

HARMONIC TIDAL ANALYSIS AT A FEW STATIONS USING THE LEAST SQUARES METHOD

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ABSTRACT

Using the least squares method, harmonic analysis has been performed on hourly water level records of 29 days at several stations depicting different types of non-tidal noise. For a tidal record at Mormugao, which was free from storm surges (low frequency noise range less than 10 cm), we found that in general the amplitudes and phases were in agreement with those published by the International Hydrographic Bureau (IHB), Monaco, within ± 1 cm and $\pm 3^\circ$ respectively. Incidentally this happens to be the range of agreement between the results of harmonic analysis by two methods, *viz.* the least squares method and the Admiralty semi-graphic method - for a tidal record at Bedwell (in Canada). For the records at Madras, Visakhapatnam, Tuticorin and Paradip all of which were contaminated by occurrences of storm surges exceeding 20 cm, the amplitudes and phases of the principal constituents M_2 , S_2 , N_2 , K_1 and O_1 were in agreement with those published by IHB within ± 2 cm and $\pm 15^\circ$ respectively, whereas for other constituents the tidal constants, specially the phases, were generally not reliable.

Key-words : Harmonic analysis, tides, least squares

INTRODUCTION

The use of the least squares method (LSM) of harmonic analysis of tides/tidal streams is becoming very popular. Poole, Rabinovich, Spielvogel and Harvey (1980) used LSM to study deep sea tides and tidal streams during the Second Soviet-American Tsunami Expedition (August-October, 1978) and compared the results with those obtained by using the response method of Munk and Cartwright (1966). Cheng and Gartner (1985) used LSM for harmonic analysis of tides and tidal currents in South San Francisco Bay, California. LSM started out as an alternative to the classical method of Doodson (1927). In certain circumstances LSM appears to be the only method applicable. For example Evans and Pugh (1982) used LSM for analysing clipped sea-level records, which occur at sites where the gauge dries at low levels, and Zetler, Schuldt, Whipple and Hicks (1965) used LSM on data randomly spaced in time to conform to daylight hours with applications to remotely sensed data (via satellites) in mind.

MATERIAL AND METHODS

Using the LSM described in the Appendix, harmonic analysis was performed on 13 records of 29 days of hourly observations, *viz.* 1 record each at Mormugao and Bedwell

(in Canada); 3 consecutive records at Madras, Visakhapatnam and Tuticorin; and 2 consecutive records at Paradip (see Table I).

Table I – Tidal records (29 days long) analysed by LSM

Rec. No.	Station	Central day
1.	Bedwell (Canada)	26.4.1905
2.	Mormugao	28.2.1979
3.	Madras	8.10.1968
4.	- do -	6.11.1968
5.	- do -	5.12.1968
6.	Vishakapatnam	8.10.1968
7.	- do -	6.11.1968
8.	- do -	5.12.1968
9.	Tuticorin	8.10.1968
10.	- do -	6.11.1968
11.	- do -	5.12.1968
12.	Paradip	6.11.1968
13.	- do -	5.12.1968

Table II – Constituents used in Least Squares determination and satellite constituents for which allowance was made in correcting phases and amplitudes

S.No.	Constituent	Speed*	Satellites
1	M ₂	28.98	
2	S ₂	30.00	K ₂ , T ₂
3	N ₂	28.44	ϑ_2
4	μ_2	27.97	2N ₂
5	L ₂	29.53	λ_2
6	K ₁	15.04	P ₁
7	O ₁	13.94	
8	J ₁	15.59	
9	Q ₁	13.40	ρ_1
10	M ₁	14.50	
11	M ₄	57.97	
12	MS ₄	58.98	MK ₄ , MT ₄

* Approx. speed of constituent in degrees per solar hour

Twelve constituents (M₂, S₂, N₂, μ_2 , L₂, K₁, O₁, J₁, Q₁, M₁, M₄, MS₄) satisfying the Rayleigh criterion $\Delta f > 1/T$ where f is the frequency difference between the constituents and T is the record length, were used in the least squares determination of amplitudes and phases (See Table II). The amplitudes and phases of S₂, N₂, μ_2 , L₂, K₁ and Q₁, were corrected for the effect of (K₂, T₂), ϑ_2 , 2N₂, λ_2 , P₁ and ρ_1 respectively on the basis of equilibrium theory using trigonometric relations similar to those derived in Dronkers (1964). The tidal constants for the latter constituents were then inferred, again, on the basis of equilibrium theory.

RESULTS AND DISCUSSION

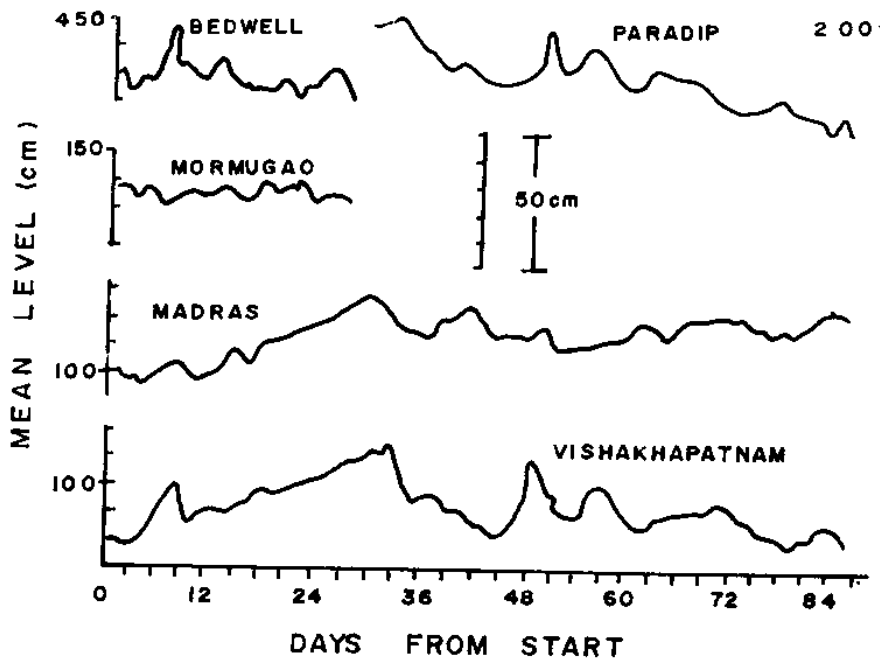


Fig. 1. Variation of mean level at different stations obtained using the low-pass filter F49.

To obtain an idea of the low-frequency variability, the records were subjected to a low-pass filter. Fig. 1 shows the mean level at Bedwell, Mormugao, Madras, Visakhapatnam and Paradip on application of the 49 point "tide-killer" filter F 49 of Groves, the filter being an improved version of the D series of filters constructed by Groves (1955). It may be seen that at Mormugao, the variations of the mean level are within the range of 10 cm and consist solely of small amplitude oscillations of periodicities 3-6 days which are of non-tidal origin. Besides the small amplitude oscillations, the low frequency variation at Madras, Vishakhapatnam, Tuticorin and Paradip consists of occurrences of storm surges exceeding 20 cm superimposed on linear trends. The linear trends are consistent with the annual cycle of mean sea level observed along the east coast of India by Shetye and Almeida, (1985). Before performing harmonic analysis the tidal records were checked for sinusoidal nature by viewing different sections of each record on a graphics terminal. It was found that the records at Bedwell, Mormugao and Paradip were sinusoidal. The records at Madras and Visakhapatnam were asymmetric during incidence of storm surges, and at Tuticorin the sinusoidal nature broke down completely during neap tides. We may, therefore, look upon the tidal record at Mormugao as representing noise-free conditions, and the tidal records at Madras, Visakhapatnam, Tuticorin and Paradip as contaminated by occurrences of storm surges exceeding 20 cm. We shall compare the tidal constants derived from harmonic analysis of these 29 day records with those published by the International Hydrographic Bureau (IHB), Monaco on the basis of more than 5 years records. The tidal record at Bedwell is used only for validation of our LSM.

