Method for Plotting Sound Ray Paths in the Potential Layer from Bathythermograms

T. C. S. RAO

National Institute of Oceanography, Dona Paula 403004

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Vertical distribution of temperature in the oceans influences greatly the sound propagation, particularly in bending the rays from their straight line paths. The changes in temperature structure are more common in the surface layers due to heating, cooling, mixing, etc., and contribute for the formation of several layers with different temperature gradients and thicknesses. Plotting of sound ray paths for such bathythermograms becomes difficult and also time-consuming if all these layers are to be considered separately. A method has been evolved which reduces the layers in the potential layer to a maximum of 3 layers irrespective of the number of layers present in the potential layer. The method is applied to several bathythermograph records and the ray paths plotted by this method are found almost similar to those plotted for the original structure of the record.

F OR problems relating to underwater acoustics it is essential to know the sound ray path and the intensity of the echo. These basically depend on (i) design characteristics of the sound source and receiver, (ii) environmental factors of the sea surface and the bottom, and (iii) physical and chemical properties of the sea water. The influence of these parameters in underwater sound propagation and detection is discussed extensively by **Urick¹**. Effect of changes in temperature on the sound ray path is studied in this paper mainly to evolve a method that simplifies the procedure for plotting ray paths for a given bathythermogram.

From the distribution of temperature with depth (up to a depth a bathythermograph can be operated) the ocean waters can be divided into 2 principle layers: (i) potential layer, and (ii) thermocline. Potential layer is defined as a layer that occurs between the sea surface and top of the thermocline². It is also sometimes called the surface layer. Defant³ defined the thermocline as a layer in which the average rate of fall in temperature is of the order of 5°C in 100 m. Many a time the potential layer is subdivided into number of layers with different thicknesses and temperature gradients due to the surface heating, cooling and mixing. In such a case it is rather difficult and also time-consuming to plot the ray paths passing through all these layers. Studies have been undertaken to classify the thickness and temperature gradient of the layers in potential layer and the successive combination of the layers to minimize the number of layers that are to be finally considered for plotting the ray paths.

Temperature Distribution and Its Effect on the Ray Path

Temperature, pressure and salinity are the principle physical and **chemical** properties of the sea water that effect the sound propagation in the sea. Variation in temperature with depth has the greatest effect on the sound transmission, namely the bending of sound rays from their straight line paths. Increase in pressure with depth introduces a constant velocity gradient of the order of 0.018 m/sec/m and the changes in the salinity in open oceans are generally small and may be neglected for plotting the ray paths.

Vertical changes in temperature and pressure introduce linear velocity gradients with depth. In a medium in which the velocity of sound changes linearly with depth, the sound rays can be shown to be arcs of circles which will have constant radius of **curvature⁴**. The horizontal distance (X) covered by the ray (Fig. 1) while passing through a layer of thickness (h) can be obtained from

$$\mathbf{X} = h \ \mathbf{C}\left(\eta_{\underline{\theta}} + \frac{\theta}{2}\right) \qquad \dots (1)$$

where θ_0 and θ_1 are the angles made by the ray with the horizontal at the top and bottom of the layer respectively and they can be obtained from Snells Law as

$$\frac{\cos \theta_0}{v_0} = \frac{\cos \theta_1}{v_1} \dots = a \text{ constant for any one ray}$$

In this expression the ray constant is reciprocal of the sound velocity in the layer in which the ray becomes horizontal, i.e. where $\cos \theta = 1$. This expression is the basis of ray computation used by



Fig. 1 — Ray path diagram for continuous fall in temperature with depth

most analog and digital computers, since it enables a particular ray to be 'traced out' by following it through successive layers into which the velocity profile may have been divided. When the potential layer consists of several layers, the total horizontal distance covered by the ray within the potential layer can be obtained from

$$X_{n} = h_{1} \cot\left(\frac{\theta_{0} + \theta_{1}}{2}\right) + h_{2} \cot\left(\frac{\theta_{1} + \theta_{2}}{2}\right) + \cdots + h_{n} \cot\left(\frac{\theta_{n-1} + \theta_{n}}{2}\right) \qquad (2)$$

Sound velocities are read from the **temperature**sound velocity tables⁵ for the corresponding temperatures from the bathythermograms. For plotting the ray path it is necessary that at least one angle is known. When there is no isothermal layer or positive gradient layer in the potential layer, the starting angle of the ray in the first layer is considered as zero. When an isothermal layer or a positive layer occurs within the potential layer, the angle at the bottom of that layer is taken as zero. Other than by computers, the ray paths are also drawn using the templates or the instruments for plotting the refracted **rays⁶**.

