

Derivatives and Dike Anomaly Interpretation

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Summary – In mining geophysics, dike-like features are of particular interest because intrusive igneous rocks and layered metamorphic rocks very frequently carry magnetic minerals. The present paper deals with an accurate method of determination of the dike parameters through analysis of horizontal derivative profiles prepared from the ground- or aero-magnetic profiles.

1. Introduction

The dipping dike equation and the quantitative methods of interpreting it continues to attract attention of geophysicists and a variety of valuable techniques have been appearing in geophysical literature from time to time. Using a few specific points on a magnetic profile on the one hand and superposition of an entire profile on pre-computed curves on the other hand formed two major groups of classification. Also different procedures have been developed for computing first and second vertical derivatives, upward and downward continuation of anomalies. The first vertical derivative picture is easier to interpret as shown by RAO and PRAKASA RAO [9]³⁾. Second derivatives of the anomaly bring the shape characteristics into sharper focus (VACQUIER *et al.* [11]). Depths to the tops of the anomaly-causing bodies can be determined with acceptable accuracy using downward continuation methods according to ROY [10]. For computing first and second vertical derivatives, upward and downward continuations formulae are in general, elaborate and calculations take considerable time if worked on a desk calculator. But transforming a ground or aero-magnetic profile into a horizontal derivative profile is easier. According to the centre point formula if $\Delta F_{(x-p, 0)}$ and $\Delta F_{(x+p, 0)}$ are anomaly values on the ΔF profile on either side of $\Delta F_{(x, 0)}$ at a small distance p , then the horizontal derivative at $(x, 0)$ is given by

$$\Delta F'_{(x, 0)} = \left. \frac{\partial(\Delta F)}{\partial x} \right|_{(x, 0)} = \frac{1}{2p} [-\Delta F_{(x-p, 0)} + \Delta F_{(x+p, 0)}]. \quad (1)$$

In this paper, the horizontal derivative profile instead of the ground or aero-magnetic curve is analysed to obtain the three crucial parameters of a dike, namely, width,

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³⁾ Numbers in brackets refer to References, page 129.

depth and angle Q , which governs the relative amplitudes of the symmetrical and asymmetrical components of the magnetic profile. Some of the ideas used in this paper have been developed by two of us (B. S. R. RAO and T. K. S. PRAKASA RAO) and others, especially the location of origin (RAO *et al.* [8]) and splitting of the anomalies into two components (POWELL [7], HUTCHISON [4] etc). One advantage in using a horizontal derivative profile is that prior knowledge of the zero datum level is not required. Linear regional anomalies will have negligible effects on the shape of the horizontal derivative profile.

2. Regional anomalies

The magnetometer does not measure solely the effect of the body to be investigated. It also measures the effect of the regional anomalies and the field of the earth in addition. Therefore effect of the regional anomalies and the field of the earth have to be eliminated. Various methods have been recommended for obtaining residual fields but as VAJK [12] pointed out, no unique determination of the regional effect is possible. There will be always a personal factor in the determination of the regional.

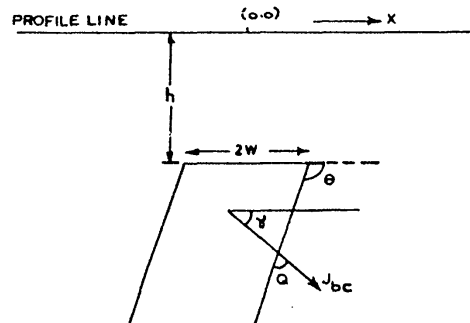


Figure 1

Even though the assumption of a linear regional field is not always satisfactory in magnetic interpretation, its elimination is comparatively easier. Let us consider its effect on the anomaly field caused by a dike-like feature. Splitting up and isolation of symmetrical part in a ground- or aero-magnetic profile result in the suppression of the regional effect (NAUDY [2]). Linear regional effects are even more effectively suppressed provided the horizontal derivative profile is decomposed into its symmetrical and asymmetrical parts. As is the usual practice, when a linear regional field is assumed and subtracted from the values of the magnetic field, it may result in incomplete or improper elimination of the regional field. The resulting residue of the regional effect can either be suppressed to a negligible level or rendered neutral by computing the horizontal derivative curve. The following mathematical treatment will explain our point of view more clearly.

