

## Sediment - Size Distribution of Innershelf Off Gopalpur, Orissa Coast Using EOF Analysis

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**Abstract:** Earlier studies on grain-size analysis of sediment samples collected over the innershelf of Gopalpur coast have revealed two significant depositional environments *viz* beach and dune. Sediment samples from water depth of < 15 m show the representation of dune type of environment and the samples beyond 15 m in the study area are characterised of beach type of environment suggesting submergence of two physiographic units of beach and dune due to Holocene transgression. Empirical Orthogonal Function (EOF) analysis carried out for the above units on grain-size distribution of the sediments in the  $\phi$  - size class revealed that the first three  $\phi$  - dependant eigen function *viz.*  $c_1(\phi)$ ,  $c_2(\phi)$  and  $c_3(\phi)$  account for more than 95% of variance predominantly represented by the first - dependent function  $c_1(\phi)$  alone. This first function  $c_1(\phi)$  of deeper sediments (15 - 30 m depth) contribute 76.34 % of the total distribution representing coarser sands, in general, and characterized by predominance of very coarse sand to the extent of 40%. Shallow sediments (0- 15 m depth) contribute 70% of the distribution—representing medium to fine sand with the dominance of 40% of fine sand. This eigen function  $c_1(\phi)$  showed significant differences in the concentration of grain size particles across the innershelf reflecting the change in the energy condition that prevailed in the area during the emergence of the innershelf.

**Keywords:** Sediment characteristics, Innershelf, Depositional environment, EOF analysis, Gopalpur, Orissa coast.

### INTRODUCTION

Surface sediment samples were collected over the innershelf off Gopalpur covering the water depth up to 30 m. Physical observation of these samples itself has indicated that they may belong to two different depositional environments, although they occur in a shallow marine environment at present. Quite contrary to normal sediment size distribution of seaward fining, a remarkable coarse sandy sediment occurs at >15 m water depth below which it is fine to medium sand. Based on the grain size statistical parameters of these sediments, Mohana Rao et al. (1989) inferred submergence of ancient beach and dune sediments due to the Holocene transgression. They considered the sediments that occur up to 15 m water depth as palaeo-dune and beyond 15 m depth as palaeo-beach, which are now occurring on the seabed as marine sediments. As these two groups of sediments (beach and dune) belong to two depositional regimes prior to their submergence, they were subjected to many process variables such as wind, waves, tides and currents that acted

independently or in combination. Mohana Rao et al. (2001) also reported the occurrence of a submerged beach ridge in this region.

As these two groups of sediments (0-15 and 15-30 m) were considered as belonging to two depositional environments, we have attempted to apply empirical methods for environmental classification with the premise that they originally belonged to two physiographic units. Empirical Orthogonal Function (EOF) analysis is a statistical technique generally used to find the variability in array of data related to different fields. The eigen vector approach facilitates the representation of a large quantity of data in terms of small number of orthogonal components accounting for a large fraction of total variance in the original data and has certain advantages for analytical work (Klovan, 1966).

This method has been widely used in many types of distributions by different workers. It is important to point out that EOF may or may not have physical significance or interpretation but depends certainly on the data examined. However, various researchers from different disciplines

employed the technique for different purposes. Stidd (1967) used eigen vectors to represent the seasonal variation of rainfall over Nevada and found that the first three eigen vectors, in order of importance have features in common with the three natural cycles of precipitation. Winant et al. (1975) and Aubray (1978) used this technique to beach data and attributed the first three eigen vectors to beach profiles, mean seasonal cycles and high frequency random oscillations respectively. Ramana Murty et al. (1986) used this method to grain-size data to identify the different energy environments along central west coast of India. Generally, these studies are done for the well-defined boundaries of physiographic units such as prominent headlands, man-made littoral barriers, submarine canyons and other shoreline features. In some instances the boundaries of a unit are well defined, whereas in other

cases what is now considered to be a definite unit might after further study prove not be such. All these factors have encouraged us to test this technique on the grain size analysis of the inner shelf samples that were interpreted as submerged beach and dune environments from grain size statistical parameters (Mohana Rao et al. 1989).

