

## A Negative Resistance Oscillator Using a Bistable Circuit

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The working of an oscillator which uses a transistor bistable circuit as the negative resistance element is explained. An expression for the negative resistance exhibited by the circuit is derived and experimentally verified.

### 1. Introduction

AN ANALYSIS of an electronic circuit producing sustained oscillations will invariably show that the circuit exhibits a negative resistance as a part of its static or dynamic characteristics. Analyzing the circuit in this way, it may be said that the three essential parts of an oscillator are: (i) a power source, (ii) a means of storing energy, and (iii) a negative resistance element. If the means of storing energy is an LC circuit, the LC circuit has a natural oscillation frequency given by the relation :

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

at which the energy stored in the circuit is transferred back and forth between the inductance and capacitance. However, resistive losses in the circuit absorb part of this circulating energy and the oscillations of current and voltage in the LC circuit are damped out. Hence, the role of the negative resistance element is to release energy from the power source to the LC

network to make up for the energy losses. In this paper, after briefly explaining the working principle of a transistor negative resistance oscillator, an expression has been derived for the dynamic negative resistance exhibited by the negative resistance element, viz. a transistor bistable circuit while working in the active region. Based on this expression, the building up of the oscillations in the circuit is explained and the result is experimentally verified.

### 2. Transistor Negative Resistance Oscillator

A transistor bistable switching circuit exhibits an unstable negative resistance between the two stable states<sup>1,2</sup>. As a two-terminal negative resistance element, this circuit has been successfully used for constructing an oscillator<sup>3,4</sup> on the above principle, for certain instrumentation needs. The circuit diagram of the oscillator is shown in Fig. 1. A typical composite curve<sup>5</sup> of this bistable circuit is shown in Fig. 2. It has four distinct regions, viz. region 1 where  $T_1$  is saturated and  $T_2$  is cut off, region 2 where  $T_1$  is active and  $T_2$  is cut off, region 3 where both  $T_1$  and  $T_2$  are active and region 4 where  $T_1$  is cut off and  $T_2$  is active or saturated. At the beginning of the fourth region,  $T_2$  may be in the active region and at the end

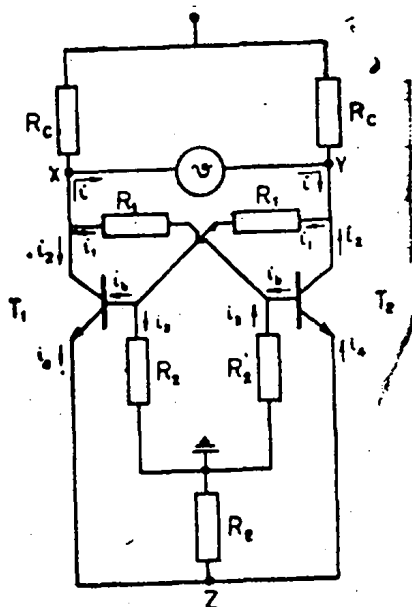


Fig. 1 — Transistor negative resistance oscillator

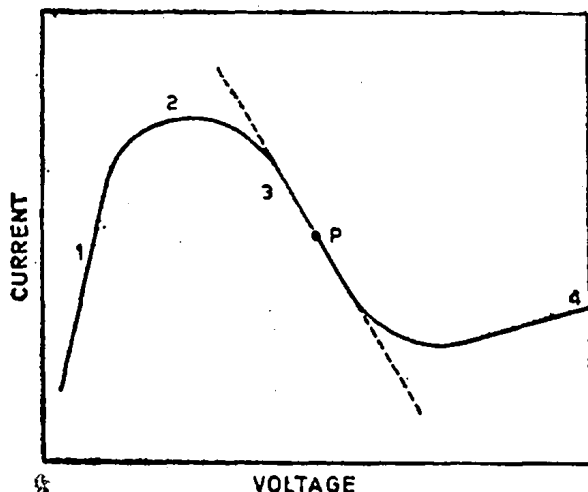


Fig. 2 — Composite curve of the bistable circuit

(highest voltage) it will be saturated. As the transistors  $T_1$  and  $T_2$  switch alternately from conduction to non-conduction and vice versa, they operate through the negative resistance region, i.e. region 3, twice in each cycle. During this period, energy is released to the LC tank circuit from the power source to compensate for the inherent resistive losses.

**3. Expression for the Negative Resistance**

Fig. 3 shows the transistor bistable circuit working in the linear region. To derive an expression for the output resistance  $R_s$  seen between the two collector points X and Y, the following assumptions are made:

- (i) Both transistors are identical in electrical characteristics.
- (ii) Both transistors are working in the active region and around the point P in Fig. 2.
- (iii) Point Z is a virtual ground point, since the total current  $2 I_e$  flowing through  $R_E$  remains constant.
- (iv) The effect of  $r_{bb'}$ , the base spread resistance, is negligible at low values of  $I_e$ .

