spreading parameter estimated either way is almost same with correlation coefficient (r) of 0.98. Even though the above equations relating spreading and wave parameter is not non-dimensional they can be used to derive an unknown parameter involved from the known one.

3.5 Differences in the spreading parameter values

It would be of interest to know different values of the spreading parameter viz., s_1 , s_2 and s_3 . The Fourier coefficients related parameters ' s_1 ' and ' s_2 ' estimated based on equation (1) were compared and found that the values of ' s_1 ' were always smaller than those of ' s_2 ' which was consistent with the observations of Cartwright [16], Mitsuyasu *et al.* [17], Hasselman *et al.* [18], Tucker [19] and Wang [21]. The difference was attributed to the noise in the system, which affects the second moments involved in calculation of ' s_2 ' and also to the limitation of resolution of the buoy data [34]. The spreading parameters ' s_U ' obtained from equation (3) was found to be not reliable for locations 1 and 2, where the waves were not locally generated and the difference in wave direction and wind direction was large [26]. Also the wind speed and its direction observed at the coast will be modified due to the sea and land breeze effects. Hence ' s_U ' is not considered in the present study. Spreading parameter ' s_L ' estimated from wave steepness was also found to deviate from the other spreading parameters. Values of spreading parameter ' s_3 ' obtained at all locations are compared in Figure 3 with the corresponding ' s_1 ' value. Value of the spreading parameter ' s_3 ' estimated from equation (5) was found to be fairly comparable with ' s_1 '. It shows that ' s_1 ' generally provides an envelop to spreading parameter ' s_3 '.

The variation of spreading parameter at peak frequency with correlation coefficient between ' s_1 ' and ' s_3 ' shows (Figure 4) that as the value of the spreading parameter increases, the correlation coefficient increases. The average correlation coefficient between ' s_1 ' and ' s_3 ' was 0.7, 0.63, 0.69 and 0.7 for

locations 1 to 4. It was also observed that as the significant wave height, mean wave period or spectral peakedness parameter increases, high correlation was observed between the spreading parameter ' s_3 ' estimated based on empirical equation (5) and ' s_1 ' based on first order Fourier coefficients. For high waves ($H_S > 1.5$ m), the average correlation coefficient was 0.9. The design condition for most of the structures is guided by the storm generated severe sea and the directional distribution will be narrow. The study shows under such conditions the spreading parameter can be related to significant wave height, mean period and water depth through the non-linearity parameter and can be estimated with high correlation coefficient. At location 1, the maximum H_S observed was 5.69 m during the passage of a storm and the value of ' s_1 ' and ' s_3 ' estimated were 41.3 and 40.6. At location 3, the maximum H_S observed was 3.29 m during the passage of a storm and the value of ' s_1 ' and ' s_3 ' estimated were 57 and 52.3. The advantage of the proposed equation is that it allows the design engineer to use design wave height and wave period to obtain ' s ' without going for the input information of wind and time series data on waves.

4. Conclusions

1. The mean spreading angle was found to be slightly lower than the directional width.
2. The value of spreading parameter ' s_2 ' was found to be larger than that of ' s_1 '. The spreading parameter ' s_1 ' determined from the first order Fourier coefficients seemed to provide an enveloping curve to spreading parameter ' s_3 '.

3. The spreading parameter, s_1 , at peak frequency was found to increase with the increasing value of non-linearity parameter, F_C . Following equation is recommended to estimate the spreading parameter for all the locations around Indian coast.

$$s_1 = 5.28 (F_C)^{0.59} (f/f_p)^b \quad \text{with } b=5 \text{ for } f \leq f_p \text{ and } b=-2.5 \text{ for } f > f_p$$

4. The average correlation coefficient between ' s_1 ' and ' s_3 ' was 0.7, 0.63, 0.69 and 0.7 for the locations 1 to 4. As the significant wave height, mean wave period or spectral peakedness parameter increased, high correlation was observed between the spreading parameter ' s_1 ' and ' s_3 '.