Table III – Results of harmonic analysis for Bedwell Harbour, B.C. Lat. 48°45'N,
Long. 123°14'W, Record length 29 days, central day 26 April 1905

Sr. No.	Consti- tuent	H (ft)		g (deg)	
		LSM*	SGM [#]	LSM*	SGM [#]
1	M ₂	1.89	1.86	135	135
2	S ₂	0.46	0.45	151	153
3	N ₂	0.42	0.40	109	106
4	K ₂ \$	0.13	0.12	151	153
5	K ₁	2.55	2.56	159	158
6	O ₁	1.44	1.33	145	145
7	P ₁ \$	0.84	0.85	159	158
8	M ₄	0.05	0.06	007	008
9	MS ₄	0.03	0.03	028	035

* Values we obtained by using the least squares method, # Values derived in Suthons (1959) using the Admiralty semi-graphic method, \$ Constants for these constituents are inferred.

Table IV – Results of harmonic analysis for Mormugao, Lat. 15°25'N, Long. 73°48'E
Record Length 29 days, central day 28 February 1979

Sr. No.	Consti- tuent	H(cm)		κ (deg)		g(deg)
		LSM*	IHB [#]	LSM*	IHB [#]	LSM*
1	M ₂	56.6	55.2	298	300	310
2	S ₂	20.4	19.5	331	332	348
3	N ₂	12.9	13.2	280	282	288
4	μ ₂	1.4	1.7	251	250	257
5	L ₂	1.6	1.5	334	320	349
6	K ₁	35.4	31.2	042	046	051
7	O ₁	14.2	15.9	048	049	050
8	J ₁	4.0	2.0	051	055	063
9	Q ₁	3.3	3.3	051	052	050
10	M ₁	1.4	0.9	080	074	086
11	M ₄	1.3	1.4	004	006	028
12	MS ₄	0.5	0.7	058	048	087
13	K ₂ \$	5.5	5.7	331	323	348
14	T ₂ \$	1.2	2.4	331	316	348
15	ϑ ₂ \$	2.4	3.0	280	280	288
16	2N ₂ \$	1.5	1.9	251	240	257
17	λ ₂ \$	0.4	0.4	334	213	349
18	P ₁ \$	11.7	9.1	042	044	050
19	p ₁ \$	0.7	-	051	-	050

* Values we obtained by using the least squares method, # Values reported by the International Hydrographic Bureau, Monaco on basis of 5 years data from 1884-1889, \$ Constants for these constituents are inferred in LSM.

Table III gives the results of harmonic analysis for the record at Bedwell. The data for Bedwell was taken from Suthons (1959), who presents the Admiralty semi-graphic method (SGM) of harmonic tidal analysis. We see that in general the amplitudes and phases obtained by our LSM are in agreement with those obtained by Suthons (1959) using SGM within ± 0.03 ft (± 1 cm) and $\pm 3^\circ$ respectively. Substantial differences of 1.1 ft and 7° exist in the amplitude of O_1 and phase of MS_4 respectively. The difference in phase of MS_4 is most likely due to the smoothing of the data in the SGM. The difference in amplitude of O_1 may be due to an unnoticed error creeping in the involved computations in the SGM using different stencils. Comparisons for μ_2 , L_2 , Q_1 , etc. are not made as Suthons (1959) leaves computations for these constituents incomplete due to some limitations. We may conclude from the above discussion that the results of harmonic analysis by the LSM are consistent with those obtained by the SGM. The hindcasting skill of the least squares determination of amplitudes and phases is 0.9851, indicating that 98.51% of the variance of the data is explained by the 12 constituents included in the least squares determination.

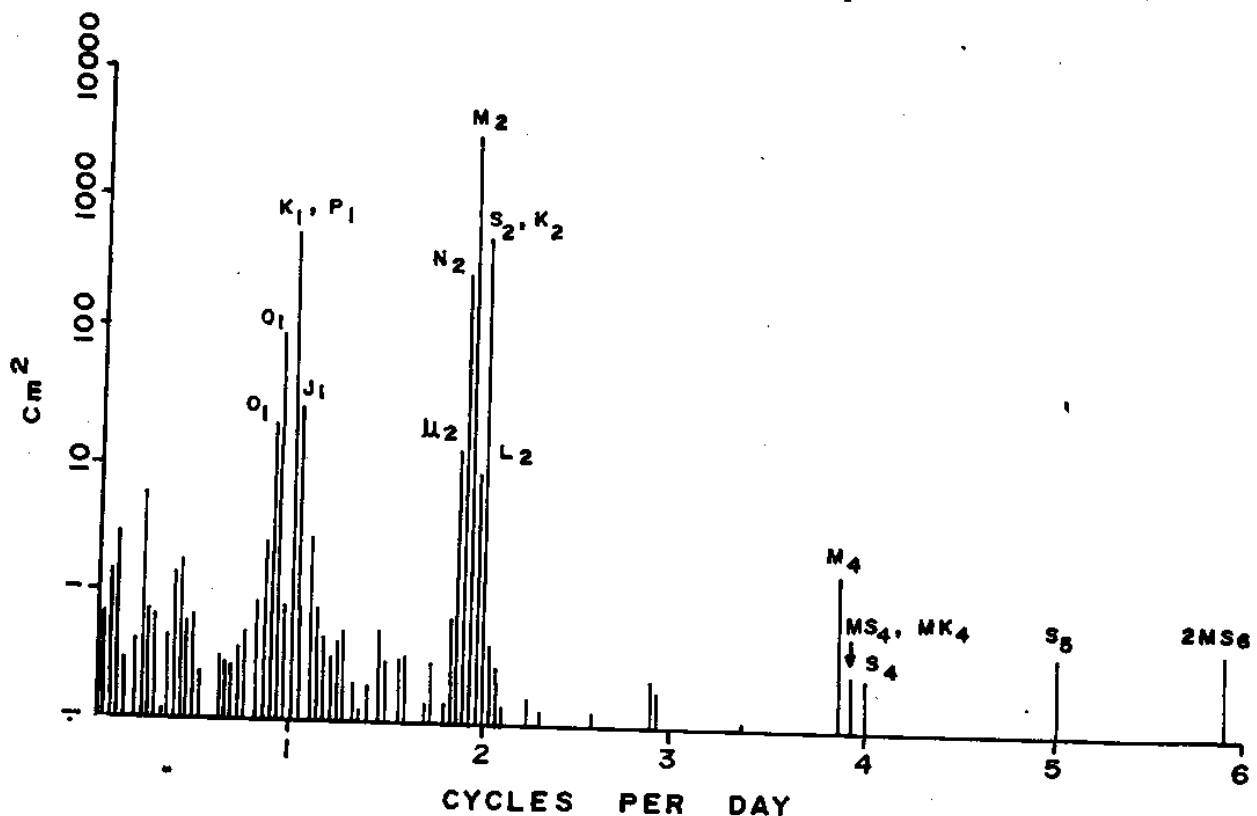


Fig.2. Periodogram analysis of Mormugao observations, 14.2.1979 to 15.3.1979

Table IV presents the results of harmonic analysis for the record at Mormugao. We see that, in general, the amplitudes and phases obtained by our LSM are in agreement with those reported by IHB on the basis of 5 years data from 1884-1889, within ± 1 cm and $\pm 3^\circ$ respectively, which as we have first seen, is the range of agreement between the LSM and SGM of harmonic analysis of short period observa-

