



WAVE GROUPS OFF - BOMBAY COAST: A CASE STUDY

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ABSTRACT

A few wave records obtained from Bombay High during the monsoon period are analysed to determine the extent of wave group formation. Average correlation between the successive wave heights is found to be +0.27, indicating that the wave heights do have 'memory'. The average run length and total run are calculated for both $H > H_m$ and $H > H_s$. The wave group formation is found to be more pronounced in the growing sea state than decaying sea state.

1. INTRODUCTION

A sequence of high waves is called a wave group and formation of the same is termed as wave grouping. In recent years, the study of wave grouping has gained significant importance. This phenomenon is considered as an important factor in the analysis related to the stability of rubble mound break waters, overtopping of waves over the sea walls and slow drift oscillations of moored vessels, floating structures etc. Laboratory experiments (Johnson et al. 1978) showed that the different variety of damaging effects are attributed to the wave grouping phenomena, even if the energy spectra and wave height distributions are identical.

2. THEORETICAL ANALYSIS

2.1 Length of run

Wave groupness is essentially a random phenomenon. Goda (1970) proposed a theory of statistical run length, assuming that there is no correlation between the successive wave heights. The run length is defined as the number of successive waves which exceed certain magnitude. The probability of group formation with run length j , above a certain level, H_L is evaluated as,

$$P_1(j) = p^{(j-1)} \times q \quad (1)$$

This is because the run with the length j , is the

phenomenon that $(j-1)$ consecutive wave exceed a certain magnitude H_L , and the $(j+1)$ th wave fails to exceed H_L . Since successive waves are assumed to be independent, the joint probability is given by the product of the individual probabilities.

Assuming that the wave heights follow a Rayleigh distribution, p becomes equal to 0.135 and $q = 1 - 0.135 = 0.865$, for $H_L = H_s$ and for $H_L = H_m$, $p = q = 0.5$.

By knowing $P_1(j)$, the mean length of run can be evaluated as it's first moment.

$$j_1 = E(j) = \sum_{j=1}^{\infty} j \times P_1(j) = 1/q \quad (2)$$

Rye (1974), Goda (1976), Dattatri (1983) have carried out studies on this aspect two compare to theoretical prediction with the field observations.

2.2 Total run

Another statistics in group analysis is that the total run defined as the number of wave heights between the exceedance of the same level by succeeding group of waves (Goda 1970, Ewing 1973). The probability of total run with length of run j , denoted by $P_2(j)$ is given by,

$$P_2(j) = (pq/(p-q)) \times (p^{j-1} - q^{j-1}) \quad (3a)$$

$$j_2 = 1/p + 1/q \quad (3b)$$

Rye (1974) carried out the study for Utaria wave data to determine the length of total run.

2.3 Wave height correlation

The statistical theory of run length is developed on the assumption that there is no correlation between the successive wave heights. The correlation coefficient can be evaluated by using the formula (Rye 1974).

$$\phi(k) = 1 / \phi(0) \times 1/N-k \times \sum_{i=1}^{N-k} (H_i - H_m)(H_{i+k} - H_m) \quad (4)$$

$$\text{where, } \phi(0) = 1 / N \times \sum_{i=1}^N (H_i - H_m)^2$$

N = number of waves in actual record

H_m = mean wave height

k = number of lags between the waves in sequence.

The values of correlation coefficient give the extent of 'memory' between successive waves as it deviates from zero. If the coefficient becomes equal to zero, the correlation vanishes. The positiveness indicates that the larger waves tend to be followed by large waves and small waves tend to be followed by other smaller waves.

3. DATA COLLECTION AND ANALYSIS

The wave data utilised for analysis were that obtained from Bombay High on the West Coast of India during the monsoon period in the month of July 1983. Water depth at the location of wave rider buoy was about 70m. 46 time series records on waves are analysed by zero-crossing method, using Norsk-Data-100 computer at the Institute. Fig.1 shows a typical plot of sequence of successive waves indicating philosophy of data analysis. For records analysed the range of H_s was 1.85 to 3.98m. The spectral width parameter s varied from 0.78 to 0.89. The wave height and wave period statistics, joint distribution of wave height and wave period are described in detail in Bhat et al.(1986).

Total of 46 wave records of each 20 minutes duration wave were divided into two groups according to whether H_m was recorded during increasing wave height condition (G) or decreasing wave height condition (D). Fig.2, shows the variation of H_m for the month of July 1983. This is done to compare the statistical properties of sea state with special reference to wave groupiness. A plot of correlation coefficient $\phi(1)$ for 46 wave records is also drawn to compare with the variation of H_m (Fig.3).