Classification of Bathythermograms

Classification of layers in a bathythermogram suggested in this method mainly involves (i) classification of layers for their thickness, (ii) classification of layers for their temperature gradients, and (iii) successive combination of layers for obtaining a best agreed combination that will give the ray which closely resembles the ray path drawn for the original structure of the bathythermogram.

Thickness classification — Thick layer : A layer is considered as a thick layer if it is more than **10** m thick. Thin layer: A layer is considered thin layer if it is equal to or less than 10 m.

Gradient classification — Isothermal layer (IT) : The rate of fall in temperature is less than $0.1^{\circ}C$ in 10 m or the rate of increase in temperature is less than $0.02^{\circ}C$ in 10 m.

Weak positive layer (WP): The rate of increase is between 0.02 and 0.25 °C in 10 m. Strong positive layer (SP): The rate of increase in temperature is more than 0.25 °C in 10 m. Weak negative layer (WN): The rate of fall in temperature is between 0.1 and 0.3 °C in 10 m. Strong negative layer (SN): The rate of fall in temperature is more than 0.3 °C in 10 m.

Combination of layers — For obtaining the best combination of layers more than 1250 models comprising all the possible combinations of the layer thicknesses and gradients mentioned above were built up for the 2 layer case and 3 layer case. The ray paths are drawn for all these models with the help of the templates, first by considering each layer as a separate layer and then for different combinations of layers in each model. Ray paths are then compared and that combination of layers for which the ray path closely follows the ray path drawn by considering each layer as a separate layer is taken as the agreed case. All possible combinations of layers of their thicknesses and temperature gradients as well as the final agreed cases for each of these combinations are given as A, B, C and D in Tables 1 and 2 for the 2 layer cases and 3 layer cases respectively. The notations A, B, C and D in the tables stand for: A, all layers in the potential layers are to be taken as single layer; **B**, first layer as a separate layer and the last 2 layers as 1 layer; C, first 2 layers as 1 layer and the last layer as a separate layer; and D, each layer as a separate layer.

For some combinations of layers more than one agreed case are recorded in the tables. Detailed study of the models has revealed that in certain combinations of the layers that fall in the same thickness and gradient, classification gives rise to more than one agreed case depending on the relative thicknesses and gradients of the layers. The conditions for choosing the correct agreed cases are given in Table 3.

When there are more than 3 layers in the potential layer, it is found that they can be finally brought down to 3 or less number of layers with the help of the tables. Detailed procedure for the successive combination of layers is illustrated in the example given below.

Consider a bathythermogram for which the temperature distribution with depth in the potential layer is given in Fig. 2. The potential layer extends up to 80 m depth and it can be divided into 6 layers with uniform temperature gradient in each layer. The final agreed case for this bathythermogram can be obtained from the tables following the procedure given below.

(a) Layers are classified for their thickness and temperature gradient from the classification given above and noted in column 4 of Fig. 2.

TABLE	1 - Two	LAYERS	IN	THE	Po	TENTIAL	LAYER	
SI No	Gradient	n	Thickness classification					
110.	classificatio	11	Thick Thick (a)	Th Tl (ick hin b)	Thin Thick (¢)	Thin Thin (d)	
1	IT WN		D		A	А	D	
2			А		D	А	А	
3	IT SP		A&D		D	А	D	
4	WN IT		D		A	А	D	
5	WN		D		A	А	D	
6	WP WN SP		D		D	D&A	D	
7	WP		D		A	D	А	
8	WP		D		A	А	А	
9			А		A	А	А	
10	SP		D		D	D	D	
11	SP		D		D	D	D	
12	WN SP WP		А		A	A&D	A&D	

IT = isothermal; SP = strong positive; WN = weak negative; and WP = weak positive.