Now, by applying a small ac voltage  $v$  between the points X and Y, and determining the value of the total input current  $i$ , the output resistance  $R_s$  is calculated as:

$$R_s = \frac{v}{i} \quad \dots(1)$$

The various values of the currents and their directions are shown in Fig. 3. Then the following equations hold good.

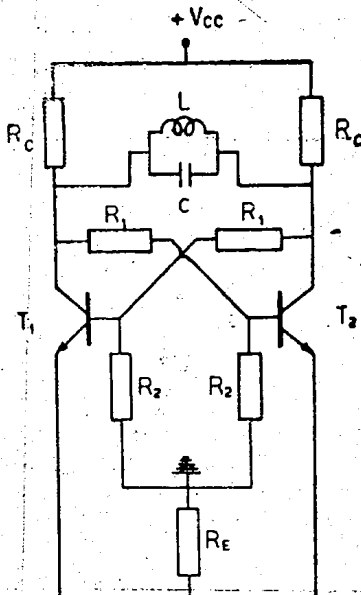


Fig. 3—Bistable circuit in the linear region

$$v = 2 (R_1 i_1 + R_2 i_2) \quad \dots(2)$$

$$i = (i_1 - i_2) \quad \dots(3)$$

$$i_1 = (i_b + i_3) \quad \dots(4)$$

$$i_2 = h_{fe} i_b \quad \dots(5)$$

$h_{fe}$  being the small signal current gain factor and

$$i_3 = r_e i_4 / R_2 \quad \dots(6)$$

where  $r_e$  is the intrinsic emitter resistance given by the expression  $r_e = (kT/q I_e)$ ,  $k$  being the Boltzmann's constant,  $T$  the absolute temperature,  $q$  the electronic charge and  $I_e$  the emitter current corresponding to the quiescent point P in the composite curve shown in Fig. 2. If  $I_e$  is expressed in milliamperes,  $r_e$  is approximately equal to  $(25/I_e)$  ohms and

$$i_4 = (1 + h_{fe}) i_b \quad \dots(7)$$

From the above equations

$$i_3 = \frac{(1 + h_{fe}) r_e}{h_{fe} R_2} \cdot i_2 \quad \dots(8)$$

$$\text{and } i_1 = \left[ \frac{i_2}{h_{fe}} + \frac{(1 + h_{fe}) r_e i_2}{h_{fe} R_2} \right]$$

$$\text{or } i_1 = \left[ \frac{R_2 + (1 + h_{fe}) r_e}{h_{fe} R_2} \right] \cdot i_2 \quad \dots(9)$$

From Eq. (3) :

$$i = i_2 \left[ \frac{R_2 + (1 + h_{fe}) r_e - h_{fe} R_2}{h_{fe} R_2} \right] \quad \dots(10)$$

From Eqs. (2), (8) and (10)

$$v = 2i \left[ \frac{R_1 R_2 + (1 + h_{fe}) r_e R_1 + (1 + h_{fe}) r_e R_2}{R_2 + (1 + h_{fe}) r_e - h_{fe} R_2} \right] \quad \dots(11)$$

and since  $\frac{v}{i} = R_s$

$$R_s = -2 \left[ \frac{R_1 R_2 + (1 + h_{fe}) (R_1 + R_2) r_e}{h_{fe} R_2 - (1 + h_{fe}) r_e - R_2} \right] \quad \dots(12)$$

and since  $\frac{r_e}{R_2} \ll 1$  and  $h_{fe} \gg 1$ ,

$$R_s \approx -2 \left[ \frac{R_1}{h_{fe}} + r_e \left( 1 + \frac{R_1}{R_2} \right) \right] \quad \dots(13)$$

The above expression gives the value of the negative resistance, in the neighbourhood of the point P in the composite curve. From the shape of the composite curve, it is evident that far away from the point P (point of inflexion), the value of the negative resistance will be much different from and higher than that given by Eq. (13). Also, in the derivation it is assumed that  $i$  is so small that it does not change the value of  $I_e$  appreciably and as such,  $r_e$  remains constant. So, for larger current swings, the negative resistance will not remain constant as given

by Eq. (13) but it will change around that value. However, Eq. (13) gives the minimum obtainable value of the negative resistance for a given value of  $I_e$ . This value is also given by the tangent line drawn at the point P (point of inflexion) in the composite curve shown in Fig. 2. Similar analysis of the bistable circuit that uses twin power supplies and grounded emitters is also possible.

