

## EFFECT OF EMBEDMENT ON VIBRATION OF A MODEL FOUNDATION OF COASTAL STRUCTURES\*

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### ABSTRACT

For the design of massive shallow and deep foundations of transit sheds, storage tanks, silos and other structures in the coastal zone, the effect of embedment is not taken into consideration. In this paper a study is made on the foundations which are subjected to dynamic loads. The scope of this paper is limited to examine by model studies the displacement and resonant frequency characteristics of a foundation partially or fully embedded within the soil medium. A concrete block of size 15 cm diameter  $\times$  30 cm height was embedded in compacted sand at various depths. For three different values of constant exciting force on the footing, the resulting vibration characteristics were measured. It was observed that there is a considerable reduction in the maximum amplitude and resonant frequency. From a theoretical model in which the effective spring constant is increasing, the experimental results were analysed. A tentative criteria has been suggested for the analytical design of the footing based on this model.

### INTRODUCTION

While the design of massive foundations of coastal structures for static loading is reasonably well established, it becomes rather complex if dynamic loadings are also associated with the foundation. In recent years, there has been a considerable growth in the knowledge relating to the behaviour of foundation soil system, subjected to dynamic loading. Excessive vibrations of foundations or any other structure may lead not only to discomfort but also to structural damage and loss of efficiency. The effect of embedment on foundation vibration has not received as wide recognition as certain other aspects of dynamics of foundations, perhaps, because its omission results in conservative assessment. However, in all evaluations of economical and safe design, every parameter should be realistically assessed, at least to a known degree of conservatism.

A limited amount of experimental studies on embedded foundations for constant exciting force and frequency dependent force are available in the literature (Chae, 1970; Fry, 1963; Krishnaswamy and Anandkrishnan, 1974; Novak, 1970 and Pauw, 1953) without any definite conclusion. The scope of this paper is limited to examining by a model, the displacement and resonant frequency characteristics of a footing, partially or fully embedded within the soil medium. Attempts have also been made to correlate the observed changes in dynamic parameters with several hypothesis associated with known theoretical models for the sandy soil. The effects of these characteristics on the design criteria is also indicated.

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One of the simple and widely used models, adopted for the design of machine foundations, consists of discrete system in which the foundation is assumed to be a rigid mass resting on the soil idealised as a system consisting of spring and dashpot. Using this model, it is possible to obtain solutions for the embedded cases also, provided the appropriate assumptions are made for the effect of resistance along the sides of the footing. Along the sides, for resistance, Coulomb damping or viscous damping can be assumed. In considering the combined effect of resistance from the sides and damping from the bottom, it is generally convenient to approximate the Coulomb damping force to an equivalent viscous damping force by the expression

$$C_{eq} = \frac{4F}{\pi \omega A} \quad \dots \quad (1)$$

where  $\omega$  = the frequency of the vibration,  $F$  = the friction force and,  $C_{eq}$  = equivalent viscous damping constant.

Den Hartog (1931) has given the exact solution for a discrete model combining viscous and coulomb damping for a constant exciting force. Anandakrishnan and Krishnaswamy (1970) have shown the exact solution for the combined viscous and friction damping using frequency dependent force. In this paper, the test results are analysed using the exact solution of Den Hartog rather than the equivalent viscous damping.