Acknowledgments

The author thanks Director of the institute for providing facilities and the colleagues who were involved in the data collection programme. This paper is NIO contribution number 4113.

References

- [1] Longuet-Higgins MS, Cartwright DE, Smith ND. Observations of the directional spectrum of sea waves using the motions of a floating buoy. *In Ocean Wave Spectra*, Prentice Hall, New York 1963:111-136.
- [2] Borgman LE. Maximum entropy and data-adaptive procedures in the investigation of ocean waves, *Second workshop on maximum entropy and bayesian methods in applied statistics*, Laramie 1982:111-117.
- [3] Isobe M, Kondo K, Horikawa K. Extension of MLM for estimating directional wave spectrum. *Symposium on Description and Modelling of Directional Seas*, Technical University, Denmark 1984; A-6: 1-15.

- [4] Oltman-Shay J, Guza RT. A data adaptive ocean wave directional spectrum estimator for pitch and roll type measurements, *Journal of Physical Oceanography* 1984; 14: 1800-1810.
- [5] Kobune K, Hashimoto N. Estimation of directional spectra from the maximum entropy principle, *Proceedings 5th International Conference on Offshore Mechanics and Arctic Engineering*, Tokyo, Japan 1986; vol. I: 80-85.
- [6] Kuik AJ, Vledder G, Holthuijsen LH. A method for the routine analysis of pitch and roll buoy wave data, *Journal of Physical Oceanography* 1988; 18: 1020-1034.
- [7] Pierson WJ, Neuman G, James RW. *Practical methods for observing and forecasting ocean waves by means of wave spectra and statistics* 1955; Pub. No. 603. US Naval Hydrographic Office.
- [8] Cote LJ, Davis O, Markes W, MCGough RJ, Mehr E, Pierson WJ, Ropek JF, Stephenson G, Vetter RC. The directional spectrum of a wind generated sea as determined from data obtained by the Stereo Wave Observation Project, *Meteor. Pap.* 2(6), 1960; New York University, College of Engineering.
- [9] Donelan MA, Hamilton J, Hui WH. Directional spectra of wind generated waves, *Philosophical Transactions of Royal Society*, London 1985; A 315: 509-562.
- [10] Niedzwecki JM, Whatley CP. A comparative study of some directional sea models, *Ocean Engineering* 1991; 18 (12): 111-128.
- [11] Ewans KC. Observations of the directional spectrum of fetch-limited waves, *Journal of Physical Oceanography* 1998; 28: 495-512.
- [12] Hwang PA and Wang DW. Directional distributions and mean square slopes in the equilibrium and saturation ranges of the wave spectrum, *Journal of Physical Oceanography* 2001; 31: 1346-1360.
- [13] Hwang PA, Wang DW, Walsh EJ, Krabill WB and Swift RN. Airborne measurements of the wave number spectra of ocean surface waves. Part

- II: Directional distribution, *Journal of Physical Oceanography* 2000; 30: 2768-2787.
- [14] Wang DW and Hwang EJ. Evolution of the bimodal directional distribution of ocean waves, *Journal of Physical Oceanography* 2001; 31: 1200-1221.
- [15] Banner ML and Young IR. Modeling spectral dissipation in the evolution of wind waves. Part I: Assessment of existing model performance, *Journal of Physical Oceanography* 1994; 24: 1550-1571.
- [16] Cartwright, D.E. The use of directional spectra in studying the output of a wave recorder on a moving ship, *In Ocean Wave Spectra*, Prentice Hall, New York 1963; 203-219.
- [17] Mitsuyasu H, Tasai F, Suhara T, Mizuno S, Ohkusu M, Honda T, Rikiishi K. Observations of the directional spectrum of ocean waves using a cloverleaf buoy, *Journal of Physical Oceanography* 1975; 5: 750-760.
- [18] Hasselmann DE, Dunckel M, Ewing JA. Directional wave spectra observed during JONSWAP 1973, *Journal of Physical Oceanography* 1980; 10: 1264-1280.
- [19] Tucker MJ. Directional wave data: The interpretation of the spread factors, *Deep Sea Research* 1987; 3(4): 633-636.
- [20] Wang DW, Tang CC, Steck KE. Buoy directional wave observations in high seas, *Proc. Oceans'89*, Seattle 1989; 1416-1420.
- [21] Wang DW. Estimation of wave directional spreading in severe seas, *Proceedings of the Second International Offshore and Polar Engineering Conference*, San Francisco, III 1992; 146-153.
- [22] Ewing JA, Laing AK. Directional spectra of seas near full development, *Journal of Physical Oceanography* 1987; 17: 1696-1706.
- [23] Kumar VS, Deo MC, Anand NM, Kumar KA. Directional spread parameter at intermediate water depth, *Ocean Engineering* 2000; 27: 889-905.