4. RESULTS AND DISCUSSION

4.1 Run length statistics

Successive wave heights for a typical wave record analysed, is shown in fig.1, with $H > H_s$ as level of significance and corresponding run length and total run are marked. The mean length of run considering all records is 2.29 for $H > H_m$ and 1.38 for $H > H_s$ showing a pronounced group formation than that predicted by theory. The corresponding theoretical values are 2.00 and 1.16 for $H > H_m$ and $H > H_s$ respectively. This is more so, particularly in higher ranges of run length. But considering the fact that the theory assuming no correlation between the successive wave heights, which is not really so, the agreement between the data and the theory can be termed as reasonably good. Table 1 gives reproduced values of the mean length of run, evaluated by Rye (1974), Goda (1976) and Dattatri (1983) for comparison. Their studies do conform to the similar conclusion.

4.2 Probability of group formation

Equation 1 gives the probability of group formation corresponding to the given length of run. This equation gives a straight line relation on a semi-logarithmic plot. Fig.4 and Fig.5 are such plots with values of field observations incorporated on it indicating that the theory predicts very low values particularly in the higher range of run lengths. Similarly, Table 2 which gives the frequency of distribution of the length of ordinary run for $H > H_m$ and $H > H_s$, also indicates the above conclusion. This result is in consistent with Goda's (1970) result and also in accordance with the conclusions of Wilson and Baird (1972).

4.3 Generation and decay cases

The data set is divided into two cases as growing and decaying sea state (Fig.2 and 3). The statistical properties of group formation are compared for the sea states by undertaking similar analysis as above. Table 1 shows that the mean length of run is higher in the case of wave generation than the wave decay, considering both the levels of significance. Fig.4 and Fig.5 also give the trend of probability of group formation to be higher in the case of generation. This phenomenon may be due to the energy input to the sea waves from wind during generation. This would result in generating more high waves with greater tendency to exceed certain level of significance. Hence the probability

of wave group formation seems to be less in the case of decay wherein which the energy redistribution and transformation essentially occurs. This is conforming to the conclusion of Rye (1974).

4.4 Dependency of Wave heights

The correlation coefficient is evaluated to study the dependency between the successive wave heights using equation 4. Fig.3 is a plot of correlation coefficient ϕ (l) for 46 records. The average value is found to be +0.27 indicating the existence of dependency with the positiveness confirming the trend of high waves to be followed by higher waves only. Another statistics of interest is the analysis with conditional run length containing H_{max} (table 2). The average value of conditional run lengths is 4.6l for $H > H_m$ and 2.1l for $H > H_s$. In other words, the highest waves does not appear singly, but is accompanied by several high waves. This is also confirmed by Goda (1976). Similar study on generation and decay cases are carried out (fig.3 and Table 3 and 4). The average values are found to be greater for growing sea condition than decaying. This again clearly exhibits the persistancy of 'memory' of generation. Fig.6 is plot of mean length of run for $H > H_m$ and $H > H_s$ versus correlation coefficient ϕ (l). Though the spread of values do not show any definite difference between the two cases, it essentially indicates a trend of data towards a general increase in the mean run length with an increase in correlation coefficient ϕ (l) of the successive waves.

Longuet-Higgins (1957) has shown theoretically that wave group formation tends to be more pronounced for narrow band process. Swell phenomenon is the criteria which occurs only during the decaying sea state and is described by narrow band spectrum. But for the present data it is found that there is no systematic relation between the spectral width parameter and the mean run length.

Goda (1970) showed that the controlling factor for run length is the peakedness parameter Q_p . Fig.7 is a plot of mean run length j versus peakedness parameter Q_p for $H > H_m$ and $H > H_s$. Considering the total data set it appears to indicate a very general increase of mean run length j with the increase in peakedness parameter Q_p . It can also be noticed that the general trend of mean j to increase with the increase in Q_p is more in generating stage than the decaying. Thus the more pronounced peakedness of Q_p during the wave growth may be

a factor which influences the group formation. It is obvious that theoretical values of mean run length is independent of Q_p as it assumes the narrow band approximation.

4.5 Total run

The mean length of total run is evaluated for total set of data, generation and decay cases. The mean values are 4.06, 4.06 and 4.06 for H , H_m and H_s and 8.67, 8.57 and 8.77 for H , H_m and H_s respectively. As this value gives the frequency of occurrence of groupiness, the lesser value indicates the higher group formation. The mean length of run considering all the records is 1.4l and total run is 8.67 for H , H_m and H_s . Hence the number of waves between two groups is equal to $8.67/1.38 = 7.29$. But the theory predicts a value equals 7.4l which is slightly more than the field observation. This again shows the higher group formation in the case of field observation than that predicted by theory.