If ΔF is the magnetic anomaly generated by a dike-like body, say, and δF is the measured magnetic field which includes a linear regional field (or the residue, if it is subtracted to some extent by assumption) given by R , then

$$\delta F = (\Delta F \pm R) = \Delta F \pm (m x + C). \quad (2)$$

Where m is the gradient of the regional and C is a constant. Differentiating with respect to x , we get

$$\delta F' = (\Delta F' \pm m) \quad (3)$$

Splitting into symmetrical and asymmetrical parts,

$$S' = \frac{1}{2} [\delta F'(x) + \delta F'(-x)] \quad (4)$$

$$= \frac{1}{2} [\Delta F'(x) + \Delta F'(-x)] \pm m \quad (5)$$

$$A' = \frac{1}{2} [\delta F'(x) - \delta F'(-x)] \quad (6)$$

$$= \frac{1}{2} [\Delta F'(x) - \Delta F'(-x)]. \quad (7)$$

That means the symmetrical horizontal derivative component of the measured magnetic field is in excess by m , which is equal to the uniform gradient of the regional field, over the symmetrical horizontal derivative component of the magnetic field due to the dike alone [Equations (4) and (5)]. The asymmetrical component is free from the regional effect as can be seen from equations (6) and (7). In fact, linear regional anomalies tilt the regionally free magnetic profiles. On the other hand, if horizontal derivative profiles are constructed, the resulting constant regional gradient m in equation (5) shifts the datum level upward or downward from the actual datum level for which there is no regional anomaly.

It is for this reason, solutions for depth and width of a dike are derived here from the horizontal derivative equation. This method has the added advantage over the existing dike anomaly interpretations in that the linear regional gradients will have no disturbing effect on the origin, that is, the position corresponding to $x=0$.

3. The dipping dike equation

If a magnetized tabular body has infinite length, extends to infinite depth, has parallel sides and a horizontal upper surface and is uniformly magnetized, its total intensity anomaly is given by

$$\Delta T = 2 J_{bc} \sin \theta \left\{ \sin Q \left[\tan^{-1} \frac{x+w}{h} - \tan^{-1} \frac{x-w}{h} \right] + \frac{\cos Q}{2} \log \frac{(x-w)^2 + h^2}{(x+w)^2 + h^2} \right\}. \quad (8)$$

Where J_{bc} is the intensity of magnetization in the plane of the profile, θ is the dip of the dike and Q is a compound angle involving the inclination of the earth's magnetic field and θ . w is the half-width and h is the depth to the top of the dike.

The horizontal derivative of ΔT is obtained by differentiating expression (8) with respect to x ,

$$\frac{\partial(\Delta T)}{\partial x} = 2 J_{bc} \sin \theta \left\{ \sin Q \left[\frac{h}{(x+w)^2 + h^2} - \frac{h}{(x-w)^2 + h^2} \right] + \cos Q \left[\frac{(x-w)}{(x-w)^2 + h^2} - \frac{(x+w)}{(x+w)^2 + h^2} \right] \right\} \quad (9)$$