- [24] Deo MC, Gondane DS, Kumar VS. Analysis of wave directional spreading using neural networks, *Journal of Waterway, Port, Coastal and Ocean Engineering* 2002; 128 (1): 30-37.
- [25] Stephen FB, Tor Kollstad. Field trials of the directional waverider, *Proceedings of the First International Offshore and Polar Engineering Conference, Edinburgh, III, 1991*; 55-63.
- [26] Kumar VS. Analysis of directional spreading of wave energy with special reference to Indian coast, Unpublished Ph. D Thesis, Indian Institute of Technology, Bombay, India 1999; 150p.
- [27] Swart D H, Loubser CC. Vocoidal theory for all non-breaking waves, *Proc. 16th coastal engineering conference 1978*; vol.1: 467-486.
- [28] Goda Y, Miura K, Kato K. On-board analysis of mean wave direction with discus buoy, *Proceedings International Conference on wave and wind directionality- Application to the design of structures, Paris 1981*; 339-359.
- [29] Longuet-Higgins MS. On the statistical distribution of the heights of the sea waves, *Journal of Marine Research* 1952; 11(3): 245-266.
- [30] Rao CVKP, Baba M. Observed wave characteristics during growth and decay: a case study, *Continental Shelf Research* 1996; 16(12): 1509-1520.
- [31] Benoit M. Practical comparative performance survey of methods used for estimating directional wave spectra from heave-pitch-roll data, *Proceedings Conference Coastal Engineering* 1992; 62-75.
- [32] Besnard JC, Benoit M. Representative directional wave parameters - Review and comparison on numerical simulations, *International symposium: Waves - Physical and numerical modeling, IAHR, Vancouver 1994*; 901-910.
- [33] Goda Y. Random seas and design of maritime structures, *University of Tokyo press, Tokyo 1985*; 420p.
- [34] Tucker MJ. Interpreting directional data from large pitch-roll-heave buoys, *Ocean Engineering* 1989; 16(2): 173-192.

Table 1 Details of wave data collection sites

Location	Site (off)	Water depth (m)	Distance from coast (km)	Period	Percentage of data used for analysis
1	Goa	23	10.5	June 96- May 97	91.1
2	Nagapattinam, Tamil Nadu	15	5.0	March 95- February 96	81.0
3	Visakhapatnam	12	2.0	December 97 – November 98	81.2
4	Gopalpur, Orissa	15	2.0	January 94- December 94	57.6

Table 2 Minimum, maximum and mean value of H_s and H_{max}

Location	Significant wave height (m)			Maximum wave height (m)		
	Minimum	Maximum	Mean	Minimum	Maximum	Mean
1	0.15	5.69	1.07	0.26	10.14	1.89
2	0.21	1.82	0.66	0.34	3.21	1.15
3	0.26	3.29	0.88	0.47	5.99	1.54
4	0.21	2.52	0.81	0.33	4.59	1.38

Table 3 Probability distribution of directional width with significant wave height for location 1