5. CONCLUSIONS

The wave group analysis shows that

1. The theory of probability of group formation, though assumes no dependency between successive waves, seems to give a reasonable fit to the data.
2. The group formation is found to be more pronounced in the case of growing sea state.
3. The sea waves do have 'memory'.

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LIST OF SYMBOLS

- G = Spectral width parameter
 $\phi(k)$ = Correlation coefficient with k number of lags
 j_1 = Run length of wave height that exceed a certain level
 j = Total run length
 p^2 = Probability density function
 H_1 = Wave height
 H_L = Any level of significance of wave height
 H_m = Mean wave height
 H_s = Significant wave height
 P_1 = Probability distribution function for ordinary runs
 P_2 = Probability distribution function for total runs

Table 1: Comparison of Mean Length of Runs evaluated by different authors

Location/Author	$H > H_m$			$H > H_s$		
	Total	Growth	Decay	Total	Growth	Decay
Present data Bombay	2.29	2.30	2.29	1.38	1.43	1.35
Dattatri(1983) Mangalore	2.23			1.34		
Goda (1976) Japan Coast				1.42		
Rye (1974) Utsria				1.34	1.41	1.26
Theoretical values	2.00	2.00	2.00	1.16	1.16	1.16

Table 2: Frequency distribution of length of run of wave heights(total 46 records)

j Run Length	Ordinary Runs				Runs containing H_{max}			
	$H > H_m$		$H > H_s$		$H > H_m$		$H > H_s$	
	obs.	cal.	obs.	cal.	obs.	cal.	obs.	cal.
1	569	672.5	459	561.4	1	23.0	13	39.8
2	339	336.3	149	75.7	7	11.5	10	5.4
3	198	168.1	28	10.4	9	5.8	7	0.7
4	108	84.1	10	1.3	7	2.9	5	0.1
5	62	42.1	3	0.2	10	1.4	1	
6	28	21.1			5	0.7		
7	19	10.5			3	0.4		
8	9	5.4			1	0.2		
9	5	2.6				0.1		
10	5	1.3			1			
11	1	0.7			1			
12	2	0.3			1			
Total	1345	1345	649	649	46	46	46	46
j mean	2.29	2.00	1.38	1.16	4.61	2.00	2.11	1.16

Table 3: Frequency distribution of length of run of wave heights (generation 17 records)

j Run length	Ordinary Runs				Runs containing H_{max}			
	$H > H_m$		$H > H_s$		$H > H_m$		$H > H_s$	
	obs.	cal.	obs.	cal.	obs.	cal.	obs.	cal.
1	212	252.0	167	208.5		8.5	3	14.7
2	123	126.0	52	28.1	1	4.3	6	2.0
3	78	63.0	16	3.8	3	2.2	5	0.3
4	42	31.5	5	0.5	3	1.1	2	
5	25	15.8	1	0.1	6	0.5	1	
6	11	7.9			1	0.3		
7	7	3.9			1	0.1		
8	2	2.0			1			
9	1	1.1						
10	1	0.5						
11	1	0.2			1			
12	1	0.1						
Total	504	504	241	241	17	17	17	17
j mean	2.30	2.00	1.43	1.16	5.00	2.00	2.53	1.16

Table 4: Frequency distribution of length of run of wave heights (decay 29 records)

j Run length	Ordinary Runs				Runs containing H_{max}			
	$H > H_m$		$H > H_s$		$H > H_m$		$H > H_s$	
	obs.	cal.	obs.	cal.	obs.	cal.	obs.	cal.
1	357	420.5	292	352.9	1	14.5	10	25.1
2	216	210.3	97	47.6	6	7.3	13	3.4
3	120	105.2	12	6.4	6	3.6	2	0.4
4	66	52.6	5	0.9	4	1.8	3	0.1
5	37	26.3	2	0.2	4	0.9	1	
6	17	13.1			4	0.5		
7	12	6.5			2	0.3		
8	7	3.8				0.1		
9	4	1.9						
10	4	1.0			1			
11	0	0.5						
12	1	0.4			1			
Total	841	841	408	408	29	29	29	29
j mean	2.29	2.00	1.35	1.16	4.38	2.00	2.03	1.16