tions. Substantial differences of 14° and 6° exist in the phases of L_2 and M_1 respectively. These may be due to the differences in u (See Eq. 1 in the Appendix) arising from formulae given in Doodson and Warburg (1941) that we have used and tabulated values of $(E + u)$ in Schureman (1958) for these constituents. Just as in case of Bedwell, substantial phase difference exists in case of MS_4 . The hindcasting skill for Mormugao is 0.9922, explaining thereby the remarkable consistency between the results of the LSM and values reported by IHB. Fig. 2 shows a periodogram of the tidal data for Mormugao. For the periodogram we used 30 days of hourly observations, so that spectral lines are $1/720$ cycles per hour ($= 1/30$ cycles per day) apart. The ordinate shows the square of the amplitude. Ordinate values less than 0.1 cm^2 are not shown. We see clearly that the constituents (S_2, K_2) are represented by the same line, similar being the case for the constituents (K_1, P_1) and also (N_2, ϑ_2). This is a manifestation of the Raleigh criterion according to which only constituents separated by frequencies larger than $1/T$ can be resolved.

Table V – Results of harmonic analysis by LSM for Madras, Lat $13^\circ 06' \text{ N}$,
Long $80^\circ 18' \text{ E}$ for 3 records
(1) M-1, 29 days, central day 8.10.1968, (2) M-2, 29 days, central day
6.11.1968 and (3) M-3, 29 days, central day 5.12.1968

Sr. No.	Constituent	H(cm)				κ (deg)			
		M-1	M-2	M-3	IHB*	M-1	M-2	M-3	IHB*
1.	M_2	34.2	33.1	32.9	33.2	241	238	236	240
2.	S_2	13.1	13.5	12.4	13.9	270	267	270	270
3.	N_2	7.5	7.4	7.6	7.4	234	231	228	234
4.	μ_2	1.6	1.4	1.7	1.0	219	219	204	184
5.	L_2	1.0	1.2	1.8	1.3	306	283	247	263
6.	K_1	8.6	8.6	9.1	9.1	331	334	331	337
7.	O_1	2.6	2.2	2.6	2.9	320	328	323	325
8.	J_1	0.4	0.9	1.0	0.6	305	268	285	316
9.	Q_1	0.1	0.4	0.4	0.2	349	118	019	107
10.	M_1	0.7	1.4	0.2	0.3	021	024	347	306
11.	M_4	0.2	0.3	0.2	0.2	003	016	046	188
12.	MS_4	0.3	0.2	0.2	0.2	226	160	332	236
13.	$K_2 \#$	3.6	3.7	3.4	3.6	270	267	270	268
14.	$T_2 \#$	0.8	0.8	0.7	1.0	270	267	270	239
15.	$\vartheta_2 \#$	1.4	1.4	1.4	1.7	234	231	228	237
16.	$2N_2 \#$	1.7	1.5	1.8	1.3	219	219	204	218
17.	$z \#$	0.3	0.3	0.5	0.9	306	283	247	295
18.	$P_1 \#$	2.8	2.8	3.0	2.9	331	334	331	339
19.	$p_1 \#$	0.0	0.1	0.1	-	349	118	019	-

* Values reported by the International Hydrographic Bureau, Monaco

Constants for these constituents are inferred in LSM.

Table VI – Results of harmonic analysis by LSM for Vishakapatnam, Lat 17° 41'N, Long 83° 17'E for 3 records
 (1) V-1, 29 days, central day 8.10.1968, (2) V-2, 29 days, central day 6.11.1968 and (3) V-3, 29 days, central day 5.12.1968

Sr. No.	Constituent	H(cm)				K(deg)			
		V-1	V-2	V-3	IHB*	V-1	V-2	V-3	IHB*
1.	M ₂	48.0	47.1	45.5	44.8	244	247	256	254
2.	S ₂	21.4	20.2	21.9	19.8	275	273	281	286
3.	N ₂	10.4	10.4	8.3	9.4	241	238	237	248
4.	2	2.1	2.1	1.8	0.9	219	222	182	260
5.	L ₂	0.4	2.3	2.0	1.7	285	286	336	259
6.	K ₁	10.4	11.0	10.8	10.9	335	336	338	342
7.	O ₁	3.6	3.5	2.6	4.2	327	332	324	332
8.	J ₁	0.3	1.1	1.1	0.8	321	299	326	345
9.	Q ₁	0.3	0.7	0.4	0.4	074	097	140	331
10.	M ₁	1.3	1.0	0.6	0.4	016	044	073	303
11.	M ₄	0.3	0.4	3.3	0.4	139	099	095	320
12.	MS ₄	0.3	0.4	2.4	0.3	170	173	104	354
13.	K ₂ #	5.8	5.5	5.9	5.9	275	273	281	278
14.	T ₂ #	1.3	1.2	1.3	1.4	275	273	281	269
15.	2 #	1.9	1.9	1.5	2.6	241	238	237	213
16.	2N ₂ #	2.3	2.2	1.9	1.6	219	222	182	233
17.	2 #	0.1	0.6	0.5	0.7	285	286	336	261
18.	P ₁ #	3.5	3.6	3.6	3.1	335	336	338	341
19.	2 #	0.1	0.1	0.1	-	074	097	140	-

* Values reported by the International Hydrographic Bureau, Monaco on the basis of 6 years data from 1879-1885

Constants for these constituents are inferred in LSM

Tables V and VI present the results of harmonic analysis for the records at Madras and Visakhapatnam respectively. We see that the amplitudes and phases of the principal constituents M₂, S₂, N₂, K₁ and O₁ are in agreement with values reported by IHB within ± 2 cm and $\pm 15^\circ$ respectively. For the other constituents used in the least squares determination, the tidal constants, specially the phases are not at all reliable.

Thus for records contaminated by occurrences of storm surges exceeding 20 cm, there is a deterioration in the value of the tidal constants for the principal constituents, and the tidal constants, specially the phases, for other constituents are not reliable. This is due to the nonlinear effects of storm surges.

To establish this contention we repeated the harmonic analysis for a 60 days record for Paradip after subjecting to a band-pass filter with unit response in the period band 30-5 hrs, and zero response elsewhere to linearly remove the non-tidal

frequencies. This was achieved by first fourier transforming the record, then multiplying the fourier transforms by 1 or zero depending on whether the fourier transforms correspond to the band 30-5 hrs or not and finally computing the inverse fourier transforms. We found that the changes in constants were within ± 0.05 cm and $\pm 0.5^\circ$ in amplitude and phase respectively, except for Q_1 which inexplicably changed in phase by 39° , as a consequence of pre-filtering the record which linearly removes the low and high frequency noise.