		TABLE	2 Three	LAYERS	IN THE P	OTENTIAL LAYER			
S1	Gradient				Thickness	classification			
No.	classificati	on Thick Thick Thick (a)	Thick Thick Thin (b)	Thick Thin Thick (c)	Thin Thick Thick (d)	Thick Thin Thin (e)	Thin Thick Thin (f)	Thin Thin Thick (g)	Thin Thin Thin (h)
1	IT	D	A	A	D	Α	В	Α	D
	WN								
2		Α	В	С	Α	Α	C & D	С	Α
-	ŴŊ								
3	WP IT	С	D	С	С	С	D	С	D
•	ŴN								
4	SP IT	с	С	С	С	Α	С	С	А
	WP								
5		С	A	Α	С	Α	С	С	с
•	WP								
6		с	С	С	С	С	С	С	С
	WP								
7	IT	C	Α	С	С	Α	С	С	С
	SP								
8	IT	Α	Α	Α	С	Α	Α	С	Α
	SP WN								
9	IT	Α	Α	Α	Α	Α	А	Α	Α
	SP WP								
10	WN	D&A	B&C	A & B	C	В	С	Α	А
	TT WN								
11	WN	С	C	С	С	С	C	С	С
	WP								
1 2	WN	С	C	C	C	C	C	С	С
	SP								
13	W N WD	A & D	A&C	C & B	Α	С	A	Α	Α
	IT					•			
14	WN WP	Α	A	Α	A	A	A	Α	Α
	WN	0	0	C	c	C	C	0	
15	WN WP	C	U	t	ι	t	ι	ť	С
	SP	C	C	C	C	C	۸		-
16	SP	U	t	C	U	ť	л	A	A
4 -	IT WN	٨	R & A	Δ	А	Α	А	Δ	7
17	SP	A	Dun	~					А
10	W N WN	C	С	С	С	С	С	C	c
10	SP	v						Ŭ	C
30	WP WP	С	D	С	Α	С	D	С	С
00	IT								
20	W N WP	С	C	С	С	С	С	С	с
21	WP WP	С	C	С	С	С	С	С	с
22	WP	B & D	В	D	B & D	В	В	D	A
	W N IT								
23	WP	Α	D	Α	D	Α	D	Α	A
	WN WP								
24	WP	С	Α	С	С	Α	Α	С	А
	WN SP								

		TABLE	2 - Three	LAYERS IN	THE POTENTIA	L LAYER-	Contd		
SI	Gradient				Thickness c	lassification			
NO.	classification	Thick Thick Thick (a)	Thick Thick Thin (b)	Thick Thin Thick (c)	Thin Thick Thick (d)	Thick Thin Thin (e)	Thin Thick Thin (f)	Thin Thin Thick (g)	Thin Thin Thin (h)
2 5	WP SP IT	С	С	С	С	С	С	С	С
26	WP SP	А	А	А	А	А	А	А	А
27		С	А	А	С	А	А	С	А
28	SP IT	В	В	В	В	В	В	В	В
29	WN SP IT	С	А	С	С	Α	А	С	А
30	SP IT	А	А	А	А	A	А	А	А
3 1	SP SP WN	В	С	С	В	С	В	С	С
32	SP WN	D	D	С	D	A	А	А	А
33	SP WN	А	A	A	А	А	А	А	А
34	SP SP WP	С	С	С	С	С	С	C	С
3 5	SP WP	С	С	С	C&D	С	В	С	С
36	SP WP SP	С	С	С	С	С	С	С	С
	IT 😑 isoth	ermal; SF	' = strong po	ositive; WN =	- weak negat	ive; and WF) = weak p	oositive.	

(b) First, the top 3 layers are taken and by referring to Table 2 (19c), the agreed case for this type of combination of layers is given as C according to which the top 2 layers are to be considered as one layer and the 3rd layer as a separate layer.

(c) The other 3 layers in the bathythermogram are taken and referring to Table 2 (7g), the agreed case is given as C.

(d) From (b) and (c), the 6 layers in the potential layer are now reduced to 4 layers and they are reclassified for their thickness and gradients and are noted in column 5 (Fig. 2).

(e) Top 3 layers in column 5 are taken and referring to Table 2 (24a), the agreed case is given as C.

(f) From (e), the number of layers in the potential layer are reduced to 3 layers and are reclassified as given in column 6 (Fig. 2). (g) Referring the 3 layers in column 6 to Table 2

(16a), the agreed case is given as C. (h) From (g), the number of layers in the potential (i) Referring these 2 layers to Table 1 (7a), the

agreed case is given as D, according to which these layers are to be taken as separate layers.