H_s (m)	directional width at peak frequency (Deg)							Total
	0-10	10-20	20-30	30-40	40-50	50-60	60-70	
0.0-0.5	-	0.059	0.125	0.030	0.006	0.002	0.001	0.223
0.5-1.0	-	0.115	0.248	0.032	0.006	-	-	0.401
1.0-1.5	-	0.076	0.082	0.010	0.003	-	-	0.171
1.5-2.0	-	0.065	0.021	0.003	-	-	-	0.089
2.0-2.5	-	0.024	0.006	-	-	-	-	0.030
2.5-3.0	0.001	0.041	0.001	-	-	-	-	0.043
3.0-3.5	-	0.026	0.002	-	-	-	-	0.028
3.5-4.0	-	0.006	0.001	-	-	-	-	0.007
4.0-4.5	-	0.002	-	-	-	-	-	0.002
4.5-5.0	-	0.006	-	-	-	-	-	0.006
Total	0.001	0.420	0.486	0.075	0.015	0.002	0.001	1.000

Table 4 Probability distribution of directional width with significant wave height for location 2

H _s (m)	directional width at peak frequency (Deg)							Total
	0-10	10-20	20-30	30-40	40-50	50-60	60-70	
0.0-0.5	0.001	0.133	0.115	0.032	0.011	0.004	0.002	0.298
0.5-1.0	0.008	0.347	0.200	0.027	0.002	0.002	-	0.586
1.0-1.5	-	0.067	0.043	0.001	-	0.001	-	0.112
1.5-2.0	-	0.004	-	-	-	-	-	0.004
Total	0.009	0.551	0.358	0.060	0.013	0.007	0.002	1.000

Table 5 Probability distribution of directional width with significant wave height for location 3

H _s (m)	directional width at peak frequency (Deg)							Total
	0-10	10-20	20-30	30-40	40-50	50-60	60-70	
0.0-0.5	-	0.083	0.023	0.005	0.001	-	-	0.112
0.5-1.0	0.013	0.401	0.127	0.011	0.001	-	-	0.553
1.0-1.5	0.015	0.215	0.041	0.002	-	-	-	0.273
1.5-2.0	0.005	0.052	0.003	-	-	-	-	0.060
2.0-2.5	-	0.001	-	-	-	-	-	0.001
2.5-3.0	-	-	-	-	-	-	-	-
3.0-3.5	-	0.001	-	-	-	-	-	0.001
Total	0.033	0.753	0.194	0.018	0.002	-	-	1.000

Table 6 Probability distribution of directional width with significant wave height for location 4

H _s (m)	directional width at peak frequency (Deg)							Total
	0-10	10-20	20-30	30-40	40-50	50-60	60-70	
0.0-0.5	0.015	0.226	0.061	0.008	-	-	-	0.310
0.5-1.0	0.015	0.248	0.099	0.011	0.001	-	-	0.374
1.0-1.5	0.006	0.176	0.052	0.001	-	-	-	0.235
1.5-2.0	0.003	0.071	0.002	-	-	-	-	0.076
2.0-2.5	0.002	0.001	0.001	-	-	-	-	0.004
2.5-3.0	-	0.001	-	-	-	-	-	0.001
Total	0.041	0.723	0.215	0.020	0.001	-	-	1.000

List Figures

Figure 1. Variation of mean spreading angle (θ_k) with directional width (σ).

Figure 2. Variation of maximum spreading parameter ' s_3 ' estimated based on non-linearity parameter and ' s_4 ' estimated from multiple regressions.

Figure 3. Variation of ' s_1 ' and ' s_3 ' at peak frequency with time.

Figure 4. Variation of correlation coefficient (r) between s_1 and s_3 with spreading parameter, s_1 , at peak frequency.