It is seen that occurrence of storm surges exceeding 20 cm affect the accuracy of tidal constants and that the process of prefiltering the data does not cause any appreciable change in the tidal constants.

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REFERENCES

- Cheng, R.T. and J.W. Gartner, 1985. Harmonic analysis of tides and tidal currents in South San Francisco Bay, California. *Estuarine, Coastal and Shelf Science*, 21: 57-74.
- Doodson, A.T., 1927. The analysis of tidal observations. *Philosophical Transaction of the Royal Society, London* A227: 223-279.
- Doodson, A.T. and Warburg, 1941. *Admiralty manual of tides*. HMSO, London, 270 pp.
- Dronkers, J.J., 1964. *Tidal computations in rivers and coastal waters*. North Holland Pub. Co., Amsterdam, 518 pp.
- Evans, J.J. and D.T. Pugh, 1982. Analysing clipped sea-level records for harmonic tidal constituents. *Int. Hydrogr. Rev.*, 59 : 115-122.
- Groves, G.W., 1955. Numerical filters for discrimination against tidal periodicities. *Transaction of American Geophysical Union*, 36: 1073-1084.
- Munk, W.H. and D.E. Cartwright, 1966. Tidal spectroscopy and predication. *Philosophical Transaction of the Royal Society, London* A259: 533-581.
- Poole, S.L., A.B. Rabinovich, L.Q. Spielvogel and R.R. Harvey, 1980. Study of ocean tides in the region of the Kuril-Kamchatka and Japan trenches. *Oceanology*, 20: 655-659.
- Schureman, P., 1958. *Manual of harmonic analysis and prediction of tides*. U.S. Dept. of Commerce, Special Publication No. 98.

- Shetye, S.R. and A.M. Almeida, 1985. An examination of the factors that influence the monthly mean sea level along the coast of India. *IOC/UNESCO Workshop on Regional Cooperation in Marine Science in the Central Indian Ocean and Adjacent seas and Gulf, Colombo, 8-13 July 1985, Workshop Rept. No. 37 - Supplement.*
- Suthons, C.T., 1959. The the admiralty semi-graphic method of harmonic tidal analysis. *Admiralty Tidal Handbook No. 1, N.P. 122(1)*, Hydrographic Department, Admiralty.
- Zetler, B.D., M.D. Schuldt, R.W. Whipple and S.D. Hicks, 1965. Harmonic analysis of tides from data randomly spaced in time. *Journal of Geophysical Research*, **70**: 2805-2811.

APPENDIX

Least Squares Method

The height of the tide at any time may be predicted harmonically from known amplitudes and phases of individual constituents by the following formula (see Dronkers, 1964) :

$$h(t) = H_0 + \sum_{n=1}^{NC} f_n H_n \cos[W_n t + (E_n + u_n) - g_n] \quad (1)$$

in which

h = height of the tide at any time t

t = time reckoned in hours from some initial epoch such as beginning of year of prediction

H_0 = mean height of water level above datum used for predictions

H_n = mean amplitude of any constituent A

f_n = nodal correction for reducing mean amplitude H_n to year of prediction

W_n = speed of constituent in degrees per solar hour

$(E_n + u_n)$ = value of equilibrium argument of constituent A at Greenwich at the origin of time, i.e. where $t = 0$

g_n = Greenwich phase of constituent A

NC = number of constituents required for adequate description of the tide

Constituent speed W_n distinguishes the different constituents. The harmonic constants H_0 , H_n and g_n are generally available from publications of the International Hydrographic Bureau, Monaco. E_n , u_n and f_n depend solely on the origin of time and can be easily determined (See Doodson and Warburg, 1941). Thus the only variable on the right hand side of Eq (1) is t , varying which we can determine the tidal curve. Thus the forward problem : given the harmonic constants H_n and g_n , to determine the tide — is easily solved.

We shall now discuss the inverse problem : given NH hourly observations to determine the tidal constants H_n and g_n . As already mentioned 3 methods are available for the determination of harmonic constants. The classical approaches of Doodson (1927) uses a system multipliers for determining the constants, and was appropriate in the early days when computations had necessarily to be made by hand. With the advent of computers, and their ability for inverting large matrices, the LSM has become the standard method for determining tidal constants, though the response method of Munk and Cartwright (1966) is increasingly being used. We shall now briefly review the LSM.

By means of the addition formula for the cosine function, Eq (1) can be written as

$$h(t) = H_0 + \sum_{n=1}^{NC} A_n \cos W_{nt} + \sum_{n=1}^{NC} B_n \sin W_{nt} \quad (2)$$

where, $A_n = f_n H_n \cos (E_n + u_n - g_n)$ and $B_n = -f_n H_n \sin (E_n + u_n - g_n)$ then if A_n and B_n are known, the tidal constants are given by

$$H_n = \sqrt{A_n^2 + B_n^2} / f_n \quad (3)$$

$$g_n = E_n + u_n - \arctan (-B_n / A_n) \quad (4)$$

If we set $t = 0$ to coincide with some time origin, say, 0000 hrs on the central day at which hourly observations are available, then Eq (2) leads to NH linear equations in which the 2NC unknown are A_n and B_n . The number of hourly observations NH and the number of constituents NC are suitably chosen such that NH is much larger than 2NC, i.e. the ratio 2NC/NH is small, so that the system of equations generated by Eq (2) is grossly overdetermined.

Let h_1, h_2, \dots, h_{NH} and $\hat{h}_1, \hat{h}_2, \dots, \hat{h}_{NH}$ respectively represent the original series of tidal observations and the series predicted by Eq (2). Values of (A_n, B_n) for $n = 1, 2, \dots, NC$ are then determined by solving the normal equations obtained by minimizing the value of

$$\sum_{i=1}^{NH} (h_i - \hat{h}_i)^2.$$

Once A_n and B_n are determined, the tidal constants H_n and g_n are obtained from Eqs (3) and (4). The hindcasting skill (HCS) of (A_n, B_n) is defined as follows :

$$HCS = \frac{\sum_{i=1}^{NH} (h_i - H_0)^2}{\sum_{i=1}^{NH} (\hat{h}_i - H_0)^2} \quad (5)$$

where H_0 is the mean height. HCS which takes values between 0 and 1, determines the reliability of the least squares solution (A_n, B_n) . If HCS is very close to 1, then the values of A_n, B_n are very reliable, provided that the ratio 2NC/NH is small. On the contrary if HCS is close to zero, then A_n, B_n and consequently H_n, g_n are not at all reliable.

If the ratio 2NC/NH is not small, then a phenomenon known as artificial predictability may take place. Essentially it means that even if the (A_n, B_n) have a HCS close to 1, so that Eq (2) correctly predicts the tide for a particular sample comprising of the NH hourly observations used in the LSM, for any other sample, comprising say, the next or earlier NH hourly observations, Eq (2) will fail in correctly determining the tide using above values of (A_n, B_n) as gauged by the HCS obtained using Eq (5) for this sample, if the ratio 2NC/NH is large. In the field of tidal

prediction, the main concern is in predicting future values of tide. Therefore ensuring that the ratio $2NC/NH$ is sufficiently small is of paramount importance.