Thus the 6 layers in the potential layer are

finally reduced to 2 layers for which the final ray path is plotted (dashed line in Fig. 3). The ray path for the original 6 layers is also plotted (solid line in Fig. 3). It can be seen that both the ray paths follow almost the same path and the difference in the horizontal distances covered by these 2 ray paths at the bottom of the potential layer is comparatively negligible. The method was applied to more than 70 bathythermograms and the errors in the horizontal ranges are found to be within 300 m. Some of these bathythermograms along with their classification and ranges are given in Table 4.

Strong negative gradient - Sometimes the strong negative gradient layers occur in the potential layer and behave like thermocline for the purpose of plotting the sound ray paths. A few models were studied by introducing strong negative layers, temperature gradient more than $0.3^{\circ}C$ in 10 m, in the potential layer and the ray paths are drawn. These studies have revealed the following:

(a) If the ray path does not become tangential to any of the layers below the strong negative layer, it can be continued into regions below this layer with the gradient of the strong negative layer. type of case is denoted by E in Table 4. This

TABLE 3 -SELECTION OF CORRECT AGREED CASE IS GIVENTABLE 4 — SOM GRMS, TIMORE THAN ONE AGREED CASE IS GIVENTABLE 4 — SOM GRMS, TIAgreed case Agreed No. casesThicknessTab. 1 (3a)D1st layer > 2nd layer A For all combinations other than aboveNo. of layers and thickness classi- fication in the potential layerTab. 1 (3a)D1st layer > 2nd layer A For all combinations other than aboveNo. of layers and thickness classi- fication in the potential layerTab. 1 (12c & D (12 & 2 & D)-**-**d)AFor all combinations other than above2Tab. 2 (2f)D \leq 30 m C2Tab. 2 (10a)A2nd layer > 1st or 3r' layer above2Tab. 2 (10b)C2nd layer > 1st layer above2Tab. 2 (10c)B1st layer > 2nd layer A For all combinations other than above2Tab. 2 (13a)D1st layer > 2nd layer aboveThick Thick A For all combinations other than above3Tab. 2 (13b)C1st layer > 2nd layer A For all combinations other than above3Thick Thick ThickTab. 2 (13c)B1st layer > 2nd layer A For all combinations other than above3Thick Thick ThickTab. 2 (13c)B1st layer > 2nd layer A For all combinations other than above3Thick Thick ThickTab. 2 (13c)B1st layer > 2nd layer A For all combinations other than above3Thick Thick <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th></th> <th>-</th> <th></th>								-	
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Thin			Graule	int — 0 .35				3	Thick
	<u> </u>								Thin



Fig. 2 - Classification and successive combination of layers





No. of thickn ficatio poten	layers and less classi- on in the tial layer	Gradient classifica- tion	Agreed case	Direct ray range at the potential layer depth m				
				Agreed case	Original case			
2	Thick		D	4800	4800			
2	Thin	WN	А	3700	3700			
2	Thick	WN	А	1200	1200			
2	Thin Thick	IT IT	А	1100	1100			
2	Thick	SP IT	D	1600	1600			
2	Thin Thin	$_{ m IT}^{ m WN}$	А	2600	2600			
3	Thick Thick Thick	WP IT WN	D	4000	4000			
3	Thick Thick Thick	IT IT WN	А	3200	3200			
3	Thin Thick Thick	IT IT WN	Е	2900	2900			
3	Thick Thin Thick	SN IT W P	С	3000	2800			
3	Thick Thick Thick	W N IT SP	С	2650	2600			
3	Thick Thick Thick	IT IT SP	С	2400	2200			
3	Thick Thick Thin	IT IT SP	С	2300	2050			
3	Thick Thin Thick	IT IT SP	С	3600	3600			
3	Thick Thick Thin	WN IT SP	А	2950	2750			
3	Thick Thick Thick	WN IT SP	Е	2600	2600			
3	Thick Thick Thin Thiak	IT SN	Ε	2800	2850			
3	Thick Thin Thick	IT SN WN	Е	3600	3600			
3	Thin Thick Thick	IT SN SN	Е	2100	2100			
3	Thick Thick Thick Thick	WN IT WN	А	2200	2250			
3	Thin Thin Thin Thick	WN WP WN	А	2500	2400			
3	Thick Thick Thick	WP SN IT	Е	3100	3100			
3	Thick Thin Thick	SN IT SN	Ε	1500	1650			
3	Thick Thick Thick	SN SP WN	А	2600	2600			