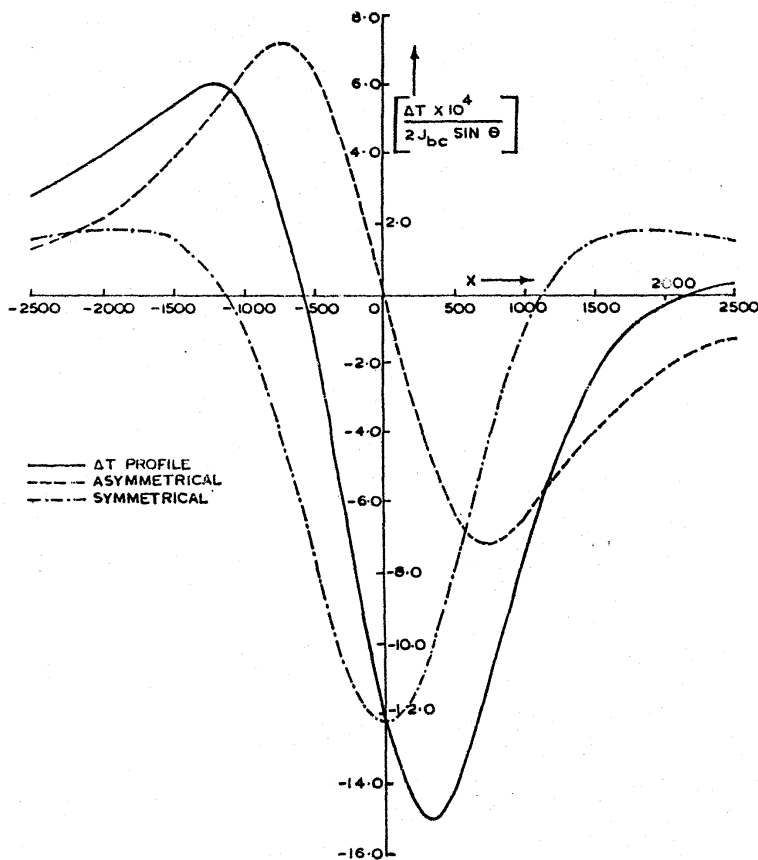


Figure 2

As in the case of ΔT expression, it can be seen that the horizontal derivative expression is also a combination of a symmetric and an asymmetric function. In order to break down $\Delta T'$ into its symmetric and asymmetric parts, the position corresponding to $x=0$ is to be found. A template (Fig. 3), designed by RAO *et al.* [8] may be used to locate the origin. The template, drawn on a transparent sheet is placed on the horizon-

tal derivative profile in such a manner that the base line of the profile coincides with XX line of the template as in Fig. 4. The template is then moved along the base line of the profile until two segments of the profile occupy the same compartment on opposite XX line of the template. The template is then moved along the base line of the profile until two segments of the profile occupy the same compartment on opposite sides of the template's centre. Then if the template is rotated about its centre slightly,

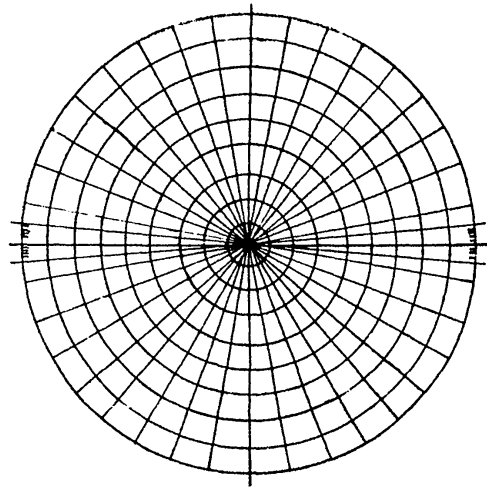


Figure 3

the two segments of the profile come to corners or sides of opposite compartments where it is easy to identify the two symmetrical points. When the two symmetrical points thus identified are joined, the line passes through the base line exactly at $x=0$. This procedure is perfectly valid because the horizontal derivative profile is unsymmetrical throughout except at two points on either side of the origin.

The position of the symmetrical points may be calculated by equating the symmetrical part of the horizontal derivative expression to zero. That is when

$$\left[\frac{(x-w)}{(x-w)^2 + h^2} - \frac{(x+w)}{(x+w)^2 + h^2} \right] = 0 \quad (10)$$

and

$$x^2 = (w^2 + h^2). \quad (11)$$

For use in future calculations, let us term this as x_d in Fig. 5.