In literature, often, instead of the Greenwich phase g_n , the local phase K_n is given. The phases g_n and K_n are related as follows :

$$g_n = K_n + pL - W_n S / 15$$

where L is the longitude of the place; S is the time meridian of the place, both reckoned as degrees west of Greenwich; $p=1, 2$ and 4 respectively for diurnal, semi-diurnal and ter-diurnal constituents; and W_n is the speed of the constituent in degrees per solar hour.-

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Key-words: Harmonic analysis, tides, least squares

INTRODUCTION

The use of the least squares method (LSM) of harmonic analysis of tides/tidal streams is becoming very popular. Poole, Rabinovich, Spielvogel and Harvey (1980) used LSM to study deep sea tides and tidal streams during the Second Soviet-American Tsunami Expedition (August-October, 1978) and compared the results with those obtained by using the response method of Munk and Cartwright (1966). Cheng and Gartner (1985) used LSM for harmonic analysis of tides and tidal currents in South San Francisco Bay, California. LSM started out as an alternative to the classical method of Doodson (1927). In certain circumstances LSM appears to be the only method applicable. For example Evans and Pugh (1982) used LSM for analysing clipped sea-level records, which occur at sites where the gauge dries at low levels, and Zetler, Schuldt, Whipple and Hicks (1965) used LSM on data randomly spaced in time to conform to daylight hours with applications to remotely sensed data (via satellites) in mind.

MATERIAL AND METHODS

Using the LSM described in the Appendix, harmonic analysis was performed on 13 records of 29 days of hourly observations, viz. 1 record each at Mormugao and Bedwell

(in Canada); 3 consecutive records at Madras, Visakhapatnam and Tuticorin; and 2 consecutive records at Paradip (see Table I).

Table I – Tidal records (29 days long) analysed by LSM

Rec. No.	Station	Central day
1.	Bedwell (Canada)	26.4.1905
2.	Mormugao	28.2.1979
3.	Madras	8.10.1968
4.	- do -	6.11.1968
5.	- do -	5.12.1968
6.	Vishakapatnam	8.10.1968
7.	- do -	6.11.1968
8.	- do -	5.12.1968
9.	Tuticorin	8.10.1968
10.	- do -	6.11.1968
11.	- do -	5.12.1968
12.	Paradip	6.11.1968
13.	- do -	5.12.1968

Table II – Constituents used in Least Squares determination and satellite constituents for which allowance was made in correcting phases and amplitudes

S.No.	Constituent	Speed*	Satellites
1	M ₂	28.98	
2	S ₂	30.00	K ₂ , T ₂
3	N ₂	28.44	ϑ_2
4	μ_2	27.97	2N ₂
5	L ₂	29.53	λ_2
6	K ₁	15.04	P ₁
7	O ₁	13.94	
8	J ₁	15.59	
9	Q ₁	13.40	ρ_1
10	M ₁	14.50	
11	M ₄	57.97	
12	MS ₄	58.98	MK ₄ , MT ₄

* Approx. speed of constituent in degrees per solar hour

Twelve constituents (M₂, S₂, N₂, μ_2 , L₂, K₁, O₁, J₁, Q₁, M₁, M₄, MS₄) satisfying the Rayleigh criterion $\Delta f > 1/T$ where f is the frequency difference between the constituents and T is the record length, were used in the least squares determination of amplitudes and phases (See Table II). The amplitudes and phases of S₂, N₂, μ_2 , L₂, K₁ and Q₁, were corrected for the effect of (K₂, T₂), ϑ_2 , 2N₂, λ_2 , P₁ and ρ_1 respectively on the basis of equilibrium theory using trigonometric relations similar to those derived in Dronkers (1964). The tidal constants for the latter constituents were then inferred, again, on the basis of equilibrium theory.

RESULTS AND DISCUSSION

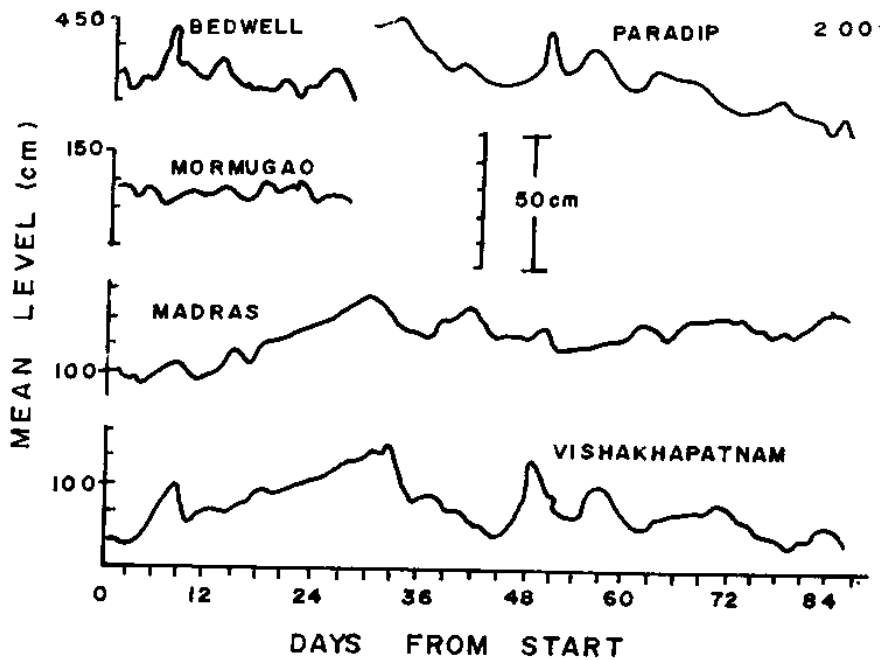


Fig. 1. Variation of mean level at different stations obtained using the low-pass filter F49.

To obtain an idea of the low-frequency variability, the records were subjected to a low-pass filter. Fig. 1 shows the mean level at Bedwell, Mormugao, Madras, Visakhapatnam and Paradip on application of the 49 point "tide-killer" filter F 49 of Groves, the filter being an improved version of the D series of filters constructed by Groves (1955). It may be seen that at Mormugao, the variations of the mean level are within the range of 10 cm and consist solely of small amplitude oscillations of periodicities 3-6 days which are of non-tidal origin. Besides the small amplitude oscillations, the low frequency variation at Madras, Vishakhapatnam, Tuticorin and Paradip consists of occurrences of storm surges exceeding 20 cm superimposed on linear trends. The linear trends are consistent with the annual cycle of mean sea level observed along the east coast of India by Shetye and Almeida, (1985). Before performing harmonic analysis the tidal records were checked for sinusoidal nature by viewing different sections of each record on a graphics terminal. It was found that the records at Bedwell, Mormugao and Paradip were sinusoidal. The records at Madras and Visakhapatnam were asymmetric during incidence of storm surges, and at Tuticorin the sinusoidal nature broke down completely during neap tides. We may, therefore, look upon the tidal record at Mormugao as representing noise-free conditions, and the tidal records at Madras, Visakhapatnam, Tuticorin and Paradip as contaminated by occurrences of storm surges exceeding 20 cm. We shall compare the tidal constants derived from harmonic analysis of these 29 day records with those published by the International Hydrographic Bureau (IHB), Monaco on the basis of more than 5 years records. The tidal record at Bedwell is used only for validation of our LSM.