#### METHODOLOGY

Surface sediment samples were collected along various perpendicular transects to the coast (Fig.1). The method of collection and analysis of samples were dealt in the previous paper (Mohana Rao et al. 1989). All samples were divided into two groups i.e. shallow (submerged dune) covering the water depth up to 15 m and deeper (submerged beach)

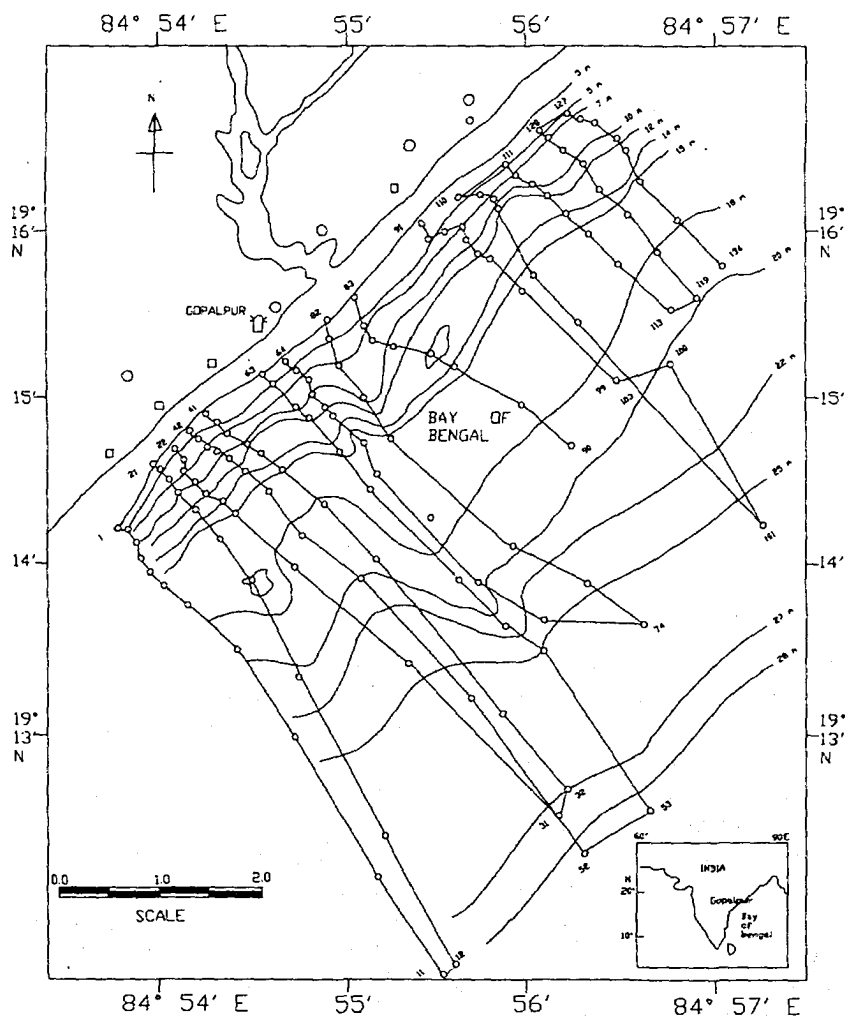


Fig.1. Sampling location and bathymetry off Gopalpur, Orissa coast.

covering the water depths between 15-30 m. For convenience, the samples related to former beach and dune are referred here as *deeper* and *shallow* respectively to avoid confusion.

