

Method for Plotting Sound Ray Paths in the Potential Layer from Bathythermograms

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Vertical distribution of temperature in the oceans influences greatly the sound propagation, particularly in bending the rays from their straight line paths. The changes in temperature structure are more common in the surface layers due to heating, cooling, mixing, etc., and contribute for the formation of several layers with different temperature gradients and thicknesses. Plotting of sound ray paths for such bathythermograms becomes difficult and also time-consuming if all these layers are to be considered separately. A method has been evolved which reduces the layers in the potential layer to a maximum of 3 layers irrespective of the number of layers present in the potential layer. The method is applied to several bathythermograph records and the ray paths plotted by this method are found almost similar to those plotted for the original structure of the record.

FOR problems relating to underwater acoustics it is essential to know the sound ray path and the intensity of the echo. These basically depend on (i) design characteristics of the sound source and receiver, (ii) environmental factors of the sea surface and the bottom, and (iii) physical and chemical properties of the sea water. The influence of these parameters in underwater sound propagation and detection is discussed extensively by **Urick**¹. Effect of changes in temperature on the sound ray path is studied in this paper mainly to evolve a method that simplifies the procedure for plotting ray paths for a given bathythermogram.

From the distribution of temperature with depth (up to a depth a bathythermograph can be operated) the ocean waters can be divided into 2 principle layers: (i) potential layer, and (ii) thermocline. Potential layer is defined as a layer that occurs between the sea surface and top of the **thermocline**². It is also sometimes called the surface layer. **Defant**³ defined the thermocline as a layer in which the average rate of fall in temperature is of the order of 5°C in 100 m. Many a time the potential layer is subdivided into number of layers with different thicknesses and temperature gradients due to the surface heating, cooling and mixing. In such a case it is rather difficult and also time-consuming to plot the ray paths passing through all these layers. Studies have been undertaken to classify the thickness and temperature gradient of the layers in potential layer and the successive combination of the layers to minimize the number of layers that are to be finally considered for plotting the ray paths.

Temperature Distribution and Its Effect on the Ray Path

Temperature, pressure and salinity are the principle physical and **chemical** properties of the sea water that effect the sound propagation in the sea. Variation in temperature with depth has the greatest effect on the sound transmission, namely

the bending of sound rays from their straight line paths. Increase in pressure with depth introduces a constant velocity gradient of the order of **0.018 m/sec/m** and the changes in the salinity in open oceans are generally small and may be neglected for plotting the ray paths.

Vertical changes in temperature and pressure introduce linear velocity gradients with depth. In a medium in which the velocity of sound changes linearly with depth, the sound rays can be shown to be arcs of circles which will have constant radius of **curvature**⁴. The horizontal distance (X) covered by the ray (Fig. 1) while passing through a layer of thickness (h) can be obtained from

$$X = h C \left(\frac{\theta_0 + \theta_1}{2} \right) \quad \dots (1)$$

where θ_0 and θ_1 are the angles made by the ray with the horizontal at the top and bottom of the layer respectively and they can be obtained from Snells Law as

$$\frac{\cos \theta_0}{v_0} = \frac{\cos \theta_1}{v_1} \dots = \text{a constant for any one ray}$$

In this expression the ray constant is reciprocal of the sound velocity in the layer in which the ray becomes horizontal, i.e. where $\cos \theta = 1$. This expression is the basis of ray computation used by

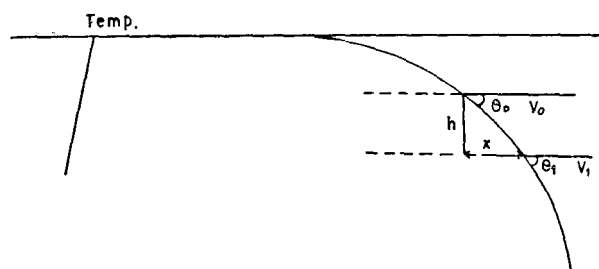


Fig. 1 — Ray path diagram for continuous fall in temperature with depth

most analog and digital computers, since it enables a particular ray to be 'traced out' by following it through successive layers into which the velocity profile may have been divided. When the potential layer consists of several layers, the total horizontal distance covered by the ray within the potential layer can be obtained from

$$X_n = h_1 \cot \left(\frac{\theta_0 + \theta_1}{2} \right) + h_2 \cot \left(\frac{\theta_1 + \theta_2}{2} \right) + \dots + h_n \cot \left(\frac{\theta_{n-1} + \theta_n}{2} \right) \dots (2)$$

Sound velocities are read from the temperature-sound velocity tables⁵ for the corresponding temperatures from the bathythermograms. For plotting the ray path it is necessary that at least one angle is known. When there is no isothermal layer or positive gradient layer in the potential layer, the starting angle of the ray in the first layer is considered as zero. When an isothermal layer or a positive layer occurs within the potential layer, the angle at the bottom of that layer is taken as zero. Other than by computers, the ray paths are also drawn using the templates or the instruments for plotting the refracted rays⁶.