After fixing the ordinate corresponding to $x=0$, splitting of the profile into its symmetrical and asymmetrical terms can be accomplished.

For convenience let us represent $\partial(\Delta T)/\partial x$ as $\Delta T'$, the symmetrical part of $\partial(\Delta T)/\partial x$ as S' and the asymmetrical part of $\partial(\Delta T)/\partial x$ as A' such that $\partial(S)/\partial x = A'$ and $\partial(A)/\partial x = S'$.

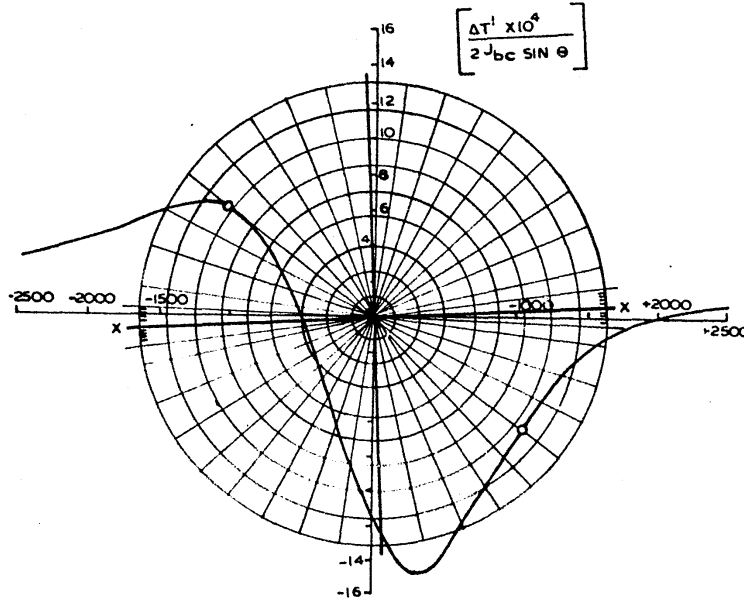


Figure 4

Thus we have

$$S' = \frac{1}{2}[\Delta T'(x) + \Delta T'(-x)] \tag{12a}$$

$$= \left[\frac{(x-w)}{(x-w)^2 + h^2} - \frac{(x+w)}{(x+w)^2 + h^2} \right] \tag{12}$$

$$A' = \frac{1}{2}[\Delta T'(x) - \Delta T'(-x)] \tag{13a}$$

$$= \left[\frac{h}{(x+w)^2 + h^2} - \frac{h}{(x-w)^2 + h^2} \right]. \tag{13}$$

4. Expressions for width and depth

Let us consider the symmetrical part of the horizontal derivative expression. It has its maximum value when $x=0$, i.e.

$$S'_{\max} = S'(0) = - \left[\frac{2w}{w^2 + h^2} \right]. \tag{14}$$

If we assume a value x such that $S'(x_{1/2}) = \frac{1}{2}S'(0)$,

$$- \left[\frac{w}{w^2 + h^2} \right] = \left[\frac{(x_{1/2} - w)}{(x_{1/2} - w)^2 + h^2} - \frac{x_{1/2} + w}{(x_{1/2} + w)^2 + h^2} \right]. \tag{15}$$

Solving for $x_{1/2}$, we get

$$[x_{1/2}]^2 = -2h^2 \pm \sqrt{4h^4 + (h^2 + w^2)^2}. \tag{16}$$

Similarly if we assume values $x_{1/4}$ and $x_{3/4}$ such that $S'(x_{1/4}) = \frac{1}{4} S'(0)$ and $S'(x_{3/4}) = \frac{3}{4} S'(0)$ we get two more relations each for h and w . If these solutions are also obtained, all the three sets may be represented graphically following the procedure outlined by KOULOMZINE *et al.* [5]. In the present paper, solutions for h and w obtained from $x_{\frac{1}{4}}$ and x_a only are given.