Computational Procedure

The size distribution of the sediment samples was represented in the form of matrix  $h(x, \phi)$  of discrete size class, where  $x$  represents the sampling location and  $\phi$  is grain - size class considered at half - $\phi$  intervals from  $-\phi$  to  $4\phi$ . Each element of this matrix is the sample with that size class expressed as percentage value of the whole sample. The elements of the data matrix  $h(x, \phi)$  were normalized with  $(n_x * n_\phi)^{-1/2}$  where  $n_x, n_\phi$  are the number of stations and grain-size classes respectively. Following Lawson and Hanson (1974), the normalized data matrix  $X(x, \phi)$  is written as product of three matrices in component form:

$$X(x, \phi) = h(x, \phi) / (n_x n_\phi)^{1/2} \tag{1}$$

$$= \sum_{k=1}^r c_k(\phi) e_k(x) l_k \tag{2}$$

$$X = 1, \dots, n_x, \phi = 1, \dots, n_\phi$$

and in Singular Value Decomposition (SVD) form:

$$X = U L V^T \tag{3}$$

The dimensions of matrices  $X, U, L,$  and  $V^T$  are  $(n_x, n_\phi), (n_x, r), (r, r)$  and  $(r, n)$  respectively. The matrices  $U$  and  $V$  are orthonormal, i.e.  $U^T U = I, V^T V = I$ , where  $I$  is the identity matrix and superscript  $T$  refers to the matrix transpose operator. The matrix  $L$  is a diagonal matrix of nonzero "singular values" of  $X$ , and  $r$  ( $r \leq \min(n_x, n_\phi)$ ) is the rank of matrix  $X$ . Obviously, the rank of all the three - component matrices will be  $r$ , and hence all diagonal elements of  $L$  are squares of singular non zero values ( $l_j^2$ ). The sets  $s(d) = \{e_1, e_2, \dots, e_p\}$  and  $s(m) = \{c_1, c_2, \dots, c_p\}$  are the orthogonal vectors (column vectors of  $U$  and  $V$ ) of the matrix  $X$  respectively and  $p$  ( $p \leq r$ ) is the number of factors to be considered. Since these functions arise from data empirically and orthogonally by decomposing in SVD form, we call them 'Empirical Orthogonal Function' (EOF). EOF depicts the underlying dynamics describing statistically to determine the direction of maximum variability. These directions called the "principal components" or EOFs of the second order statistics or "moments" of the phenomenon. More technically EOFs are the eigen vectors of the covariance matrix of the second order moments. The

geometrical meaning of SVD with respect to mathematical treatment has been detailed.

Geometrical Interpretation of SVD

The SVD has the following geometrical interpretation (Wiggins, 1972).  $u_1$  is the most "common" column vector, in the sense that it has the largest projection for all the columns in  $X$  than any other unit vector.

$$\|e_1^T X\| = \text{maximum} = \lambda_1 = l_1^2 \tag{4}$$

The norm  $\|.. \|$  is defined as the square root of the sum of squares of all elements in the matrix;

$$\|X\| = \sqrt{\sum_{ij} a_{ij}^2} \tag{5}$$

$e_2$  is the second most common in the sense that, it has the largest projection after taking the projection from the first singular vector out.

$$\|e_2^T (X - (e_1 - e_1^T X))\| = \text{maximum} = \lambda_2 = l_2^2 \tag{6}$$

The rest of the singular vectors follow in order. The same interpretation applies between the  $c_j$ s and rows of  $X$ .

The singular value,  $\lambda_1$  can be interpreted as being a measure of the variance of  $X$  accounted for by mode  $i$ . We can expand equation  $(X = U L V^T)$  as,

$$X = \lambda_1 e_1 c_1^T + \lambda_2 e_2 c_2^T + \dots + \lambda_p e_p c_p^T \tag{7}$$