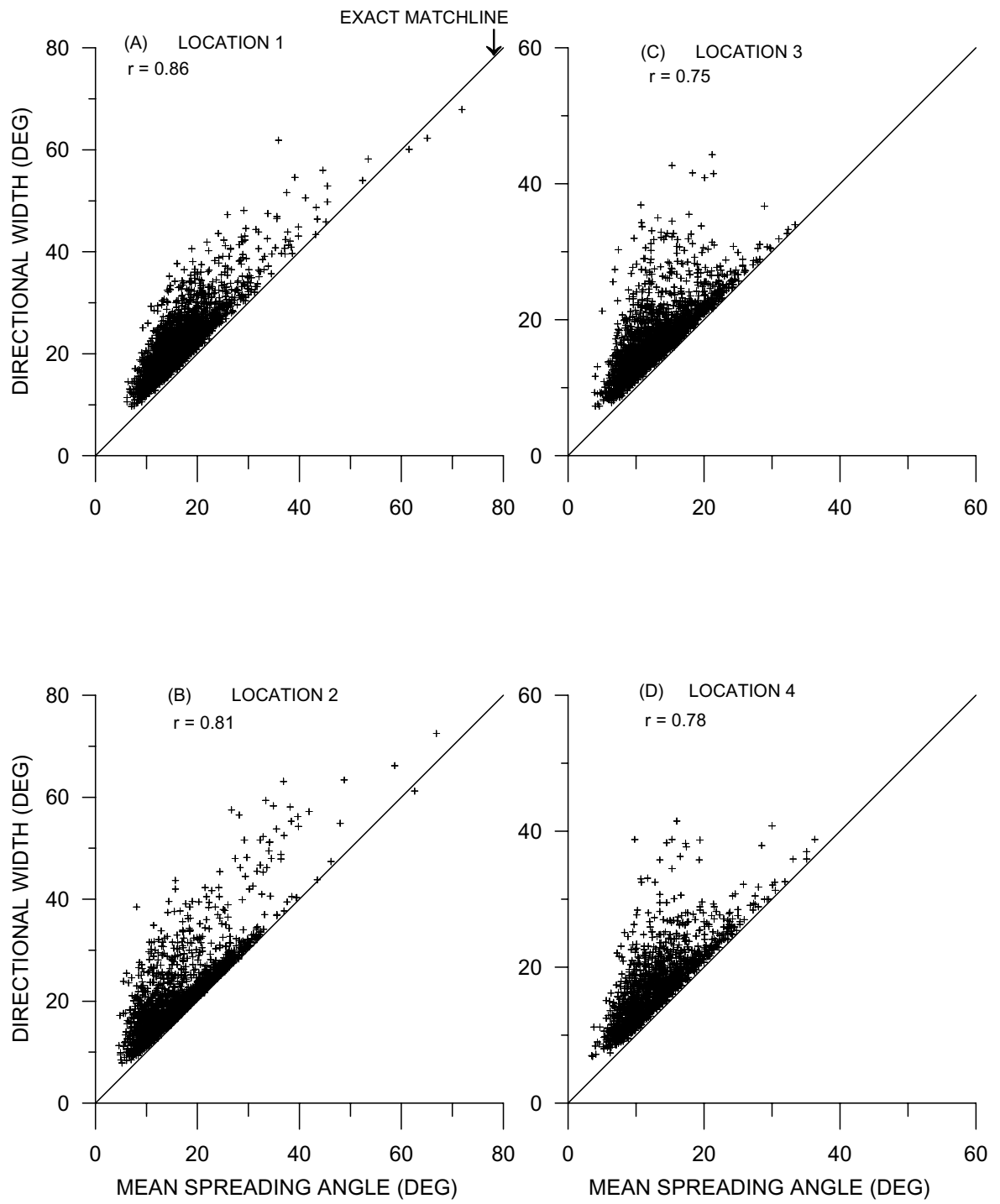


Figure 1. Variation of mean spreading angle (θ_k) with directional width (σ).