5. Expression for Q

The horizontal derivative profile crosses the distance axis at two points A and B such that

$$a + b = + 2 h \tan Q. \quad (20)$$

For large values of Q , only one cross over can be seen while the other one occurs at a larger distance rendering it difficult to identify accurately. Therefore an alternate method must be evolved by which Q can be calculated over any range.

The horizontal derivative profile is characterised by three identifiable points. They are the two symmetrical points and the origin of the profile. The derivative values at these points may be used to obtain an expression for Q .

$\Delta T'$ at $x=0$ is given by putting $x=0$ in (9)

$$\Delta T'(0) = 2 J_{bc} \sin \theta \cos Q \left[- \frac{2 w}{w^2 + h^2} \right]. \quad (21)$$

$\Delta T'_s$ at any one of the symmetrical points is obtained by putting $x^2 = w^2 + h^2$ in (9)

$$\Delta T'_s = 2 J_{bc} \sin \theta \sin Q \left[\frac{- 4 w h (w^2 + h^2)^{1/2}}{2 h^4 + 2 h^2 (2 w^2 + h^2)} \right]. \quad (22)$$

Dividing (21) by (22), we get the ratio R' .

$$R' = \frac{\Delta T'(0)}{\Delta T'_s} = \cot Q \frac{2 h}{(w^2 + h^2)^{1/2}}. \quad (23)$$

Hence

$$\cot Q = [x_a R' / 2 h] = \frac{x_a \Delta T'(0)}{2 h \Delta T'_s}. \quad (24)$$

In the above expression, all except $\cot Q$ are known and hence Q may be calculated. Usually, the geological dip, θ is a known parameter and hence the direction of magnetization γ , in the plane of the profile may be calculated because $Q = (\gamma + \psi - \theta)$ in which, ψ also is a known direction lying in the vertical plane containing the magnetic meridian in which the anomaly is measured.

Using (21), the strength of magnetization J_{bc} in the plane of the profile may also be calculated.

6. Procedure

- i) Draw the horizontal derivative profile using the centre point formula.
- ii) Locate the origin of this profile using the template.
- iii) Resolve the derivative profile into its symmetrical part using expression (12a).
- iv) Scale off x_d and $x_{1/2}$ from the symmetrical curve.
- v) Calculate h and w using expressions (18) and (19).
- vi) Read the values, $\Delta T''(0)$ and $\Delta T'_s$ on the derivative profile.
- vii) Calculate Q using expression (24).

7. Verified example

To illustrate the results obtainable with the proposed method a profile from south of the magnetic equator near Vallenar, Chile, of PARKER GAY Jr. [6] is selected. Matching was found difficult for this profile with GAY's standard curves. He admitted that the curve matching cannot be considered excellent in this particular case. One

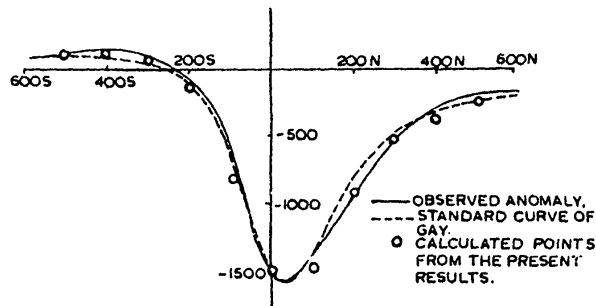


Figure 6

reason for obtaining poor fit may be the presence of regional anomalies. Figure 6 shows the observed anomaly together with a standard curve of GAY and the computed points obtained by the results of the present paper. Following are the results:

	PARKER GAY	Present authors	Drill data
Depth	110 m	103 m	97 m
Width/depth ratio	1.5	1.25	
Q	$-155''$	-155°	

This example emphasizes the advantage in calculating a horizontal derivative profile from a ground- or aero-magnetic profile. It may be concluded that the effect of incompletely eliminated regionals (both linear and low gradient irregular regionals) can be either suppressed to a negligible level or rendered neutral by computing the horizontal derivative profile.

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