Table III – Results of harmonic analysis for Bedwell Harbour, B.C. Lat. 48°45'N,
Long. 123°14'W, Record length 29 days, central day 26 April 1905

Sr. No.	Consti- tuent	H (ft)		g (deg)	
		LSM*	SGM [#]	LSM*	SGM [#]
1	M ₂	1.89	1.86	135	135
2	S ₂	0.46	0.45	151	153
3	N ₂	0.42	0.40	109	106
4	K ₂ \$	0.13	0.12	151	153
5	K ₁	2.55	2.56	159	158
6	O ₁	1.44	1.33	145	145
7	P ₁ \$	0.84	0.85	159	158
8	M ₄	0.05	0.06	007	008
9	MS ₄	0.03	0.03	028	035

* Values we obtained by using the least squares method, # Values derived in Suthons (1959) using the Admiralty semi-graphic method, \$ Constants for these constituents are inferred.

Table IV – Results of harmonic analysis for Mormugao, Lat. 15°25'N, Long. 73°48'E
Record Length 29 days, central day 28 February 1979

Sr. No.	Consti- tuent	H(cm)		κ (deg)		g(deg)
		LSM*	IHB [#]	LSM*	IHB [#]	LSM*
1	M ₂	56.6	55.2	298	300	310
2	S ₂	20.4	19.5	331	332	348
3	N ₂	12.9	13.2	280	282	288
4	μ ₂	1.4	1.7	251	250	257
5	L ₂	1.6	1.5	334	320	349
6	K ₁	35.4	31.2	042	046	051
7	O ₁	14.2	15.9	048	049	050
8	J ₁	4.0	2.0	051	055	063
9	Q ₁	3.3	3.3	051	052	050
10	M ₁	1.4	0.9	080	074	086
11	M ₄	1.3	1.4	004	006	028
12	MS ₄	0.5	0.7	058	048	087
13	K ₂ \$	5.5	5.7	331	323	348
14	T ₂ \$	1.2	2.4	331	316	348
15	ϑ ₂ \$	2.4	3.0	280	280	288
16	2N ₂ \$	1.5	1.9	251	240	257
17	λ ₂ \$	0.4	0.4	334	213	349
18	P ₁ \$	11.7	9.1	042	044	050
19	p ₁ \$	0.7	-	051	-	050

* Values we obtained by using the least squares method, # Values reported by the International Hydrographic Bureau, Monaco on basis of 5 years data from 1884-1889, \$ Constants for these constituents are inferred in LSM.

Table III gives the results of harmonic analysis for the record at Bedwell. The data for Bedwell was taken from Suthons (1959), who presents the Admiralty semi-graphic method (SGM) of harmonic tidal analysis. We see that in general the amplitudes and phases obtained by our LSM are in agreement with those obtained by Suthons (1959) using SGM within ± 0.03 ft (± 1 cm) and $\pm 3^\circ$ respectively. Substantial differences of 1.1 ft and 7° exist in the amplitude of O_1 and phase of MS_4 respectively. The difference in phase of MS_4 is most likely due to the smoothing of the data in the SGM. The difference in amplitude of O_1 may be due to an unnoticed error creeping in the involved computations in the SGM using different stencils. Comparisons for μ_2 , L_2 , Q_1 , etc. are not made as Suthons (1959) leaves computations for these constituents incomplete due to some limitations. We may conclude from the above discussion that the results of harmonic analysis by the LSM are consistent with those obtained by the SGM. The hindcasting skill of the least squares determination of amplitudes and phases is 0.9851, indicating that 98.51% of the variance of the data is explained by the 12 constituents included in the least squares determination.

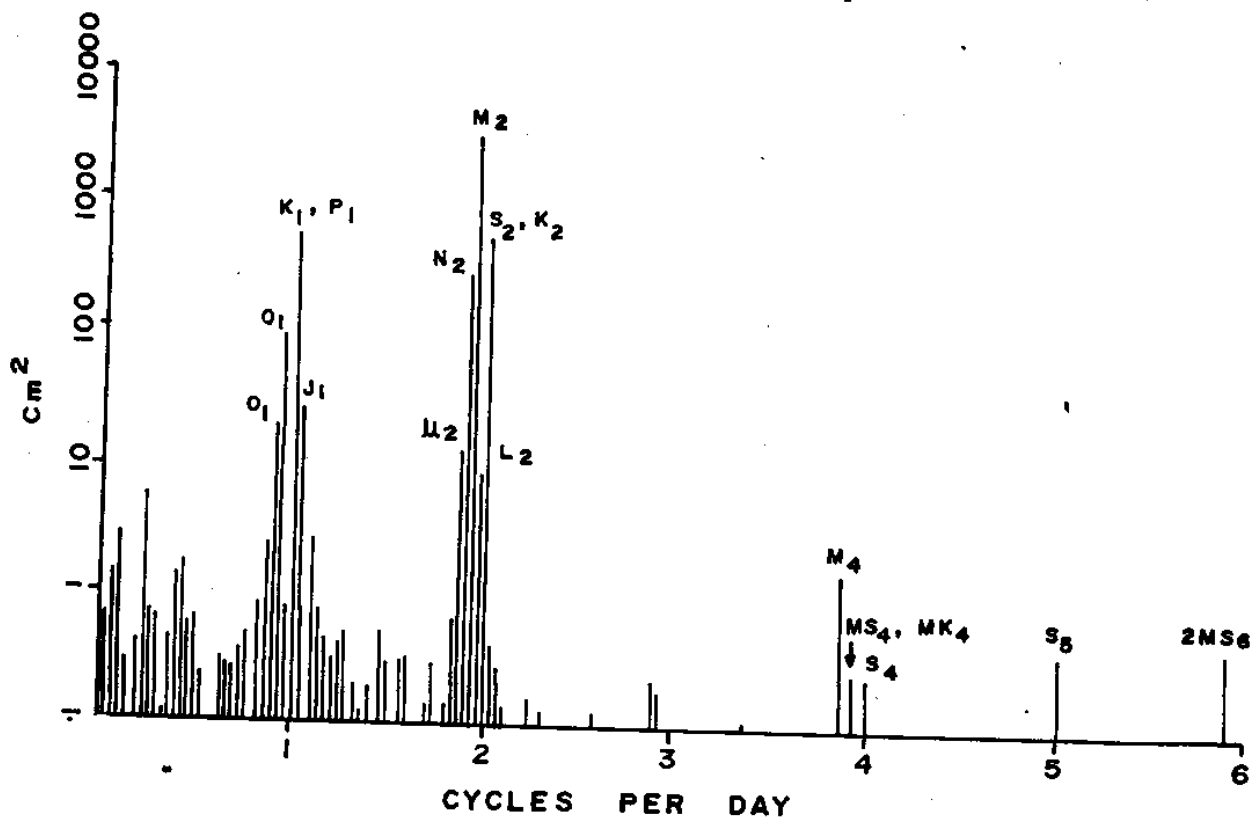


Fig.2. Periodogram analysis of Mormugao observations, 14.2.1979 to 15.3.1979

Table IV presents the results of harmonic analysis for the record at Mormugao. We see that, in general, the amplitudes and phases obtained by our LSM are in agreement with those reported by IHB on the basis of 5 years data from 1884-1889, within ± 1 cm and $\pm 3^\circ$ respectively, which as we have first seen, is the range of agreement between the LSM and SGM of harmonic analysis of short period observa-