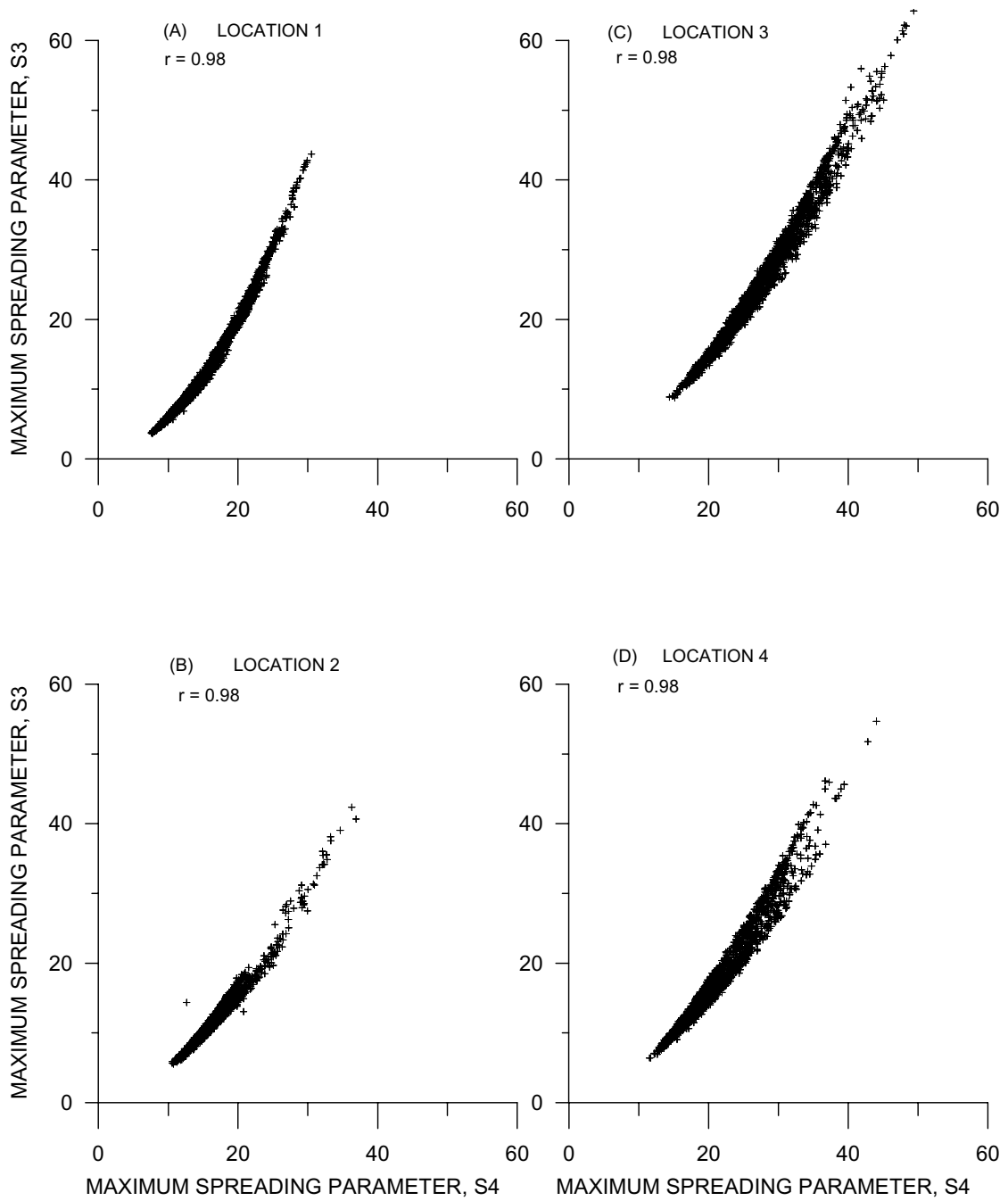


Figure 2. Variation of maximum spreading parameter 's₃' estimated based on non-linearity parameter and 's₄' estimated from multiple regressions.