Classification of Bathythermograms

Classification of layers in a bathythermogram suggested in this method mainly involves (i) classification of layers for their thickness, (ii) classification of layers for their temperature gradients, and (iii) successive combination of layers for obtaining a best agreed combination that will give the ray which closely resembles the ray path drawn for the original structure of the bathythermogram.

Thickness classification — Thick layer : A layer is considered as a thick layer if it is more than 10 m thick. Thin layer: A layer is considered thin layer if it is equal to or less than 10 m.

Gradient classification — Isothermal layer (IT) : The rate of fall in temperature is less than 0.1°C in 10 m or the rate of increase in temperature is less than 0.02°C in 10 m.

Weak positive layer (WP): The rate of increase is between 0.02 and 0.25°C in 10 m. Strong positive layer (SP): The rate of increase in temperature is more than 0.25°C in 10 m. Weak negative layer (WN): The rate of fall in temperature is between 0.1 and 0.3°C in 10 m. Strong negative layer (SN): The rate of fall in temperature is more than 0.3°C in 10 m.

Combination of layers — For obtaining the best combination of layers more than 1250 models comprising all the possible combinations of the layer thicknesses and gradients mentioned above were built up for the 2 layer case and 3 layer case. The ray paths are drawn for all these models with the help of the templates, first by considering each layer as a separate layer and then for different combinations of layers in each model. Ray paths are then compared and that combination of layers for which the ray path closely follows the ray path drawn by considering each layer as a separate layer is taken as the agreed case. All possible combinations of layers of their thicknesses and temperature gradients

as well as the final agreed cases for each of these combinations are given as A, B, C and D in Tables 1 and 2 for the 2 layer cases and 3 layer cases respectively. The notations A, B, C and D in the tables stand for: A, all layers in the potential layers are to be taken as single layer; B, first layer as a separate layer and the last 2 layers as 1 layer; C, first 2 layers as 1 layer and the last layer as a separate layer; and D, each layer as a separate layer.

For some combinations of layers more than one agreed case are recorded in the tables. Detailed study of the models has revealed that in certain combinations of the layers that fall in the same thickness and gradient, classification gives rise to more than one agreed case depending on the relative thicknesses and gradients of the layers. The conditions for choosing the correct agreed cases are given in Table 3.

When there are more than 3 layers in the potential layer, it is found that they can be finally brought down to 3 or less number of layers with the help of the tables. Detailed procedure for the successive combination of layers is illustrated in the example given below.

Consider a bathythermogram for which the temperature distribution with depth in the potential layer is given in Fig. 2. The potential layer extends up to 80 m depth and it can be divided into 6 layers with uniform temperature gradient in each layer. The final agreed case for this bathythermogram can be obtained from the tables following the procedure given below.

(a) Layers are classified for their thickness and temperature gradient from the classification given above and noted in column 4 of Fig. 2.

TABLE 1 - TWO LAYERS IN THE POTENTIAL LAYER

| Sl No. | Gradient classification | Thickness classification | | | |
|--------|-------------------------|--------------------------|----------------|----------------|---------------|
| | | Thick Thick (a) | Thick Thin (b) | Thin Thick (c) | Thin Thin (d) |
| 1 | IT WN | D | A | A | D |
| 2 | IT WP | A | D | A | A |
| 3 | IT SP | A&D | D | A | D |
| 4 | WN IT | D | A | A | D |
| 5 | WN WP | D | A | A | D |
| 6 | WN SP | D | D | D&A | D |
| 7 | WP IT | D | A | D | A |
| 8 | WP WN | D | A | A | A |
| 9 | WP SP | A | A | A | A |
| 10 | SP IT | D | D | D | D |
| 11 | SP WN | D | D | D | D |
| 12 | SP WP | A | A | A&D | A&D |

IT = isothermal; SP = strong positive; WN = weak negative; and WP = weak positive.