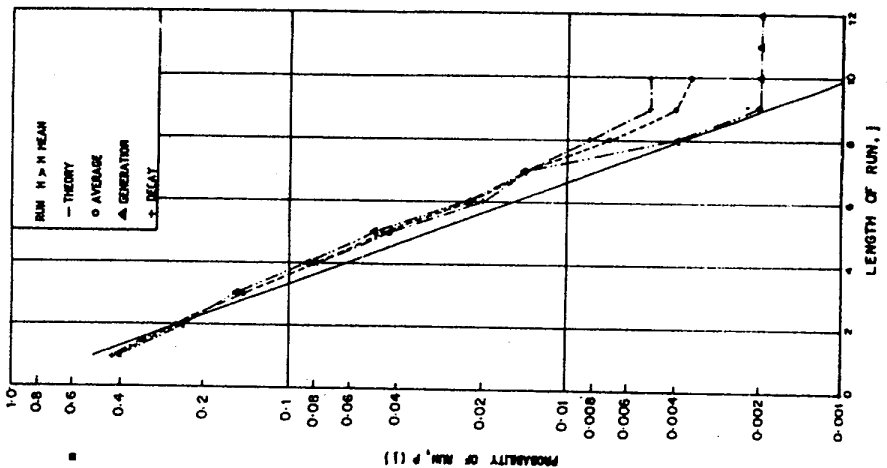
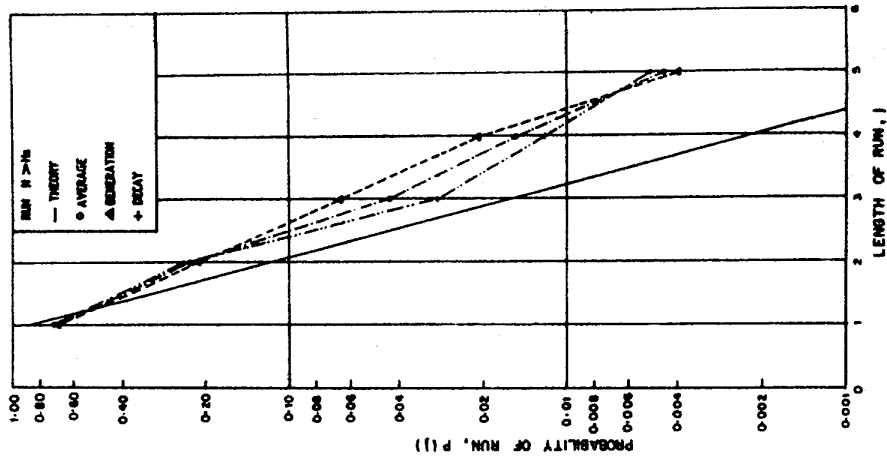
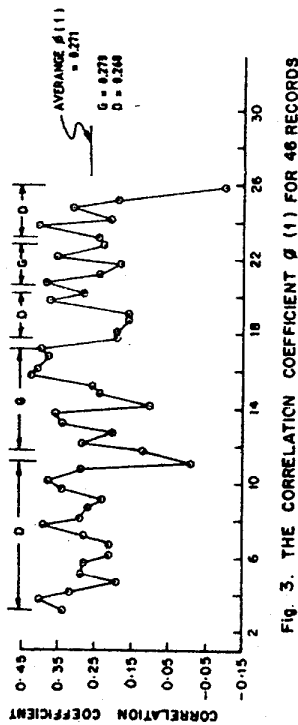
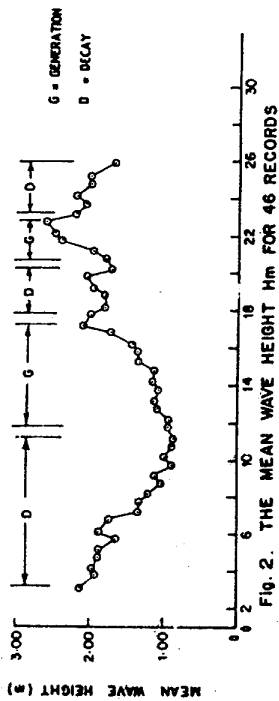
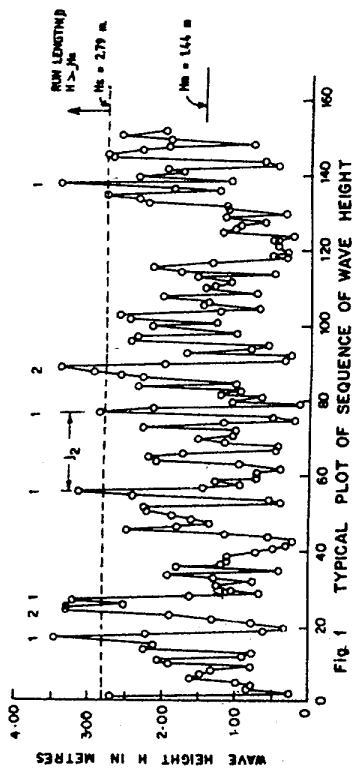