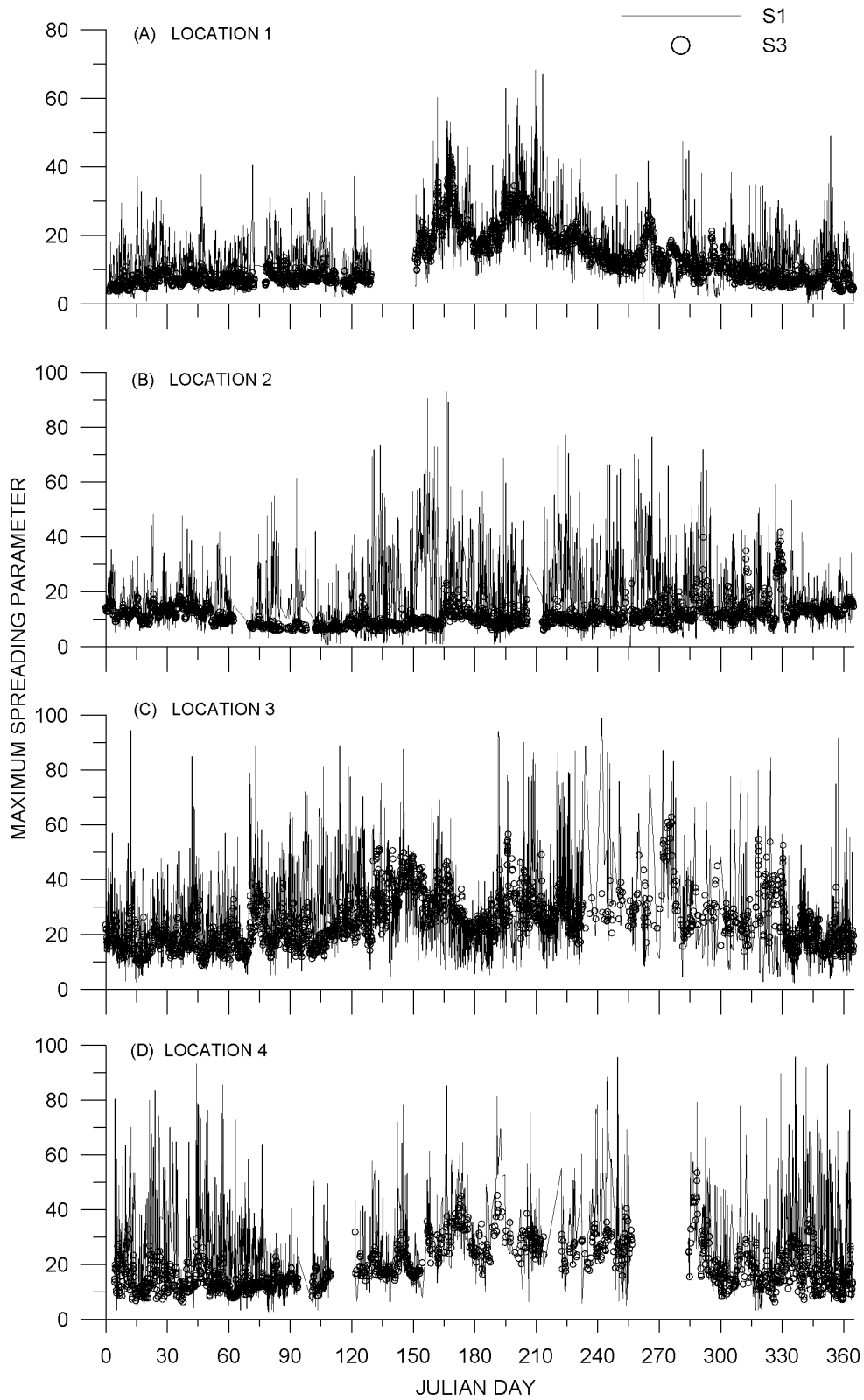


Figure 3. Variation of ' s_1 ' and ' s_3 ' at peak frequency with time.