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TABLE 2 — THREE LAYERS IN THE POTENTIAL LAYER

| Sl No. | Gradient classification | Thickness classification | | | | | | | |
|--------|--|--------------------------|----------------------|----------------------|----------------------|---------------------|---------------------|---------------------|--------------------|
| | | Thick Thick Thick (a) | Thick Thick Thin (b) | Thick Thin Thick (c) | Thin Thick Thick (d) | Thick Thin Thin (e) | Thin Thick Thin (f) | Thin Thin Thick (g) | Thin Thin Thin (h) |
| 1 | IT WN | D | A | A | D | A | B | A | D |
| 2 | IT WN WP | A | B | C | A | A | C & D | C | A |
| 3 | IT WN SP | C | D | C | C | C | D | C | D |
| 4 | IT WP | C | C | C | C | A | C | C | A |
| 5 | IT IT WP WN | C | A | A | C | A | C | C | c |
| 6 | IT WP WN | c | C | C | C | C | C | C | C |
| 7 | IT SP IT SP | C | A | C | C | A | C | C | C |
| 8 | IT IT SP WN | A | A | A | C | A | A | C | A |
| 9 | IT SP WP | A | A | A | A | A | A | A | A |
| 10 | WN IT WN WN | D & A | B & C | A & B | C | B | C | A | A |
| 11 | IT WP WN IT SP | C | C | C | C | C | C | C | C |
| 12 | WN IT SP WN WP | C | C | C | C | C | C | C | C |
| 13 | IT WN WP IT WN WP WN SP | A & D | A & C | C & B | A | C | A | A | A |
| 14 | IT WN WP WN WN WP SP | A | A | A | A | A | A | A | A |
| 15 | WN WN WP SP WN SP | C | C | C | C | C | C | C | C |
| 16 | IT WN SP WN WN SP WN SP | C | C | C | C | C | A | A | A |
| 17 | IT WN SP WN WN SP WN WP WP IT WN WP IT WP WP IT SP | A | B & A | A | A | A | A | A | A |
| 18 | WN WN SP WP WP IT WN WP IT WP WP IT SP | C | C | C | C | C | C | C | C |
| 39 | WN WP IT WN WP IT WP WP IT SP | C | D | C | A | C | D | C | C |
| 20 | WN WP IT WP WP IT SP | C | C | C | C | C | C | C | C |
| 21 | WN WP WP IT SP | C | C | C | C | C | C | C | C |
| 22 | WN IT WP WN WP WN WP WN SP | B & D | B | D | B & D | B | B | D | A |
| 23 | IT WP WN WP WN WP WN SP | A | D | A | D | A | D | A | A |
| 24 | WN WP WN WP WN SP | C | A | C | C | A | A | C | A |

TABLE 2 — THREE LAYERS IN THE POTENTIAL LAYER— *Contd*

| Sl No. | Gradient classification | Thickness classification | | | | | | | |
|--------|-------------------------|--------------------------------|-------------------------------|-------------------------------|-------------------------------|------------------------------|------------------------------|------------------------------|-----------------------------|
| | | Thick Thick Thick (a) | Thick Thick Thin (b) | Thick Thin Thick (c) | Thin Thick Thick (d) | Thick Thin Thin (e) | Thin Thick Thin (f) | Thin Thin Thick (g) | Thin Thin Thin (h) |
| 25 | WP SP IT | C | C | C | C | C | C | C | C |
| 26 | WP SP WN | A | A | A | A | A | A | A | A |
| 27 | WP SP WP | C | A | A | C | A | A | C | A |
| 28 | SP IT WN | B | B | B | B | B | B | B | B |
| 29 | SP IT WP | C | A | C | C | A | A | C | A |
| 30 | SP IT SP | A | A | A | A | A | A | A | A |
| 31 | SP WN IT | B | C | C | B | C | B | C | C |
| 32 | SP WN WP | D | D | C | D | A | A | A | A |
| 33 | SP WN SP | A | A | A | A | A | A | A | A |
| 34 | SP WP IT | C | C | C | C | C | C | C | C |
| 35 | SP WP WN | C | C | C | C&D | C | B | C | C |
| 36 | SP WP SP | C | C | C | C | C | C | C | C |

IT = isothermal; SP = strong positive; WN = weak negative; and WP = weak positive.

