

# NUMERICAL PREDICTION OF SHORELINE ADJACENT TO BREAKWATER

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## A B S T R A C T

Existing mathematical models for prediction of shoreline changes in the vicinity of a breakwater were reviewed. The analytical and numerical results obtained from these models have been compared. Under the numerical approach, two different implicit finite difference schemes were developed and used for predicting the growth of the shoreline adjacent to a breakwater. The shorelines predicted by non-linear models were compared with those obtained from linear models. It is concluded that a linearized model which possesses better numerical stability would be satisfactory for the theoretical prediction of the shoreline evolution adjacent to a breakwater.

## INTRODUCTION

Construction of any coastal structure that projects into the sea from the shoreline obstructs the movement of sand along the shore and results in deposition on the updrift side of the net annual longshore transport and erosion on the down drift side. The phenomenon of coastal processes associated with the shoreline evolution is highly complex and mathematical modelling of this process has received limited attention in the past. Further, the lack of well accepted laws of longshore and offshore-onshore sediment movement and the nonavailability of sufficient wave statistics have made the task of calibrating mathematical models very difficult. In spite of these difficulties, a reasonable estimate of the long-term shoreline evolution using the existing mathematical models would be useful and better than no estimate at all. This paper presents the simulation of shoreline

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adjacent to a breakwater using two different implicit finite difference schemes. The results thus obtained were compared with the solution of the linearized problem.

### MODEL FORMULATION

Assuming that the changes in mean sea level and beach slope are not significant and equating the rate of changes in beach volume to the total addition and/or subtraction of beach sediment in unit length of the beach (Fig. 1), the shoreline one-line model (Kraus and Harikas, 1983) becomes,

$$(b + D_c) \frac{\partial y_s}{\partial t} = - \frac{\partial Q}{\partial x} + q(x) \quad \dots \quad (1)$$

where,

- b = height of the beach berm
- D<sub>c</sub> = depth of no sediment movement
- y<sub>s</sub> = distance perpendicular to the shoreline
- t = time
- x = distance along the shoreline
- q(x) = the quantity of sand dredged or deposited on the beach.
- Q = the longshore transport rate

The longshore transport rate Q, is given by,

$$Q = A k_D^2 \cos \alpha_o \sin \alpha_b \text{ in cubic yards } \dots \dots \quad (2)$$

where  $A = 7500 \frac{\rho g^2}{32\pi} \cdot H_o^2 \cdot T$

$$\alpha_o = \text{deepwater wave angle} = \alpha - \tan^{-1} \left( \frac{dy}{dx} \right) \pm \frac{\pi}{2} \dots \quad (3)$$

$\alpha$  = wave angle w.r.t. x axis (Fig. 2)

$$\alpha_b = \text{breaking wave angle} = \beta \alpha_o \text{ (Le Mehaute and Kho, 1967)}$$

$$\beta = 0.25 + 5.5 H_o / L_o \quad \dots \quad (4)$$

$$k_D = \text{refraction coefficient} = (\cos \alpha_o / \cos \alpha_b)^{0.5}$$

g = acceleration due to gravity in ft/sec<sup>2</sup>

H<sub>o</sub> = deepwater wave height in ft.

T = wave period in sec.

$\rho$  = sea water density in lb/ft<sup>3</sup>

L<sub>o</sub> = deep water wave length in ft.

Equations (1) and (2) specifies the problem and by solving these non-linear equations, the shoreline changes can be computed.

### ANALYTICAL SOLUTION

Expressing the variables in non-dimensional form equation (1) becomes,

$$\frac{\partial \bar{y}}{\partial \bar{t}} = - \frac{\partial \bar{Q}}{\partial \bar{x}} + \bar{q}(\bar{x}) \quad \dots \quad (6)$$

where,

$$\begin{aligned} \bar{x} &= x/(b + Dc) \\ \bar{y} &= y/(b + Dc) \\ \bar{t} &= At/(b + Dc)^3 \\ \bar{Q} &= Q/A = K_D^2 \cos \alpha_o \cdot \cos \alpha_b \quad \dots \quad (7) \end{aligned}$$

Le Mehaute and Soldate (1980) obtained an analytical solution to the linearised form of the shoreline equation (6) in the case of breakwater perpendicular to the shoreline where only refraction is important and there is no change in mean sea level and beach slope.