TABLE 4 - SOME EXAMPLES OF ACTUAL BATHYTHERMO-

THEIR CLASSIFICATION AND RANGES

No. of la thicknes fication potentia	ayers and ss classi- in the al layer	Gradient classifica- tion	Agreed case	Direct ray the pote de	range at ntial layer pth m	No. of thickne ficatio potent	layers and ess classi- n in the ial layer	Gradient classifica- tion	Agreed case	Direct ray the poter de	7 range at ntial layer pth m
I	- J -			Agreed case	Original case					Agreed case	Original case
3	Thick Thin Thick	SN SP	Е	1400	1500	5	Thick Thick Thin	IT SP SN	С	4000	4000
4	Thick Thick Thick Thick Thick	IT WN SP IT	С	3300	3200	5	Thick Thick Thick Thick Thick	SP WN SN WP SN	Ε	2200	2400
4	Thick Thin	IT WN	С	3200	3000	6	Thick Thick Thick		٨		
4	Thick Thin Thick Thick Thin	VN WN SP SN	Е	1600	1550	0	Thin Thin Thin Thick Thick	TT WN WP WN SP	A	3100	3400
4	Thick Thick Thin Thin Thiak	IT SN IT WN	E	3450	3500	6	Thick Thick Thin Thick Thin	IT WP IT WN	С	3300	3100
4	Thick Thin Thin Thin	IT SP IT	Е	1850	1700	6	Thin Thin Thick Thick	SP IT IT	E	4000	4000
4	Thin Thick Thin Thin Thin	SN IT SP SN IT	Е	1150	1150		Thin Thick Thin Thin Thin	WN IT SN SP IT			
4	Thin Thick Thin Thick Thick	IT SN SP WP	А	4200	4100	6	Thick Thin Thick Thick Thin	IT WN IT SN	A	3800	3550
4	Thick Thin Thin Thick	IT WN SN SP	А	2800	2650	6	Thick Thick Thick Thick Thin	SP WP WN SP			
4	Thick Thin Thick Thick	IT SN SP W N	С	2850	2700		Thin Thick Thin Thick	SN IT WN SN	E	2100	2200
4	Thick Thick Thick Thick Thick	WP SN SP IT	В	3600	3550	7	Thin Thick Thick Thick Thin	IT SN SP SN	F	0100	2200
4	Thick Thick Thick Thick Thick	SP SN SF' IT	В	2700	2600	8	Thin Thin Thick Thick	IT SP WN IT	L	2100	2200
4	Thick Thick Thick Thick	W N SN SP	А	2900	3100	0	Thick Thick Thick	SP SN SP	E	2500	2500
5	Thin Thick Thick Thick Thin	IT IT SP IT SP	А	2000	1750		Thin Thin Thick Thick	SN SP WN WP			
5	Thin Thin Thick	WN IT WN	С	3450	3300						, a

(b) If the ray path becomes tangential to any one of the layers below the strong negative layer, then layers in between the strong neagative layer and the layer to which the ray becomes tangential can be taken as a single layer with the average gradient. The bathythermogram is then reclassified and the final agreed case is obtained with the help of the tables.

5

Thick Thick

Thick

Thick

Thick

Е

WN SN

SP

SN

IT

3100

2900

Whether the ray path becomes tangential or not to any layer can be inferred from the temperature velocity distribution. The ray becomes tangential or if the temperature or velocity in any layer below the strong negative layer is equal to or more than the temperature or velocity at the surface of the strong negative layer. In all other cases the ray path does not become tangential to any of the layers below the strong negative layers.

The conclusion for drawing the ray paths through the strong negative layers are arrived at studying a few models. The validity of these conclusions needs more detailed studies. It can be seen from Table 4 that the maximum errors in the horizontal range occur when the strong negative layer is present in the potential layer.

A method to simplify the procedure for plotting the sonic ray paths in the potential layer from a bathythermogram has been devised based on the principle of layer thickness and gradient classification and their successive combinations. This has proved useful to minimize the number of layer in the potential layer to 3 or less irrespective of the number of layers present in the bathythermogram. The method described in this paper increases the speed of sonic ray path plotting in underwater acoustic problems such as travel time corrections to the echograms,

calculation of time intervals of the bottom reflected signals in seismic experiments, echo ranging, estimation of Sonar ranges, etc. The procedure involved in applying the method is simple and can be easily followed even by semi-skilled persons like the regular operators of the equipment concerned.

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