tions. Substantial differences of 14° and 6° exist in the phases of L_2 and M_1 respectively. These may be due to the differences in u (See Eq. 1 in the Appendix) arising from formulae given in Doodson and Warburg (1941) that we have used and tabulated values of $(E + u)$ in Schureman (1958) for these constituents. Just as in case of Bedwell, substantial phase difference exists in case of MS_4 . The hindcasting skill for Mormugao is 0.9922, explaining thereby the remarkable consistency between the results of the LSM and values reported by IHB. Fig. 2 shows a periodogram of the tidal data for Mormugao. For the periodogram we used 30 days of hourly observations, so that spectral lines are $1/720$ cycles per hour ($= 1/30$ cycles per day) apart. The ordinate shows the square of the amplitude. Ordinate values less than 0.1 cm^2 are not shown. We see clearly that the constituents (S_2, K_2) are represented by the same line, similar being the case for the constituents (K_1, P_1) and also (N_2, ϑ_2). This is a manifestation of the Raleigh criterion according to which only constituents separated by frequencies larger than $1/T$ can be resolved.

Table V – Results of harmonic analysis by LSM for Madras, Lat $13^\circ 06' \text{ N}$, Long $80^\circ 18' \text{ E}$ for 3 records
(1) M-1, 29 days, central day 8.10.1968, (2) M-2, 29 days, central day 6.11.1968 and (3) M-3, 29 days, central day 5.12.1968

Sr. No.	Constituent	H(cm)				κ (deg)			
		M-1	M-2	M-3	IHB*	M-1	M-2	M-3	IHB*
1.	M_2	34.2	33.1	32.9	33.2	241	238	236	240
2.	S_2	13.1	13.5	12.4	13.9	270	267	270	270
3.	N_2	7.5	7.4	7.6	7.4	234	231	228	234
4.	μ_2	1.6	1.4	1.7	1.0	219	219	204	184
5.	L_2	1.0	1.2	1.8	1.3	306	283	247	263
6.	K_1	8.6	8.6	9.1	9.1	331	334	331	337
7.	O_1	2.6	2.2	2.6	2.9	320	328	323	325
8.	J_1	0.4	0.9	1.0	0.6	305	268	285	316
9.	Q_1	0.1	0.4	0.4	0.2	349	118	019	107
10.	M_1	0.7	1.4	0.2	0.3	021	024	347	306
11.	M_4	0.2	0.3	0.2	0.2	003	016	046	188
12.	MS_4	0.3	0.2	0.2	0.2	226	160	332	236
13.	$K_2 \#$	3.6	3.7	3.4	3.6	270	267	270	268
14.	$T_2 \#$	0.8	0.8	0.7	1.0	270	267	270	239
15.	$\vartheta_2 \#$	1.4	1.4	1.4	1.7	234	231	228	237
16.	$2N_2 \#$	1.7	1.5	1.8	1.3	219	219	204	218
17.	$\lambda_2 \#$	0.3	0.3	0.5	0.9	306	283	247	295
18.	$P_1 \#$	2.8	2.8	3.0	2.9	331	334	331	339
19.	$p_1 \#$	0.0	0.1	0.1	-	349	118	019	-

* Values reported by the International Hydrographic Bureau, Monaco

Constants for these constituents are inferred in LSM.

Table VI – Results of harmonic analysis by LSM for Vishakapatnam, Lat 17° 41'N, Long 83° 17'E for 3 records
 (1) V-1, 29 days, central day 8.10.1968, (2) V-2, 29 days, central day 6.11.1968 and (3) V-3, 29 days, central day 5.12.1968

Sr. No.	Constituent	H(cm)				K(deg)			
		V-1	V-2	V-3	IHB*	V-1	V-2	V-3	IHB*
1.	M ₂	48.0	47.1	45.5	44.8	244	247	256	254
2.	S ₂	21.4	20.2	21.9	19.8	275	273	281	286
3.	N ₂	10.4	10.4	8.3	9.4	241	238	237	248
4.	μ ₂	2.1	2.1	1.8	0.9	219	222	182	260
5.	L ₂	0.4	2.3	2.0	1.7	285	286	336	259
6.	K ₁	10.4	11.0	10.8	10.9	335	336	338	342
7.	O ₁	3.6	3.5	2.6	4.2	327	332	324	332
8.	J ₁	0.3	1.1	1.1	0.8	321	299	326	345
9.	Q ₁	0.3	0.7	0.4	0.4	074	097	140	331
10.	M ₁	1.3	1.0	0.6	0.4	016	044	073	303
11.	M ₄	0.3	0.4	3.3	0.4	139	099	095	320
12.	MS ₄	0.3	0.4	2.4	0.3	170	173	104	354
13.	K ₂ #	5.8	5.5	5.9	5.9	275	273	281	278
14.	T ₂ #	1.3	1.2	1.3	1.4	275	273	281	269
15.	V ₂ #	1.9	1.9	1.5	2.6	241	238	237	213
16.	2N ₂ #	2.3	2.2	1.9	1.6	219	222	182	233
17.	λ ₂ #	0.1	0.6	0.5	0.7	285	286	336	261
18.	P ₁ #	3.5	3.6	3.6	3.1	335	336	338	341
19.	ρ ₂ #	0.1	0.1	0.1	-	074	097	140	-

* Values reported by the International Hydrographic Bureau, Monaco on the basis of 6 years data from 1879-1885

Constants for these constituents are inferred in LSM

Tables V and VI present the results of harmonic analysis for the records at Madras and Visakhapatnam respectively. We see that the amplitudes and phases of the principal constituents M₂, S₂, N₂, K₁ and O₁ are in agreement with values reported by IHB within ± 2 cm and $\pm 15^\circ$ respectively. For the other constituents used in the least squares determination, the tidal constants, specially the phases are not at all reliable.

Thus for records contaminated by occurrences of storm surges exceeding 20 cm, there is a deterioration in the value of the tidal constants for the principal constituents, and the tidal constants, specially the phases, for other constituents are not reliable. This is due to the nonlinear effects of storm surges.

To establish this contention we repeated the harmonic analysis for a 60 days record for Paradip after subjecting to a band-pass filter with unit response in the period band 30-5 hrs, and zero response elsewhere to linearly remove the non-tidal

frequencies. This was achieved by first fourier transforming the record, then multiplying the fourier transforms by 1 or zero depending on whether the fourier transforms correspond to the band 30-5 hrs or not and finally computing the inverse fourier transforms. We found that the changes in constants were within ± 0.05 cm and $\pm 0.5^\circ$ in amplitude and phase respectively, except for Q_1 which inexplicably changed in phase by 39° , as a consequence of pre-filtering the record which linearly removes the low and high frequency noise.

It is seen that occurrence of storm surges exceeding 20 cm affect the accuracy of tidal constants and that the process of prefiltering the data does not cause any appreciable change in the tidal constants.