After eliminating  $\bar{y}$  in equation (6) and avoiding the head bars over the variable, it becomes,

$$\frac{\partial Q}{\partial t} = a \frac{\partial^2 Q}{\partial x^2} - a \frac{\partial q}{\partial x} \quad \dots \quad (8)$$

where,

$$a = K_D^2 (\beta \cos \alpha \cdot \cos \alpha_b - \sin \alpha \cdot \sin \alpha_b) / \left[ 1 + \left( \frac{\partial y}{\partial x} \right)^2 \right]$$

For a standard breakwater problem, substituting the boundary conditions (Le Mehaute and Soldate, 1980) and considering  $q(x) = 0$  the analytical solution for equation (8) is written as,

$$y(x,t) = \frac{Q_o}{a} \left[ x \operatorname{erfc} \left( \frac{x}{\sqrt{4at}} \right) - 2 \sqrt{\frac{at}{\pi}} \exp \left( \frac{-x^2}{4at} \right) \right] \dots \quad (9)$$

### NUMERICAL MODELS

Two finite difference models (Scorborough, 1979) are used for the simulation of the shoreline evolution. In the first model (Model 1), the longshore transport equation (7) and the nondimensional sediment continuity equation (6) are expressed

in finite difference form. The second model (Model 2) constructs the finite difference scheme by discretizing the equation (8). The implicit finite difference scheme is used in both the models.

### Model 1

When the changes in mean sea level and the changes in beach slope are not significant, the shoreline equation in non-dimensional form is given by equation (6).

For a given initial and boundary conditions and wave characteristics, eqn. (6) is integrated numerically together with equations (4), (5) and (7).

Using a staggered grid representation (Fig. 3), equation (6) becomes,

$$Y_{n,t+1} = B(Q_{n,t+1} - Q_{n+1,t+1}) + C_n \quad \dots \quad (10)$$

where,

$$B = \frac{\delta t}{2\delta x}$$

$$C_n = Y_{n,t} + B(Q_{n,t} - Q_{n+1,t} + 2\delta x q_{n,t})$$

Here,  $\{Q_n\}$  is specified at the grid points and  $\{q_n\}$  and  $\{y_n\}$  at the centre of grid spacing

$\delta x$  = the spacing between the grids where  $\{Q_n\}$  is defined.

$\delta t$  = the time interval chosen for integration.

$n$  = number of space step

$t$  = number of time step.

The advantage of this implicit scheme is that it permits the use of longer time steps of integration. Since this linearised equation is of the form of the diffusion equation, the limit of time step to preserve stability in an implicit numerical integration scheme is given by,

$$r = \frac{\delta t}{\delta x^2} \leq \frac{1}{2a}$$

thus by selecting longer grid spacing, a larger time step can be used.

The longshore transport  $Q$  at time step  $(t+1)$  is expressed in terms of the shoreline coordinates,  $y$ ,

$$Q = K_D^2 \cdot \cos(\alpha - \alpha_{sp}) \sin \alpha_b$$

where

$$\alpha_0 = \alpha - \alpha_{sp}$$

$\alpha_{sp}$  = the angle of shore normal to x axis

$$\therefore Q = K_D^2 (\cos \alpha \sin \alpha_{sp} \cot \alpha_{sp} + \sin \alpha \sin \alpha_{sp}) \sin \alpha_b$$

and

$$Q_{n,t+1} = E_n (Y_{n-1,t+1} - Y_{n,t+1}) + F_n \quad \dots \quad (11)$$

where

$$E_n = K_D^2 \cos \alpha \sin \alpha_{sp,t} \sin \alpha_{b,t} / \delta x$$

$$F_n = K_D^2 \sin \alpha \sin \alpha_{sp,t} \sin \alpha_{b,t}$$

Equations (10) and (11) contain two sets of unknowns  $\{Y_{n,t+1}\}$  and  $\{Q_{n,t+1}\}$ . Since boundary conditions are expressed in terms of  $Q$ , we will first solve for  $\{Q_{n,t+1}\}$ . Substitution of equation (10) in (11) gives,

$$B \cdot E_n \cdot Q_{n-1,t+1} - (1+2B \cdot E_n) Q_{n,t+1} + B \cdot E_n \cdot Q_{n+1,t+1} = E_n (C_n - C_{n-1}) - F_n \quad \dots \quad (12)$$