(b) First, the top 3 layers are taken and by referring to Table 2 (19c), the agreed case for this type of combination of layers is given as C according to which the top 2 layers are to be considered as one layer and the 3rd layer as a separate layer.

(c) The other 3 layers in the bathythermogram are taken and referring to Table 2 (7g), the agreed case is given as C.

(d) From (b) and (c), the 6 layers in the potential layer are now reduced to 4 layers and they are reclassified for their thickness and gradients and are noted in column 5 (Fig. 2).

(e) Top 3 layers in column 5 are taken and referring to Table 2 (24a), the agreed case is given as C.

(f) From (e), the number of layers in the potential layer are reduced to 3 layers and are reclassified as given in column 6 (Fig. 2).

(g) Referring the 3 layers in column 6 to Table 2 (16a), the agreed case is given as C.

(h) From (g), the number of layers in the potential layer are reduced to 2 layers.

(i) Referring these 2 layers to Table 1 (7a), the agreed case is given as D, according to which these 2 layers are to be taken as separate layers.

Thus the 6 layers in the potential layer are

finally reduced to 2 layers for which the final ray path is plotted (dashed line in Fig. 3). The ray path for the original 6 layers is also plotted (solid line in Fig. 3). It can be seen that both the ray paths follow almost the same path and the difference in the horizontal distances covered by these 2 ray paths at the bottom of the potential layer is comparatively negligible. The method was applied to more than 70 bathythermograms and the errors in the horizontal ranges are found to be within 300 m. Some of these bathythermograms along with their classification and ranges are given in Table 4.

Strong negative gradient — Sometimes the strong negative gradient layers occur in the potential layer and behave like thermocline for the purpose of plotting the sound ray paths. A few models were studied by introducing strong negative layers, temperature gradient more than 0.3°C in 10 m, in the potential layer and the ray paths are drawn. These studies have revealed the following:

(a) If the ray path does not become tangential to any of the layers below the strong negative layer, it can be continued into regions below this layer with the gradient of the strong negative layer. This type of case is denoted by E in Table 4.

TABLE 3 - SELECTION OF CORRECT AGREED CASE WHEN MORE THAN ONE AGREED CASE IS GIVEN

| Agreed case No. | Agreed cases | Thickness | Agreed case |
|------------------|--------------|---------------------------------------|-------------|
| Tab. 1 (3a) | D | 1st layer > 2nd layer | other than |
| | A | For all combinations above | |
| Tab. 1 (6c) | A | > 30 m* | |
| | D | For all combinations above | other than |
| Tab. 1 (12c & d) | D | —* | |
| | A | For all combinations above | other than |
| Tab. 2 (2f) | D | ≤ 30 m | |
| | C | For all combinations above | other than |
| Tab. 2 (10a) | A | 2nd layer > 1st or 3rd layer | |
| | D | For all combinations above | other than |
| Tab. 2 (10b) | C | 2nd layer > 1st layer | |
| | B | For all combinations above | other than |
| Tab. 2 (10c) | B | 1st layer > 3rd layer | |
| | A | For all combinations above | other than |
| Tab. 2 (13a) | D | 1st layer > 2nd & 3rd layers | |
| | A | For all combinations above | other than |
| Tab. 2 (13b) | C | 1st layer > 2nd layer | |
| | A | For all combinations above | other than |
| Tab. 2 (13c) | B | 1st layer > 3rd layer | |
| | C | For all combinations above | other than |
| Tab. 2 (22a & d) | D | 3rd layer > 2nd layer | |
| | B | For all other combinations than above | other than |
| Tab. 2 (35d) | D | —* | |
| | C | For all combinations above | other than |

*Gradient — 0.35°C in 10 m.