Fig. 5. PROBABILITY OF GROUP FORMATION

Fig. 4. PROBABILITY OF GROUP FORMATION

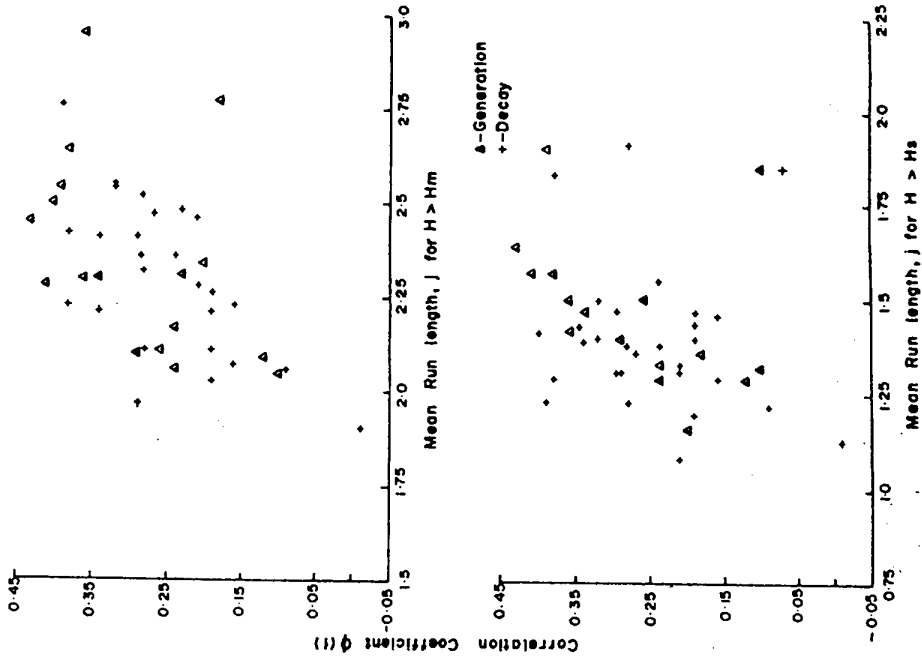


Fig 6 MEAN RUN LENGTH V/S CORRELATION COEFFICIENT $\phi(1)$

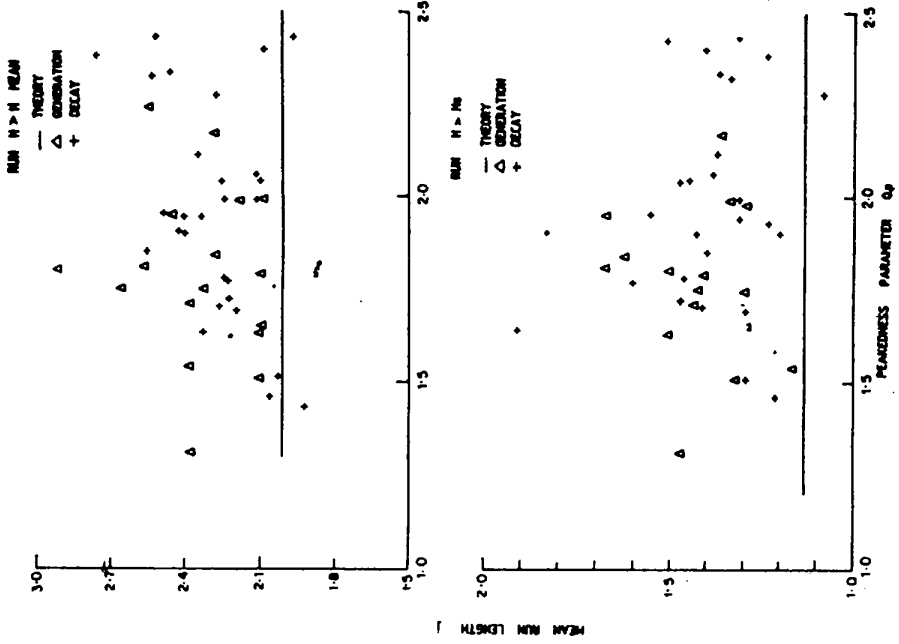


Fig. 7. MEAN RUN LENGTH V/S. PEAKEDNESS PARAMETER