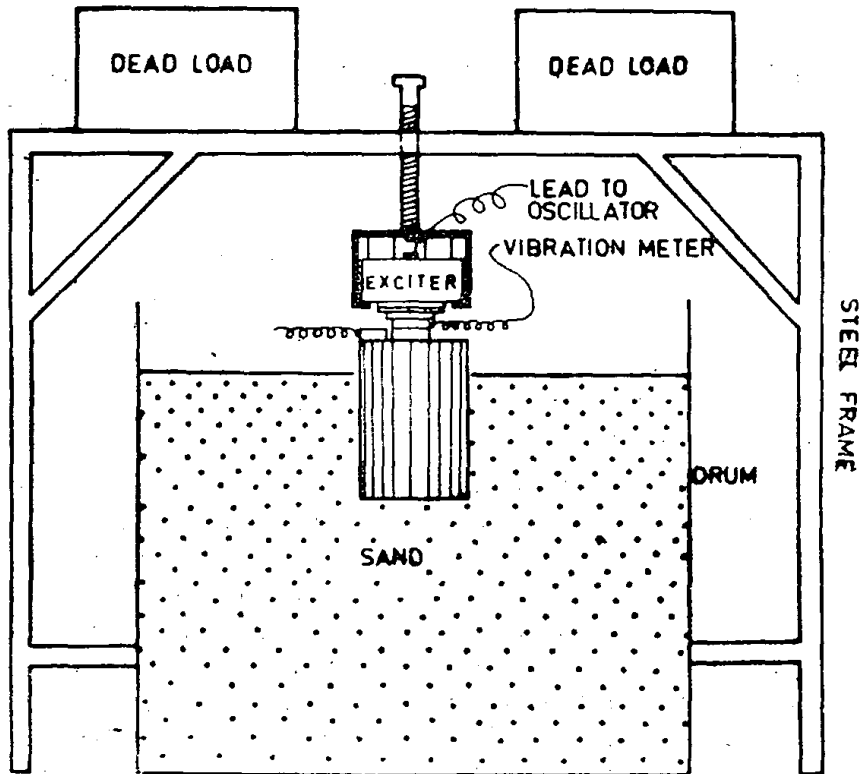


Fig. 1. Setup of embedded footing.

TEST SET UP

For the purpose of model testing, a concrete cylinder of 15 cm dia  $\times$  30.5 cm height was used. The height of the model was chosen to be larger than its diameter so as to obtain several levels of partial embedment and also to provide a higher mass ratio which can yield pronounced peaks at resonant frequency. As most of the foundations are concrete, the model was also made of the same material so that the effect of frictional resistance developed between the concrete and soil is simulated. Well

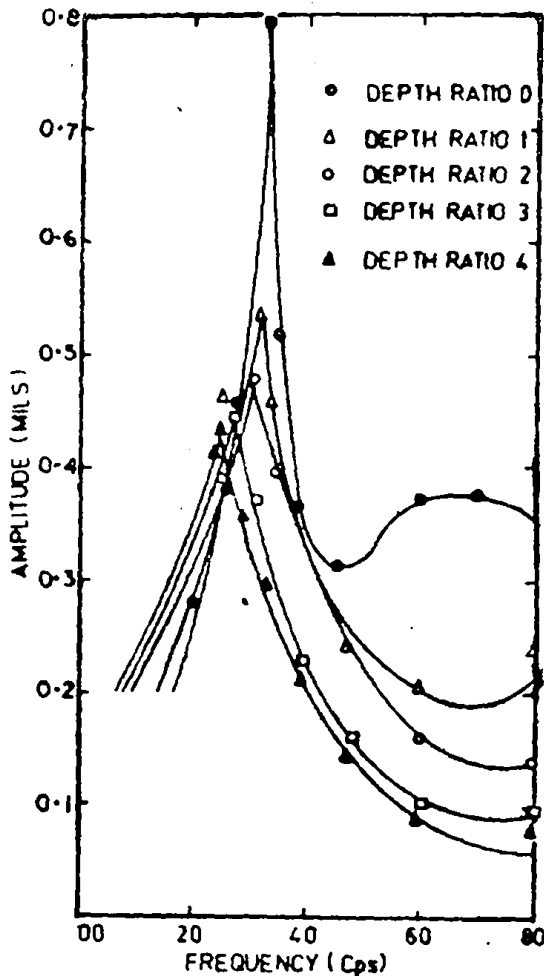


Fig. 2. Effect of embedment (under constant force  $Q_0 = 0.1$  kg).