**4. Negative Resistance Oscillations**

The bistable switching source providing the dynamic negative resistance, if connected across an LC circuit, produces oscillations under proper conditions. In Fig. 4 (a) is shown a parallel LC circuit connected across a bistable voltage source whose output resistance  $R_s$  is negative. In Fig. 4 (b), the coil resistance  $R_L$  is brought in shunt as  $R_p$ , where  $R_p = Q^2 R_L$ ,  $Q$  being the quality factor of the coil. In Fig. 4 (c), the Thevenin equivalent circuit is shown where :

$$v_t = v_s \frac{R_p}{R_p + R_s} \quad \dots(14)$$

and 
$$R_t = \frac{R_p R_s}{R_p + R_s} \quad \dots(15)$$

The transfer function of the circuit is given in Laplace transform notation as :

$$\frac{v_o}{v_s} = \frac{R_p}{R_p + R_s} \left[ \frac{SL}{(S^2 LC R_t + SL + R_t)} \right] \quad \dots(16)$$

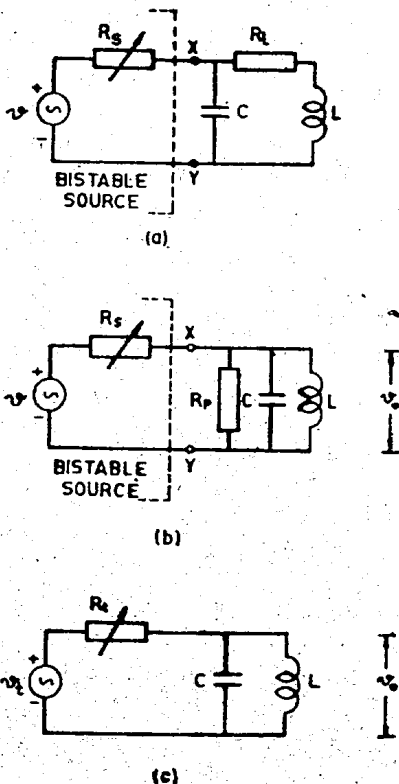


Fig. 4 -- Thevenin equivalent of the oscillator

The poles of the transfer function are :

$$S_1 = -\frac{1}{2 C R_t} + j \sqrt{\frac{1}{LC} - \left(\frac{1}{2 C R_t}\right)^2} \quad \dots(17)$$

and

$$S_2 = -\frac{1}{2 C R_t} - j \sqrt{\frac{1}{LC} - \left(\frac{1}{2 C R_t}\right)^2} \quad \dots(18)$$

These poles are complex conjugates.

Substituting for  $R_t$  in terms of  $R_p$  and  $R_s$  ( $R_s$  being negative in value) from Eq. (15) as :

$$R_t = -\frac{R_p R_s}{(R_p - R_s)} \quad \dots(19)$$

$$S_1 = \frac{(R_p - R_s)}{2 C R_p R_s} + j \sqrt{\frac{1}{LC} - \frac{(R_p - R_s)^2}{4 C^2 R_p^2 R_s^2}} \quad \dots(20)$$

and

$$S_2 = \frac{(R_p - R_s)}{2 C R_p R_s} - j \sqrt{\frac{1}{LC} - \frac{(R_p - R_s)^2}{4 C^2 R_p^2 R_s^2}} \quad \dots(21)$$

As an example, when  $R_p$  is nearly equal to  $R_s$  and  $(R_p - R_s)$  is positive, the term

$$\left[ \frac{1}{LC} - \frac{(R_p - R_s)^2}{4 C^2 R_p^2 R_s^2} \right]$$

will be positive, and hence the time response is also oscillatory. In addition, the real part is also positive which means that the amplitude of oscillation increases with time, the magnitude of the negative resistance also increases. This means the positive value of the real part continuously decreases, i.e. the rate of increase of amplitude continuously decreases. This shows that, to start with  $-R_s$  has to exist and the value of  $(R_p - R_s)$  has to be positive for initiating the oscillations. Because of the nonlinearity in the negative resistance characteristic, ultimately, the real part of the poles becomes zero, i.e.  $R_s$  becomes equal to  $R_p$ , and the frequency of this constant amplitude oscillations is given by :

$$f_0 = \frac{1}{2\pi \sqrt{LC}} \quad \dots(22)$$

**5. Experimental Verification**

Experimental verification of the conclusions arrived above was done as follows. In order to keep  $r_e$  and hence  $I_e$  constant, a constant current source has been used in the place of the resistor  $R_E$  as shown in Fig. 5. By controlling the voltage  $V_B$ ,  $V_E$  and thus  $I_e$  was controlled. A potentiometer  $R_{sh}$  was connected in shunt across the LC circuit and its value was adjusted till the oscillations just vanished. The experiments were repeated for different values of  $I_e$  as set by  $V_E$ . The results are given in Table 1. It can be seen from