Each term represents contributions from a different mode. The variance that mode  $i$  accounts for is calculated from the norm of its contribution to  $X$ :

$$\begin{aligned} \|\lambda_1 e_1 c_1^T\| &= \lambda_1^2 \sum_{ik} e_{ik}^2 c_{ik}^2 \\ &= \lambda_1^2 \left( \sum_k e_{ik}^2 \sum_l c_{il}^2 \right) \\ &= \lambda_1^2 (\|e_i\| = \|c_i\| = 1) \end{aligned} \tag{8}$$

Thus the first mode accounts for the largest variance among all modes ( $\lambda_1 = \text{maximum}$ ) and the second mode for the second largest and so on. On the other hand, the total variance of  $X$  is equal to the sum of all modal variance (Lasons and Hanson, 1974).

$$\|X\|^2 = \lambda_1^2 + \lambda_2^2 + \dots + \lambda_p^2 \tag{9}$$

*Closeness ratio:* The ratio between the sum of the factor

model and that of the data matrix is considered as the measure of closeness of the model data (Ramana Murty et al. 1986).

$$\text{Measure of closeness} = \frac{\sum_{i=1}^p l_i^2}{\sum_{i=1}^r l_i^2} \quad (10)$$

where p is the number of factors and r is the rank of the data matrix X. The first eigen function associated with the largest eigen value represents the border features in the data, in the least square sense, while the second function and so on describe the residual mean square data. The closeness ratio is expressed in percentage in order to enable one to judge the contribution of different modes, and thereby reproduce the model profile. The computed eigen values and closeness ratios are presented in Table 1. These eigen values give energy levels and corresponding eigen vectors indicating the trend of structure/features associated with the environment.  $\phi$  - distribution of individual percentage contribution of  $j^{\text{th}}$  component of  $k^{\text{th}}$  eigen function for present analysis is given by

$$\left[ \frac{v_{jk}}{\sum_{l=1}^{No} |v_{lk}|} \right] \left[ \frac{\lambda_k}{\sum_{l=1}^r \lambda_l} \right] * 100 \quad (11)$$

where  $v_{jk}$  is column vector of the matrix V of factor k.

## RESULTS AND DISCUSSION

Table 2 shows the significant observations from Fig.2 based on the distribution of  $c_1(\phi)$ ,  $c_2(\phi)$  and  $c_3(\phi)$  functions for each of the individual depositional units of shallow regions (submerged dune) and deeper region (submerged beach). First  $c_1(\phi)$  eigen function reveals the occurrence of unimodal distribution in group 1 (shallow region) and bimodal distribution in group 2 (deeper region) sediments with different phi modal values (Fig.2) suggesting different energy conditions. The percentage distribution of  $\phi$  modal values have been computed to obtain more details of dominant features associated with different energy levels (Table 1) using equation 11.. Dominant modal value of 69.97% is observed in the characteristic features of this group of sediments (Tables 1 and 2). In this distribution, the 3.0  $\phi$  size depicting the sediment texture of very fine sand which contributes to the extent of 21.69% of 69.97% and is considered as dune environment (Mohana Rao et al. 1989). Relatively less dominance (69.97%) of first ( $\phi$ ) dependent eigen function, in the present study, when compared to the work done on dune (>95%) elsewhere by

**Table 1.** Eigen values of grain size distribution

S. No.	Shallow		Deep		Total	
	Eigen Value	Closeness ratio(%)	Eigen Value	Closeness ratio(%)	Eigen Value	Closeness ratio(%)
1	164.00	69.97	158.00	76.34	123.00	55.87
2	40.70	17.32	27.40	13.26	66.30	30.05
3	15.40	6.65	12.50	6.07	13.40	6.06
4	9.42	4.01	4.11	1.99	7.93	3.59
5	3.47	1.46	1.71	0.83	5.59	2.53
6	0.87	0.37	1.58	0.76	2.59	1.17
7	0.49	0.21	0.87	0.42	0.87	0.40
8	0.19	0.08	0.38	0.18	0.31	0.14
9	0.03	0.01	0.27	0.13	0.23	0.10
10	0.01	0.01	0.02	0.10	0.19	0.09
11	0.003	0.001	0.01	0.01	0.02	0.01

Ramana Murty et al. (1986), might be due to the admixing of transgressive sediments with the original *dune* sediment during the advance of sea. However, 69.97% dominance indicates that the original physiographic domain (sediment) is not completely changed. The open ended nature of the curve (Fig.2) towards finer fraction indicates more dynamic and variable regime of the deposit which is evidenced by its proximity to the present day shoreline and has got scope for the further settlement of finer fraction.

