

APPLICATION OF FFT METHODS TO WIND WAVE DATA SERIES BY ADDING ZEROS

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A method to add zeros (in FFT methods) to a short data sample ($< 2^m$) to make the sample size equal to 2^m is illustrated in this paper. It is found that the spectrum could be computed reliably without any undesirable side lobe effects and variance loss.

Key-words: Fast Forier Transform, Wave data,

ABSTRACT

Fast Forier Transform (FFT) methods are very widely used to analyse continuous data for its variance spectrum. The FFT algorithms are limited by the restriction that the sample size (N) should be a power of 2 (Robinson, 1967; Bendat and Piersol, 1971; IEEE, 1979). If N is less (marginally) than any 2^m ($m > 0$) N could be adjusted to the nearest 2^m value by adding zeros (Bendat and Piersol, 1971). But it was noted that this would lead to a resolution change and undesirable side lobe effects (Bendat and Piersol, 1971). In addition to this the computed variance after addition of zeros has to be corrected for the variance loss also. Presented in this communication is a method to correct for the variance loss. It is also shown that there is no undesirable side lobe effects.

Digitised (1 sec. intervals) wind wave data from a single record of 15 min. duration recorded using a ship borne wave recorder (Vethamony, Gopalakrishna and Varkey, 1982) is used for the study. A sample size (N) of 512 (2^9) values is only made use here for adding zeros in different proportions.

The spectra and other statistics are computed from profiles with zero mean. Fig. 1 (a & b) shows graphically the results of adding zeros in the time domain. N_r is the available real sample size short of any 2^m

A box car window function applied in the time domain to the original data series is defined below (Bendat and Piersol, 1971).

$$\begin{aligned} U_T / 2 (h) &= 0 & t < -T/2 \dots \dots 1 \\ &= 1 & -T/2 \leq t \leq T/2 \\ &= 0 & t > T/2 \end{aligned}$$

The spectrum is computed from the truncated data series. The above defined box car function has a Fourier transform as shown in Fig. 1 (c).

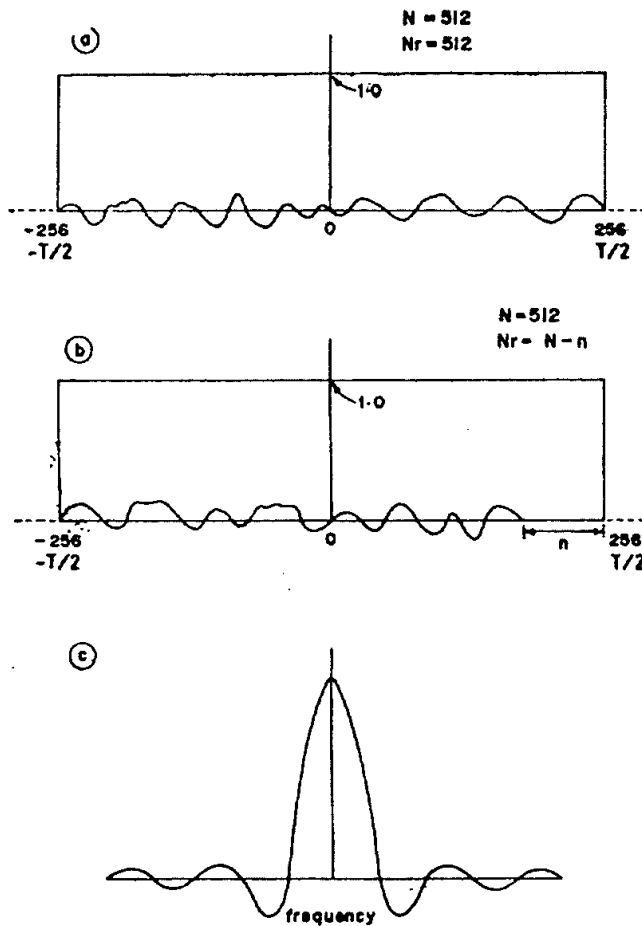


Fig 1. Box car data window applied to the full and reduced data series and the corresponding spectral window.

Adding zeros to a data series to adjust for N results in a loss of variance as shown below. From the zero mean data series of sample size N,

$$\frac{1}{N} \sum_{i=1}^N x_i^2 = S/N = \frac{\sigma^2}{(N)} \dots\dots\dots 2$$

Now, if n zeros are added to make N = 2^m in the case of small profiles,

$$\frac{1}{N} \sum_{i=1}^N x_i^2 = (S-e)/N = \frac{\sigma^2}{(N)} \dots\dots\dots 3$$

Where e is the proportionate sum of squares which would be approximately equal to (S/N)n resulting from the reduced data size. From (2) and (3) it could be seen that the real variance could be computed from the zero-added series from,

$$\sigma^2_{(N)} = \sigma^2(n) \left(\frac{N}{(N-n)} \right) \dots\dots\dots 4$$

Table I shows the results of a set of computations with different sizes of n. Mo is the 0-th spectral moment computed from the half amplitude-squared spectrum using the expression,

Table I - Variations of σ^2 , M_0 and H_s with different profiles with varying number of zeros.

Sr. No.	Sample size (N)	No. of zeros (n)	cor. factor $\frac{n}{N/(N-n)}$	Variance (σ^2)		Mo ($\sigma^2/2$)		Hs	
				with zeros	corrected	with zeros	corrected	with zeros	corrected
1	512	0	1.0000	0.1184	0.1184	0.0585	0.0585	0.9674	0.9674
2	-do-	5(%)	1.0099	0.1176	0.1188	0.0579	0.0585	0.9624	0.9674
3	-do-	26(5%)	1.0535	0.1088	0.1146	0.0536	0.0565	0.9258	0.9508
4	-do-	51(10%)	1.1106	0.0973	0.1081	0.0478	0.0531	0.8748	0.9217
5	-do-	77(15%)	1.1770	0.0936	0.1102	0.0462	0.0544	0.8595	0.9230
6	-do-	128(25%)	1.3333	0.0821	0.1095	0.0406	0.0541	0.8056	0.9304

$$M_0 = \int_1^u S(f) df \dots \dots 5$$

where, 1, u are the lower and upper frequency limits of the spectrum and S(f) spectral density estimates.

Hs is the significant wave height computed using the expression,

$$H_s = 4(M_0)^{0.5} \dots \dots 6$$

From Table I it is seen that σ^2 reduces to 0.0821 (with 25% zeros) from 0.1184 (with no zeros); a 31% reduction. It is seen that the correction applied to σ^2 practically accounts for the full variance reduction with the addition of zeros. The reduction for the corrected variance is only 1.5% from no zeros to 25% zeros. Similarly the changes in the 0-th moment (M_0) are also accounted.

The significant wave height (H_s) reduces by 16.7% from no zeros to 25% zeros. But the corrected H_s values show only a reduction of 2.8% which is of the same order as the figure for σ^2 .

The above results clearly brings out that the suggested method is suitable for analysing short data series by adding zeros provided that the data series is approximately lengthy enough for statistically stable estimates of the spectral density values. The percentage reductions in σ^2 (1.5%) and H_s (2.8%) which are approximately equal also shows that no side lobe effects are caused by the addition of zeros as also seen from Fig. 1.

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