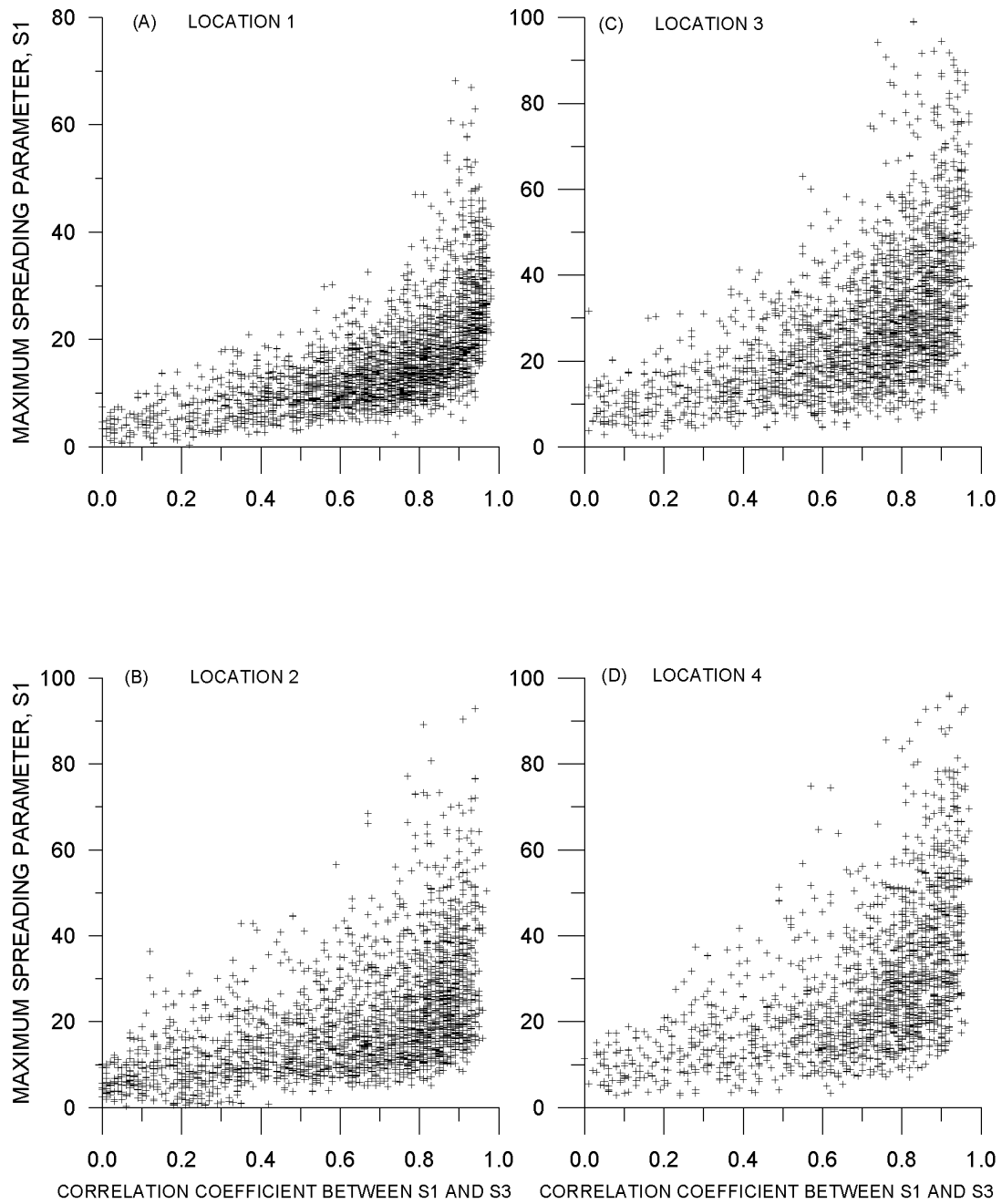


Figure 4. Variation of correlation coefficient (r) between s_1 and s_3 with spreading parameter, s_1 , at peak frequency.