The sediments belonging to the deeper region (submerged beach) show a bimodal distribution with 76.34% of dominance suggesting a mixed size distribution. The two modal values 0.5 and 1.5  $\phi$  size values (Fig.2) representing the mixed nature covers about 9% and 22.75 % respectively out of 76.34%. The coarser texture of these modal values suggests a high-energy environment such as beach deposit that is now submerged and preserved as relict physiographic domain of beach environment. As these sediments are considered as submerged beach deposits (Mohana Rao et al. 1989), the bimodal classification indicates that sediments from two sources were transported and mixed to give rise to the present environment. During the time of exposure these sediments were subjected to different types of high-energy conditions such as wave breaking, wind action and oscillating motion of the grains in the surfzone and thus formed a coarse sandy deposit. Transgressive sediments also might have played some role in producing bimodal nature. This bimodal and closed ended curve reveals the stableness of the environment, which is normally observed in the deeper regions when compared to the near shore. This is also evidenced from the closeness ratio (Table 1) in case of deeper sediments, which is relatively higher (76.34%) than the shallow sediment (69.97%).

Table 2. Salient features of EOF Analysis

Physiographic Unit	Modal Values( $\phi$ )			Percent Dominance			Remarks
	$c_1(\phi)$	$c_2(\phi)$	$c_3(\phi)$	$c_1(\phi)$	$c_2(\phi)$	$c_3(\phi)$	
Shallow	3.0	1.5 3.0	1.5 2.5 3.5	69.97	17.32	6.55	Nearly symmetric with a sharp peak at 3.0( $\phi$ ). Peak accounts for 21.69% of 69.97%. Medium to very fine sands predominant in $c_1(\phi)$ , $c_2(\phi)$ and $c_3(\phi)$ indicating seasonal and high frequency fluctuations. $c_4(\phi)$ and $c_5(\phi)$ show negligible influence for environ interpretation.
Deep	1.5	0.0 2.0	0.0 1.5 3.0	76.34	13.26	6.07	Mostly Bimodal distribution with the peaks at 0.5 and 2.0 $\phi$ indicating mixed sources of derived material. Peaks account for 9% and 22.75% of 76.39% respectively. Coarser to medium size material predominant in $c_1(\phi)$ , $c_2(\phi)$ to $c_5(\phi)$ indicate seasonal higher order fluctuations
Overall (Total)	1.5	1.5 3.0	0.0 2.2 3.5	55.87	30.05	6.06	Combination of two derived physiographic units shows trimodal distribution indicating different sources of the material.