The effect of embedment on the vibration characteristics of foundation may be governed by magnitude and type of dynamic force, depth of embedment, size and shape of the footing, type of soil media and mode of vibration. This study is confined only to the magnitude of force and depth of embedment.

RESULTS

For the experimental investigation, the concrete footing (mass of the footing = 0.0114 kg mass) was subjected to vibration for three different force levels (= 0.09, 0.18,

graded sand with a uniformity coefficient of 2.1 was used for the test. The density of this soil was nearly  $1.65 \text{ gm/cm}^3$  with a void ratio of about 0.65. This soil was contained in a steel bin 1.22 m in diameter and was compacted in layer of 7.5 cm by a rammer giving equal number of blows throughout the area. To keep the exciter in such a position that its moving table just rests on the footing without adding any weight of its own on the footing, a frame made up of steel was fabricated. The position of the exciter could be vertically adjusted by means of a central screw arrangement in the steel frame (Fig. 1). The output signal from the oscillator was fed to the amplifier which increases the voltage of the signal to a desired level for accentuating the exciter. The sinusoidal electrical pulse, converted to mechanical sinusoidal motion in the exciter, was applied to the footing. The force due to this motion remained constant for any frequency at which the exciter vibrated. The signal from the transducer, attached to the footing, was fed to the vibration meter for the measurement of amplitude, of displacement, velocity or acceleration. An oscilloscope was used to check the nature of mechanical motion of the footing.

0.36 kg approx.) at five different depths of embedment (0, 7.5, 15, 22.5, 30 cm). The embedment was achieved by pouring the sand and compacting it uniformly around the footing. Even though the observations were made for the displacement, velocity and acceleration, only the displacement values were used in plotting the frequency response curves. The values of the velocity and accelerations were used to check the harmonic nature of the displacement.

One typical frequency-displacement curve for the five different depths of embedments is shown in Fig. 2. The ratio of resonant amplitude of an embedded footing to that of the surface footing vs. a function of ratio of depth of embedment to the radius of the footing (depth ratio at different force levels) is shown in Fig. 3. Values of resonant frequency for the different depths of embedments and force levels are shown in Fig. 4.

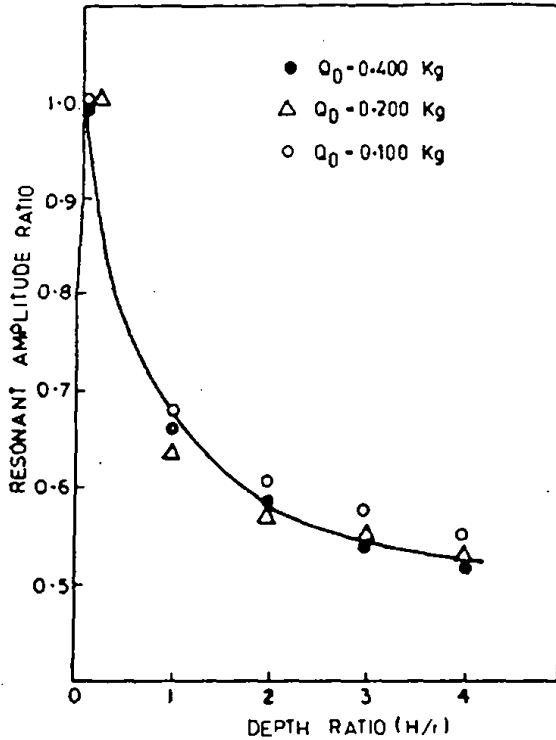


Fig. 3. Effect of depth ratio on amplitude.