TABLE 4 — SOME EXAMPLES OF ACTUAL BATHYTHERMOGRAMS, THEIR CLASSIFICATION AND RANGES

| No. of layers and thickness classification in the potential layer | Gradient classification | Agreed case | Direct ray range at the potential layer depth m | |
|---|-------------------------|-------------|---|---------------|
| | | | Agreed case | Original case |
| 2 | Thick IT | D | 4800 | 4800 |
| | Thick WN | | | |
| | Thin WN | A | 3700 | 3700 |
| | Thick IT | | | |
| 2 | Thick WN | A | 1200 | 1200 |
| | Thin IT | | | |
| 2 | Thick IT | A | 1100 | 1100 |
| | Thick SP | | | |
| 2 | Thin IT | D | 1600 | 1600 |
| | Thin WN | | | |
| 2 | Thin IT | A | 2600 | 2600 |
| | Thin WP | | | |
| 3 | Thick IT | D | 4000 | 4000 |
| | Thick WN | | | |
| | Thick IT | | | |
| 3 | Thick IT | A | 3200 | 3200 |
| | Thick WN | | | |
| | Thin IT | | | |
| 3 | Thick IT | E | 2900 | 2900 |
| | Thick WN | | | |
| | Thick SN | | | |
| 3 | Thin IT | C | 3000 | 2800 |
| | Thick WP | | | |
| | Thick WN | | | |
| 3 | Thick IT | C | 2650 | 2600 |
| | Thick SP | | | |
| | Thick IT | | | |
| 3 | Thick IT | C | 2400 | 2200 |
| | Thick SP | | | |
| | Thick IT | | | |
| 3 | Thick IT | C | 2300 | 2050 |
| | Thin SP | | | |
| | Thick IT | | | |
| 3 | Thin IT | C | 3600 | 3600 |
| | Thick SP | | | |
| | Thick WN | | | |
| 3 | Thick IT | A | 2950 | 2750 |
| | Thin SP | | | |
| | Thick WN | | | |
| 3 | Thick IT | E | 2600 | 2600 |
| | Thick SP | | | |
| | Thick SN | | | |
| 3 | Thick IT | E | 2800 | 2850 |
| | Thin SN | | | |
| | Thick IT | | | |
| 3 | Thin IT | E | 3600 | 3600 |
| | Thin SN | | | |
| | Thick WN | | | |
| 3 | Thin IT | E | 2100 | 2100 |
| | Thick SN | | | |
| | Thick SN | | | |
| 3 | Thick WN | A | 2200 | 2250 |
| | Thick IT | | | |
| | Thick WN | | | |
| 3 | Thin WN | A | 2500 | 2400 |
| | Thin WP | | | |
| | Thick WN | | | |
| 3 | Thick WP | E | 3100 | 3100 |
| | Thick SN | | | |
| | Thick IT | | | |
| 3 | Thick SN | E | 1500 | 1650 |
| | Thin IT | | | |
| | Thick SN | | | |
| 3 | Thick SN | A | 2600 | 2600 |
| | Thick SP | | | |
| | Thick WN | | | |