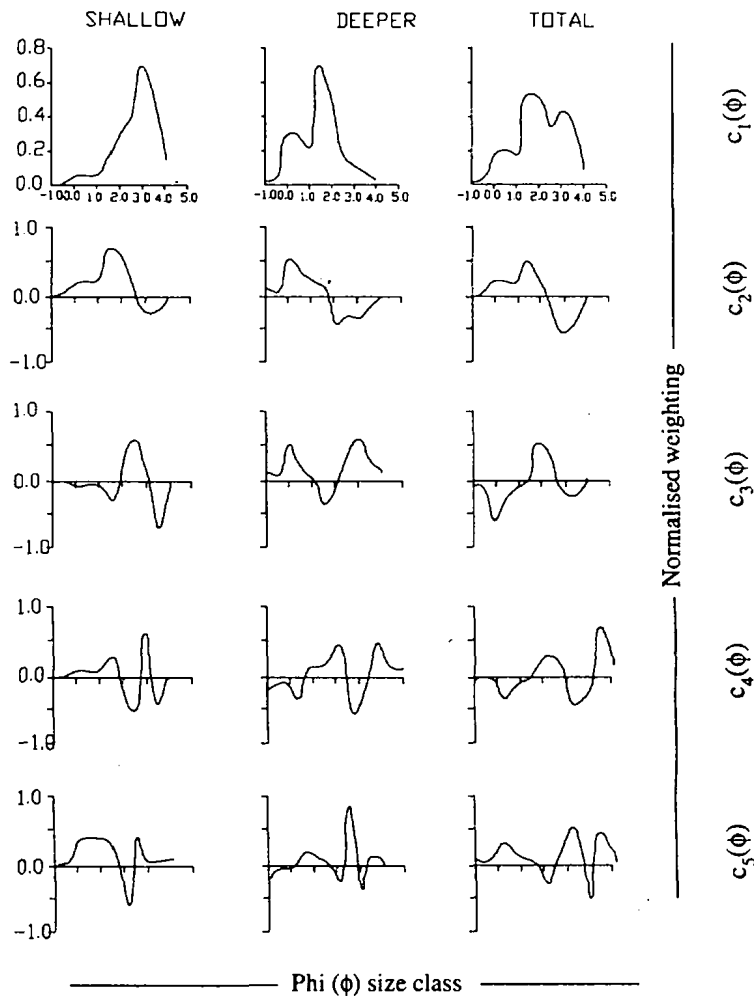


Fig.2. Five most energetic phi-dependent eigen vectors for nearshore and offshore sediments of Gopalpur.

$c_1(\phi)$  eigen function for the combination of the two physiographic domains (shallow and deeper regions) is shown as Total" in Fig 2. The open-ended trimodal nature of the curve having the dominant modal peak values around  $0.0\phi$ ,  $2.0\phi$  and  $3.0\phi$  indicates the mixing of the two domains. The open nature of this trimodal curve suggests the resultant dynamic nature of the beach and dune environments in the past and present.

The main difference of this EOF analysis with that of the conventional method is that the technique is objective and does not impose a pre-determined form to the data (discrete or indiscrete) although the resulting function can later be best-fit to any subjective curve. For example, the structures which are associated with different energies, in space as well as size class, keeping the continuity of the hypothesis to infer the dynamics of that structure which is not facilitated in the conventional techniques like Spectrum, Fourier series etc.

The second  $\phi$  dependent eigen function  $c_2(\phi)$  of the shallow region sediments (*dune*) contributes 17.32% of total variance, which reveals the seasonality. The modal values  $c_2(\phi)$  are  $1.50\phi$  and  $3.0\phi$  representing medium to very fine texture, which shows the dominance of 4.87% and 1.85% of 17.32%. Out of these percentages, the concentration of medium grain size of  $1.50\phi$  is nearly four times more than

very fine texture ( $3.0\phi$ ). This suggests the dominance of medium size grain texture due to the turbulence during the seasonal changes, which indicates high-energy conditions. During the monsoon season, high waves approach the shore and break farther offshore giving rise to multiple breaking in the form of spilling breakers. This effect is seen on the sediments of the submerged former dune environment.

The  $c_2(\phi)$  function in case of deeper region sediments contributes to the extent of 13.26% of total variance, which also reveals the seasonality. The modal values for this function are  $0.0\phi$  and  $15\phi$  representing very coarse sediment texture, which suggests high-energy environment. The influence of the  $c_2(\phi)$  eigen function reflecting the seasonal variation is relatively less on deeper sediments (submerged beach). This may be due to its location distant from the present day shore. However the limited effect that is observed on these sediments might have been caused either by wave induced mass transport or reworking of sediments in the present day environment due to long shore currents.

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