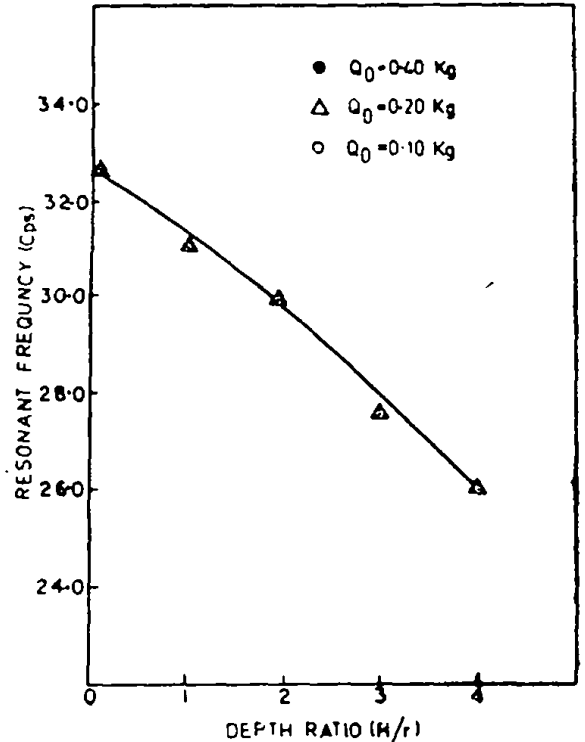


Fig. 4. Effect of depth ratio on spring constant.

### Trends

From these studies, the following trends were observed:

1. For a given constant force level, the maximum amplitude of displacement decreases with the increase in depth of embedment. The rate of decrease in the peak amplitude is much larger at smaller depths of embedment than at greater depths (Figs. 2 & 3).

2. With an increase in magnitude of dynamic force, the resonant amplitude also increases, though not in direct proportions.

3. The ratio of resonant amplitude of an embedded footing to that of the surface footing decreases at a faster rate at smaller depths of embedment as compared to larger depths (Fig. 3).

4. For a given depth of embedment, the resonant frequency is nearly the same for the different force levels (Fig. 4).

5. For a given force level, the resonant frequency decreases with the increase in depth of embedment. The rate of decrease is nearly linear (Fig. 4).

The decrease in amplitude with depth confirms the results on the effect of embedment obtained by Barkan, 1962; Chae, 1964; Novak, 1970 and Pauw, 1953. For the frequency dependent force, it was found that there is an increase in resonant frequency with depth.

#### ANALYSIS

The results of this study can be analysed on the basis of several different hypotheses. Some of the hypotheses relating to discrete model are listed below and these have been examined critically. It must be noted that none of these hypotheses may strictly be valid in such a soil which inherently exhibits nonlinear behaviour. However, certain conclusions from the trends of the behaviour would be useful for the purpose of design.

The basic parameters of the discrete model which represent the soil characteristic for vibratory loads are spring constant  $K$ , damping constant  $c$  and in some analysis, additional mass of soil assumed as vibrating in phase with the footing. Thus the maximum amplitude of motion and the resonant frequency are the functions of spring constant, damping constant and mass of the soil. To explain the various trends of the results, it may be assumed that:

(a) the spring constant is changed with depth of embedment, the rest of the parameters remaining constant, or

(b) damping constant is increasing with depth of embedment, the rest of the parameters remaining constant. Damping force may be partly viscous and partly frictional, or

(c) there is a partial increase in spring constant and partial increase in damping constant. As the analysis becomes rather complex and the degree of increase in either of these become indeterminate, these have not been discussed.

In either of the previous assumptions, the mass of soil, vibrating in phase with the footing may be taken as increasing with embedment or remaining constant.

#### *Change in spring constant*

Several authors (Chae, 1964, Kaldjian, 1969, McNeil, 1969 and Richart, Hall and Woods, 1970) have attributed the decrease in amplitude of embedded footing to an increase in the stiffness of the soil represented by spring constant. Hence this hypothesis merits a close examination.