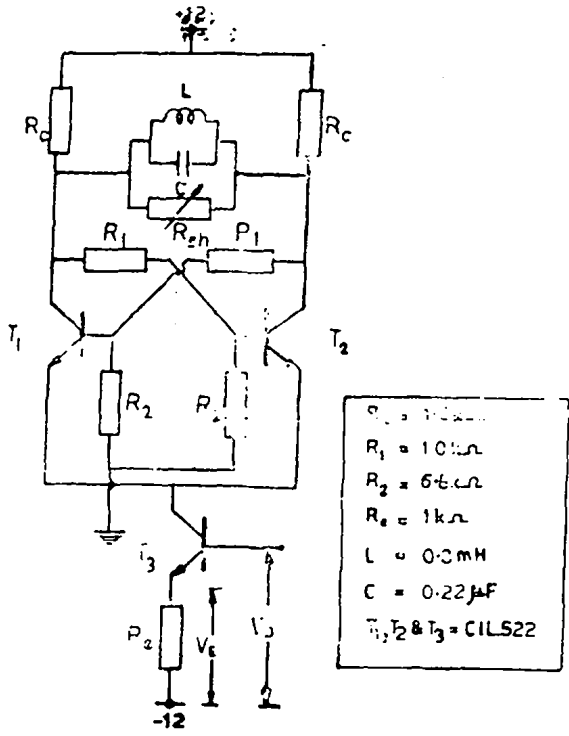


Fig. 5 — Experimental circuit

the results that, whenever  $R_p'$  (the effective shunt resistance across the LC circuit) goes below the value of  $R_s$ , oscillation stops. This is due to the fact, that in such cases, the value of  $(R_p' - R_s)$  becomes negative and the real part of the poles also becomes negative. This indicates, that the amplitude of oscillations decreases with time and consequently the oscillations die out. This proves that, in the limit, if oscillations, once started, are to continue,  $R_p'$  has to be equal to  $R_s$ . Thus the expression derived for the negative resistance is indirectly verified to be correct. The observed large variations in the value of  $(R_p' - R_s)$  as the value of  $I_e$  is increased, has been due to the difficulty experienced in just stopping the oscillations with proper adjustment of the shunt potentiometer  $R_{sh}$ .

Table 1 — Experimental Data to Verify the Expression for Dynamic Negative Resistance of the Oscillator

$[ R_s = (200 + 2.8r_o), R_p = 445 \Omega, R_p' = R_p R_{sh} / (R_p + R_{sh}) ]$

| $V_E$<br>V | $I_e$<br>mA | $R_s$<br>ohms | $R_{sh}$<br>ohms | $R_p'$<br>ohms | $(R_p' - R_s)$<br>ohms |
|------------|-------------|---------------|------------------|----------------|------------------------|
| 2.5        | 1.25        | 256           | 600              | 255            | -1                     |
| 3.0        | 1.50        | 247           | 470              | 229            | -18                    |
| 3.5        | 1.75        | 240           | 400              | 211            | -29                    |
| 4.0        | 2.00        | 235           | 300              | 193            | -42                    |
| 4.5        | 2.25        | 231           | 320              | 196            | -45                    |
| 5.0        | 2.50        | 228           | 300              | 179            | -49                    |
| 5.5        | 2.75        | 225           | 280              | 172            | -53                    |
| 6.0        | 3.00        | 223           | 250              | 160            | -63                    |

6. Conclusion

The results of the present investigation prove that, to start the oscillations first the real part of the poles has to be positive, as explained by theory. Only in the limit when constant amplitude oscillations are obtained, the real part becomes zero; when once the real part becomes negative, due to large dissipation in the tank circuit, the oscillations tend to die out. It may be mentioned, that the working of this oscillator may also be explained by the well known feedback theory.

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References

- GODDEN, A. K. & COOKS, T. A., *Electron. Engng*, 35 (1963), 751.
- MILLMAN, J. & TAUB, H., *Pulse, digital and switching wav. forms* (McGraw-Hill Book Co. Inc., New York), 1965, 476.
- SANKARANARAYANAN, P. E., *Electron. Engng*, 39 (1967), 715.
- SANKARANARAYANAN, P. E., *J. Instr. Telecommun. Engrs, New Delhi*, 16 (1970), 502.
- HULL, JAMES E., *Texas Instruments Application Report*, 1958.