For  $n = 2$  to  $N$ , eqn. (12) represents a set of  $(N-1)$  linear equations in  $(N-1)$  unknowns. The end values  $Q_1$  and  $Q_{N+1}$  are specified by boundary conditions,

$$Q_1 = 0$$

$$Q_{N+1} = Q \quad \text{i.e.} \quad \frac{\partial Q}{\partial x} = 0 \quad \text{for large } x$$

This linear system of equation is in tridiagonal form which could be solved. Then the set  $\{Y_{n,t+1}\}$  could be determined by using the relation (10). The same procedure is repeated to simulate the evolution of shorelines.

## Model 2

The second Model constructs the finite difference scheme by discretizing the equation (8). In implicit finite difference form, it becomes,

$$Q_{n,t+1} = \frac{\delta t}{2\delta x^2} [Q_{n-1,t+1} - 2Q_{n,t+1} + Q_{n+1,t+1}] + F_n \quad \dots \quad (13)$$

where,

$$F_n = -Q_{n,t} + a \frac{\delta t}{2\delta x^2} (Q_{n-1,t} - 2Q_{n,t} + Q_{n+1,t}) \\ - a \frac{\delta t}{\delta x} (q_{n,t} - q_{n-1,t})$$

Here 'a' is a function of shoreline coordinates  $\{y_n\}$  and is calculated at the previous time step  $t$ . The same staggered grid representation is used in this finite different scheme, where  $\{Q_n\}$  are specified at the grid points and  $\{q_n\}$  and  $\{y_n\}$  at the centre of grid spacing (Fig. 3). Equation (13), for  $n = 2$  to  $N$ , represents a set of  $(N-1)$  linear equation in a tridiagonal form for  $(N-1)$  unknowns which could be solved using standard procedures. Then the set  $\{y_{n,t+1}\}$  can be determined using the relation (13) and the same procedure is repeated to simulate the evolution of a shoreline.

For certain wave directions, the evolution of shoreline in regions close to the breakwater are affected by wave diffraction. For computing the diffraction coefficient  $K_D$ , the approximate procedure done by Dean and Dalrymple (1984) is adopted and incorporated in the computer program.

#### NUMERICAL EXPERIMENT

The computer program working on the two implicit finite difference models (Model 1 and Model 2) also includes,

- (i) the simulation of shoreline using the linearised equation (8), by keeping 'a' as constant (Model 3).
- (ii) the simulation of shoreline using the analytical solution of equation (9) (Model 4).

By choosing a suitable value for one of the input parameters, the shoreline can be simulated using one of the above four models.

#### RESULTS AND DISCUSSION

The simulated shoreline coordinates are given in table 1. It is observed that there is a good agreement between the numerical and the analytical solution.

Before applying the numerical model to the observed wave environment, the shorelines were simulated constantly from one direction. The results in non-dimensional form are listed in Tables 2 and 3 for  $\alpha = 45^\circ$ ,  $T = 8$  sec. and  $H_0 = 0.5$  m.

The shoreline simulations carried out for different incident wave angles show that the Model 1 generally requires

smaller  $\delta t$  (time step) than Model 2 for numerical stability. In the case of Model 3, the upper limit of  $\delta t$  was always the largest. Also for large values of  $x$ , oscillations were observed in the values of  $y$  coordinates in Model 1 but such was not the case in Model 2 and Model 3 for the same  $\delta t$ . It is thus observed that larger  $\delta t$  could be chosen by increasing the grid spacing. For example, when  $\delta x = 5$ ,  $\delta t$  can go upto 0.2, 0.5 and 2.5 for Model 1, Model 2 and Model 3 respectively, whereas for  $\delta x = 10$ ,  $\delta t$  would be respectively 5, 10 and 25 (Table 3). Further, it is observed from tables 2 and 3 that the shorelines simulated by the Models 1 and 2 are very close to each other. Though the shorelines for Model 3 differ from Models 1 and 2, the difference being very small.

Thus, this study suggests that the use of a simpler linearised model which possesses better numerical stability would be adequate for theoretically predicting the shoreline evolution adjacent to a breakwater and the use of complex non-linear models does not show any special advantage leading to significant increase in the accuracy of the results.