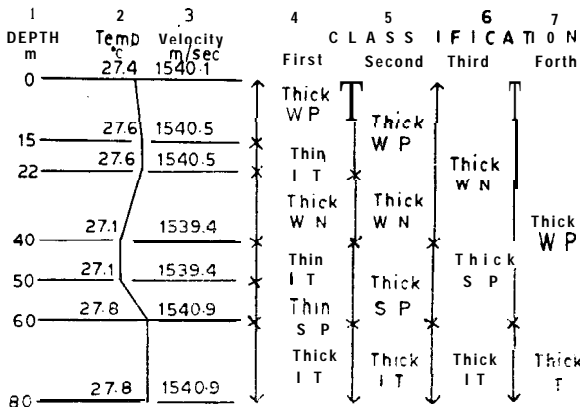


Fig. 2 — Classification and successive combination of layers

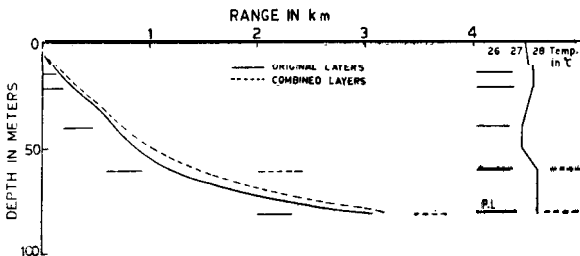


Fig. 3 -Ray path diagram

TABLE 4 — SOME EXAMPLES OF ACTUAL BATHYTHERMOGRAMS, THEIR CLASSIFICATION AND RANGES — *Contd*

| No. of layers and thickness classification in the potential layer | Gradient classification | Agreed case | Direct ray range at the potential layer depth m | |
|---|---|-------------|---|---------------|
| | | | Agreed case | Original case |
| 3 | Thick SN Thin SP | E | 1400 | 1500 |
| 4 | Thick SN Thick IT Thick WN Thick SP | C | 3300 | 3200 |
| 4 | Thick IT Thin WN | C | 3200 | 3000 |
| 4 | Thick IT Thin WN Thick WN Thin WN Thick SP Thin SN Thick IT | E | 1600 | 1550 |
| 4 | Thick IT Thin SN Thin IT Thick WN | E | 3450 | 3500 |
| 4 | Thin IT Thin SP Thin IT Thin SN | E | 1850 | 1700 |
| 4 | Thick IT Thin SP Thin SN Thin IT | E | 1150 | 1150 |
| 4 | Thick IT Thin SN Thick SP Thick WP | A | 4200 | 4100 |
| 4 | Thick IT Thin WN Thin SN Thick SP | A | 2800 | 2650 |
| 4 | Thick IT Thin SN Thick SP Thick WN Thick WP | C | 2850 | 2700 |
| 4 | Thick WN Thick SN Thick SP Thick IT Thick SP | B | 3600 | 3550 |
| 4 | Thick IT Thin SP Thick SN Thick SF Thick IT | B | 2700 | 2600 |
| 4 | Thick WN Thick SN Thick SP Thin IT | A | 2900 | 3100 |
| 5 | Thick IT Thick SP Thick IT Thin SP | A | 2000 | 1750 |
| 5 | Thin WN Thin IT Thick SP Thick IT | C | 3450 | 3300 |
| 5 | Thick WN Thick SN Thick SP Thick IT | E | 3100 | 2900 |

TABLE 4 — SOME EXAMPLES OF ACTUAL BATHYTHERMOGRAMS, THEIR CLASSIFICATION AND RANGES — *Contd*

| No. of layers and thickness classification in the potential layer | Gradient classification | Agreed case | Direct ray range at the potential layer depth m | |
|---|--|-------------|---|---------------|
| | | | Agreed case | Original case |
| 5 | Thick IT Thick SP Thin SN Thick SP | C | 4000 | 4000 |
| 5 | Thick WN Thick SN Thick WP Thin SN Thick SP Thick IT | E | 2200 | 2400 |
| 6 | Thin IT Thin WN Thin WP Thick WN Thick SP Thick IT | A | 3100 | 3400 |
| 6 | Thin IT Thin WN Thin WP Thick WN Thick SP Thick IT | C | 3300 | 3100 |
| 6 | Thin IT Thin SP Thick IT Thick IT Thin WN Thick IT Thin SN Thin SP Thick IT | E | 4000 | 4000 |
| 6 | Thick IT Thin WN Thick IT Thin SN Thick SP Thick IT | A | 3800 | 3550 |
| 6 | Thick WN Thin SP Thin SN Thick IT Thin WN Thick SN Thin IT Thick SP Thick WN | E | 2100 | 2200 |
| 7 | Thin SN Thick IT Thick SN Thick SP Thin SN Thin IT Thin SP Thick WN | E | 2100 | 2200 |
| 8 | Thick IT Thin SP Thick SN Thick SP Thin SN Thin SP Thick WN Thick WP | E | 2500 | 2500 |

(b) If the ray path becomes tangential to any one of the layers below the strong negative layer, then layers in between the strong negative layer and the layer to which the ray becomes tangential can be taken as a single layer with the average gradient. The bathythermogram is then reclassified and the final agreed case is obtained with the help of the tables.

Whether the ray path becomes tangential or not to any layer can be inferred from the temperature or velocity distribution. The ray becomes tangential if the temperature or velocity in any layer below the strong negative layer is equal to or more than the temperature or velocity at the surface of the strong negative layer. In all other cases the ray path does not become tangential to any of the layers below the strong negative layers.

The conclusion for drawing the ray paths through the strong negative layers are arrived at studying a few models. The validity of these conclusions needs more detailed studies. It can be seen from Table 4 that the maximum errors in the horizontal range occur when the strong negative layer is present in the potential layer.

A method to simplify the procedure for plotting the sonic ray paths in the potential layer from a bathythermogram has been devised based on the principle of layer thickness and gradient classification and their successive combinations. This has proved useful to minimize the number of layer in the potential layer to 3 or less irrespective of the number of layers present in the bathythermogram. The method described in this paper increases the speed of sonic ray path plotting in underwater acoustic problems such as travel time corrections to the echograms,

calculation of time intervals of the bottom reflected signals in seismic experiments, echo ranging, estimation of Sonar ranges, etc. The procedure involved in applying the method is simple and can be easily followed even by semi-skilled persons like the regular operators of the equipment concerned.

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