The value of spring constant for each of embedment was calculated assuming that the damping factor remains the same. The mass of the soil in phase with the footing was not assumed constant with the depth of embedment. The experimental results

are analysed in the light of this assumption. The value of spring constant is calculated by the expression

$$A_{max} = \frac{Q_0/K}{2D \sqrt{1-D^2}} \quad \dots \quad (2)$$

The value of damping factor is assumed constant and equal to that of surface footing. The ratio of spring constant of an embedded footing ( $K_h$ ) to that of the surface footing ( $K_s$ ) for the different depth ratios is shown in Fig. 5. The values of spring ratio ( $K_h/K_s$ ) for the different depths indicate nearly the same order of magnitude for the different force levels. The spring ratio increases with depth of embedment at a faster rate for a smaller depth. Kaldjian (1969) on the basis of finite element solution for static loading, plotted the increase in spring constant in terms of spring ratio with depth ratio. There was a very good agreement between his theoretical values and observed values as shown in Fig. 5.

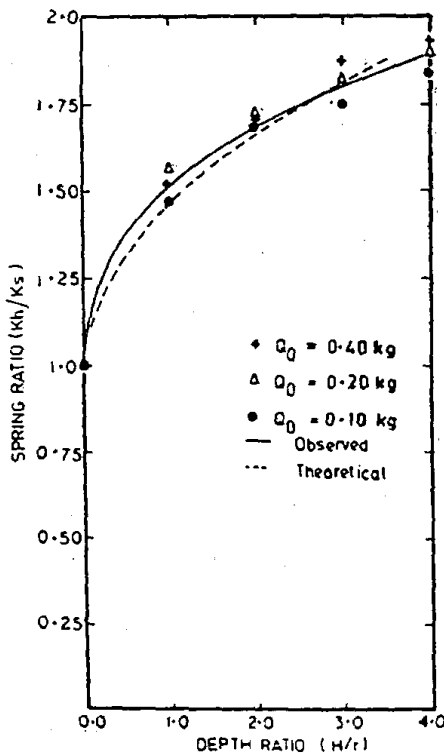


Fig. 5. Effect of depth ratio on spring constant.

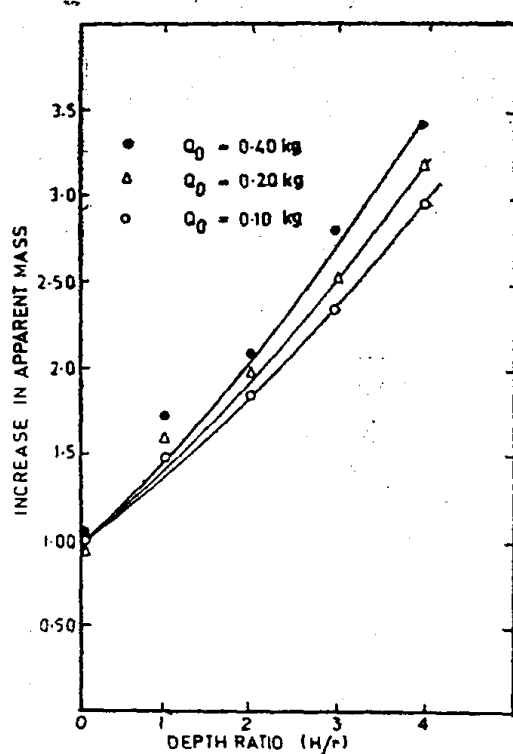


Fig. 6. Effect of depth ratio on apparent mass (for an increase in spring constant).

It seems desirable to point out that the increase in spring constant is not linear with the depth of embedment. The increase in spring constant due to embedment may be attributed to the combined effect of confinement of bottom layers due to surcharge, additional mass participating in vibrations, change in the pressure distribution below the footing and frictional effect on the sides. However, in the initial stages of embedment, the effect of confinement in increasing the soil stiffness and the change in the pressure distribution from parabolic to rigid base conditions, depending upon the force level are perhaps, more predominant than at later stages. At higher depths of embedment a greater frictional

resistance along the sides of the footing can be attributed to the increase in stiffness of the spring.