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REFERENCES

- Cheng, R.T. and J.W. Gartner, 1985. Harmonic analysis of tides and tidal currents in South San Francisco Bay, California. *Estuarine, Coastal and Shelf Science*, 21: 57-74.
- Doodson, A.T., 1927. The analysis of tidal observations. *Philosophical Transaction of the Royal Society, London* A227: 223-279.
- Doodson, A.T. and Warburg, 1941. *Admiralty manual of tides*. HMSO, London, 270 pp.
- Dronkers, J.J., 1964. *Tidal computations in rivers and coastal waters*. North Holland Pub. Co., Amsterdam, 518 pp.
- Evans, J.J. and D.T. Pugh, 1982. Analysing clipped sea-level records for harmonic tidal constituents. *Int. Hydrogr. Rev.*, 59 : 115-122.
- Groves, G.W., 1955. Numerical filters for discrimination against tidal periodicities. *Transaction of American Geophysical Union*, 36: 1073-1084.
- Munk, W.H. and D.E. Cartwright, 1966. Tidal spectroscopy and predication. *Philosophical Transaction of the Royal Society, London* A259: 533-581.
- Poole, S.L., A.B. Rabinovich, L.Q. Spielvogel and R.R. Harvey, 1980. Study of ocean tides in the region of the Kuril-Kamchatka and Japan trenches. *Oceanology*, 20: 655-659.
- Schureman, P., 1958. *Manual of harmonic analysis and prediction of tides*. U.S. Dept. of Commerce, Special Publication No. 98.

- Shetye, S.R. and A.M. Almeida, 1985. An examination of the factors that influence the monthly mean sea level along the coast of India. *IOC/UNESCO Workshop on Regional Cooperation in Marine Science in the Central Indian Ocean and Adjacent seas and Gulf, Colombo, 8-13 July 1985, Workshop Rept. No. 37 - Supplement.*
- Suthons, C.T., 1959. The the admiralty semi-graphic method of harmonic tidal analysis. *Admiralty Tidal Handbook No. 1, N.P. 122(1)*, Hydrographic Department, Admiralty.
- Zetler, B.D., M.D. Schuldt, R.W. Whipple and S.D. Hicks, 1965. Harmonic analysis of tides from data randomly spaced in time. *Journal of Geophysical Research*, 70: 2805-2811.

APPENDIX

Least Squares Method

The height of the tide at any time may be predicted harmonically from known amplitudes and phases of individual constituents by the following formula (see Dronkers, 1964) :

$$h(t) = H_0 + \sum_{n=1}^{NC} f_n H_n \cos[W_n t + (E_n + u_n) - g_n] \quad (1)$$

in which

h = height of the tide at any time t

t = time reckoned in hours from some initial epoch such as beginning of year of prediction

H_0 = mean height of water level above datum used for predictions

H_n = mean amplitude of any constituent A

f_n = nodal correction for reducing mean amplitude H_n to year of prediction

W_n = speed of constituent in degrees per solar hour

$(E_n + u_n)$ = value of equilibrium argument of constituent A at Greenwich at the origin of time, i.e. where $t = 0$

g_n = Greenwich phase of constituent A

NC = number of constituents required for adequate description of the tide

Constituent speed W_n distinguishes the different constituents. The harmonic constants H_0 , H_n and g_n are generally available from publications of the International Hydrographic Bureau, Monaco. E_n , u_n and f_n depend solely on the origin of time and can be easily determined (See Doodson and Warburg, 1941). Thus the only variable on the right hand side of Eq (1) is t , varying which we can determine the tidal curve. Thus the forward problem : given the harmonic constants H_n and g_n , to determine the tide — is easily solved.

We shall now discuss the inverse problem : given NH hourly observations to determine the tidal constants H_n and g_n . As already mentioned 3 methods are available for the determination of harmonic constants. The classical approaches of Doodson (1927) uses a system multipliers for determining the constants, and was appropriate in the early days when computations had necessarily to be made by hand. With the advent of computers, and their ability for inverting large matrices, the LSM has become the standard method for determining tidal constants, though the response method of Munk and Cartwright (1966) is increasingly being used. We shall now briefly review the LSM.

By means of the addition formula for the cosine function, Eq (1) can be written as

$$h(t) = H_0 + \sum_{n=1}^{NC} A_n \cos W_n t + \sum_{n=1}^{NC} B_n \sin W_n t \quad (2)$$

where, $A_n = f_n H_n \cos (E_n + u_n - g_n)$ and $B_n = -f_n H_n \sin (E_n + u_n - g_n)$
then if A_n and B_n are known, the tidal constants are given by

$$H_n = \sqrt{A_n^2 + B_n^2} / f_n \quad (3)$$

$$g_n = E_n + u_n - \arctan (-B_n / A_n) \quad (4)$$

If we set $t = 0$ to coincide with some time origin, say, 0000 hrs on the central day at which hourly observations are available, then Eq (2) leads to NH linear equations in which the 2NC unknown are A_n and B_n . The number of hourly observations NH and the number of constituents NC are suitably chosen such that NH is much larger than 2NC, i.e. the ratio 2NC/NH is small, so that the system of equations generated by Eq (2) is grossly overdetermined.

Let h_1, h_2, \dots, h_{NH} and $\hat{h}_1, \hat{h}_2, \dots, \hat{h}_{NH}$ respectively represent the original series of tidal observations and the series predicted by Eq (2). Values of (A_n, B_n) for $n = 1, 2, \dots, NC$ are then determined by solving the normal equations obtained by minimizing the value of

$$\sum_{i=1}^{NH} (h_i - \hat{h}_i)^2.$$

Once A_n and B_n are determined, the tidal constants H_n and g_n are obtained from Eqs (3) and (4). The hindcasting skill (HCS) of (A_n, B_n) is defined as follows :

$$HCS = \frac{\sum_{i=1}^{NH} (\hat{h}_i - H_0)^2}{\sum_{i=1}^{NH} (h_i - H_0)^2} \quad (5)$$

where H_0 is the mean height. HCS which takes values between 0 and 1, determines the reliability of the least squares solution (A_n, B_n) . If HCS is very close to 1, then the values of A_n, B_n are very reliable, provided that the ratio 2NC/NH is small. On the contrary if HCS is close to zero, then A_n, B_n and consequently H_n, g_n are not at all reliable.

If the ratio 2NC/NH is not small, then a phenomenon known as artificial predictability may take place. Essentially it means that even if the (A_n, B_n) have a HCS close to 1, so that Eq (2) correctly predicts the tide for a particular sample comprising of the NH hourly observations used in the LSM, for any other sample, comprising say, the next or earlier NH hourly observations, Eq (2) will fail in correctly determining the tide using above values of (A_n, B_n) as gauged by the HCS obtained using Eq (5) for this sample, if the ratio 2NC/NH is large. In the field of tidal

prediction, the main concern is in predicting future values of tide. Therefore ensuring that the ratio $2NC/NH$ is sufficiently small is of paramount importance.

In literature, often, instead of the Greenwich phase g_n , the local phase K_n is given. The phases g_n and K_n are related as follows :

$$g_n = K_n + pL - W_n S / 15$$

where L is the longitude of the place; S is the time meridian of the place, both reckoned as degrees west of Greenwich; $p=1, 2$ and 4 respectively for diurnal, semi-diurnal and ter-diurnal constituents; and W_n is the speed of the constituent in degrees per solar hour.-