The significance of spring constant so far discussed would be valid if the trend in the resonant frequency can be satisfactorily explained. The resonant frequency is decreasing with the depth of embedment. This decrease in resonant frequency can be attributed to the increase in the apparent mass. The values of the mass can be calculated from the equation (2) as follows:

$$f_m = f_n \sqrt{1 - 2D^2} \quad \dots \quad (3)$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \quad \dots \quad (4)$$

The increase in apparent mass is plotted in Fig. 6 with the assumption that the total mass of the model for surface footing is the same as that of the footing. It is seen that the total mass of the system (footing and soil vibrating in phase with footing) is increasing almost linearly with the depth of embedment. It may appear from the figure that the value of the apparent mass is rather large as compared to the values suggested by Barkan (1962) and Whitman (1964). But these suggested values of effective mass are related to the surface footing only. Considering the complex nature of the induced vibration within the soil mass, if the hypothesis regarding the increase in spring constant with the embedments is to remain strictly valid, it is evident that the effective mass of much higher order, as indicated above for different depth ratios, should be adopted.

#### *Change in damping constant*

While it is reasonable to assume an increase in stiffness of the imaginary spring, an alternative hypothesis from the point of view of damping has also been examined in by Jain (1971). It is conceivable that frictional forces between the sides of the footing and the soil mass are mobilised offering resistance during the vibrations and thereby reducing the maximum amplitude. The viscous damping factor of the soil at the bottom of footing in an embedded foundation can be assumed to remain the same as for surface footing. Thus, the model indicates a combined damping effect which is viscous below the footing and frictional along the sides of the footing. The reduction in amplitude is fully attributed to damping alone assuming the spring constant to remain the same.

#### APPLICATION

In the design of massive foundations, the effect of embedment is generally omitted as a conservative procedure. However, as reduction in the amplitude is considerable in large embedment, it may be of economic value to include the effect of embedment. Some general steps, to evaluate the effect of embedment on the basis of the above hypothesis are as follows:

1. from the consideration of static load and the nature of dynamic and soil conditions, the size, shape and mass of the footing can be assumed.
2. the values of spring constant ( $K_s$ ) and damping factor ( $D_s$ ) for surface footing could then be determined.

3. the value of depth ratio (depth of embedment by radius of footing) is determined. If the footing is not circular, an equivalent radius equating the area of footing to the circular footing is taken.
4. using the hypothesis, from Fig. 5 the value of the spring ratio is determined and then used in the eqn. 2 to determine the maximum amplitude. For the safe design, this amplitude must be within the permissible limits. If not, the size and mass can be changed. Resonant frequency can be determined with the help of Fig. 6. From this figure one must find out the mass of soil-footing system. Using the values of  $K_h$  and  $m$ , the natural frequency of this system can be determined and thus the resonant frequency using the value of  $D_s$ .

#### CONCLUSION

From the results of the test conducted in the laboratory, two trends in the behaviour of embedded footing are clearly established.

- (1) The maximum amplitude of the embedded footing decreases with the depth of embedment.
- (2) The resonant frequency of the embedded footing, when subjected to constant dynamic force, decreases with the depth of embedment.

Spring-dashpot model was examined to explain these trends and also to evolve a certain criteria to estimate the effect of depth of embedment on amplitude and resonant frequency. The following conclusions are arrived at :

- (1) The reduction in the amplitude due to embedment is significant and should be considered in the design of embedded footing.
- (2) The decrease in the resonant frequency under a constant force of excitation is also significant.
- (3) In using the hypothesis that spring constant is increasing with the embedment, there is a remarkable coincidence in the values of spring ratio obtained from this experiment to those computed by the theoretical analysis of Kaldjian (1969). For this hypothesis the apparant mass of the soil vibrating in phase with the footing is increasing (2 to 2.25 times for the depth ratio of 1 to 4). Natural frequency is increasing with the depth of embedment.

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