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TABLE 1

## Shoreline coordinates

$$\alpha = 45^\circ, T = 8 \text{ secs } H_0 = 0.5 \text{ m and } \bar{t} = 20$$

Shoreline coordinates, (Nondimensional)		
x coordinate	y coordinate	
	Numerical solution (Model 3)	Analytical solution (Model 4)
0.5	1.5652	1.5645
1.5	0.8281	0.8226
2.5	0.3933	0.3831
3.5	0.1676	0.1564
4.5	0.0643	0.0555
5.5	0.0223	0.0170
6.5	0.0070	0.0045
7.5	0.0020	0.0010
8.5	0.0005	0.0002
9.5	0.0001	-

TABLE 2

## Shoreline coordinates

$$\alpha = 45^\circ, T = 8 \text{ secs, } H_0 = 0.5 \text{ m, } \bar{t} = 400, \delta x = .5$$

Shoreline coordinates (non-dimensional)				
x coordinate	y coordinate			
	Model-1	Model - 2	Model - 3	Model - 4
2.5	6.8023	6.8014	6.7628	6.7577
7.5	3.5048	3.5049	3.2796	3.2429
12.5	1.4497	1.4500	1.3956	1.3337
17.5	0.3851	0.3848	0.5225	0.4638
22.5	0.0522	0.0521	0.1732	0.1349
27.5	0.0041	0.0061	0.0511	0.0325
32.5	0.0006	0.0019	0.0136	0.0065
37.5	-0.0005	0.0011	0.0032	0.0011
42.5	0.0005	0.0007	0.0007	0.0001
47.5	-0.0005	0.0004	0.0002	-
52.5	0.0005	0.0002	-	-
Non-dimen- sional inte- gration time step.	0.20	0.5	2.5	



TABLE 3

Shoreline coordinates

$\alpha = 45^\circ$ ,  $T = 8$  secs,  $H_0 = 0.5$  m,  $\bar{E} = 1000$ ,  
 $\delta x = 10$

Shoreline coordinates (non-dimensional)				
x coordinate	y coordinate			
	Model-1	Model-2	Model-3	Model-4
5	9.9749	9.9687	9.7984	9.7755
15	4.0937	4.0820	3.8011	3.6724
25	1.0602	1.0666	1.2245	1.0683
35	0.1169	0.1266	0.3326	0.2348
45	0.0066	0.0082	0.0775	0.0383
55	-0.0007	0.0006	0.0157	0.0046
65	0.0009	0.0	0.0028	0.0004
75	-0.0009		0.0005	0.0
85	0.0009		0.0	
Nondimensional integration time step	5.0	10.0	25.0	

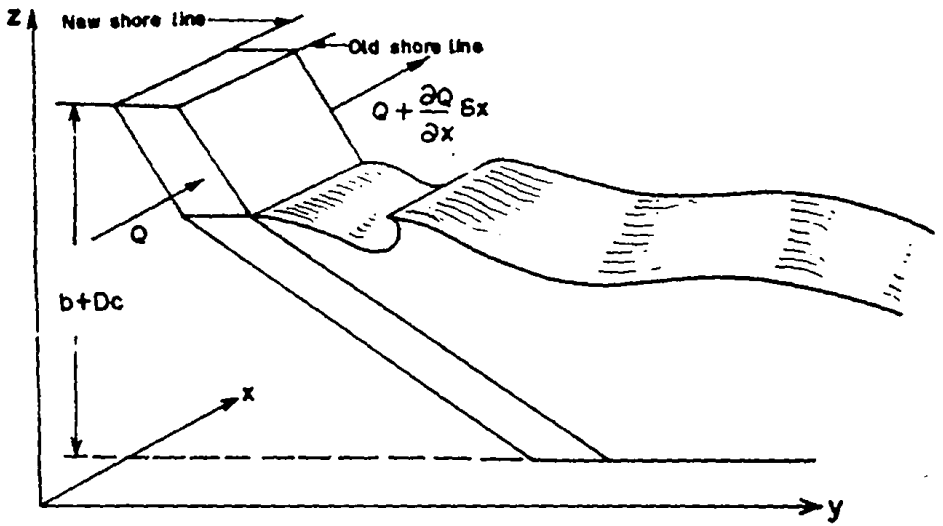


Fig.1 Schematic diagram showing the beach material balance.

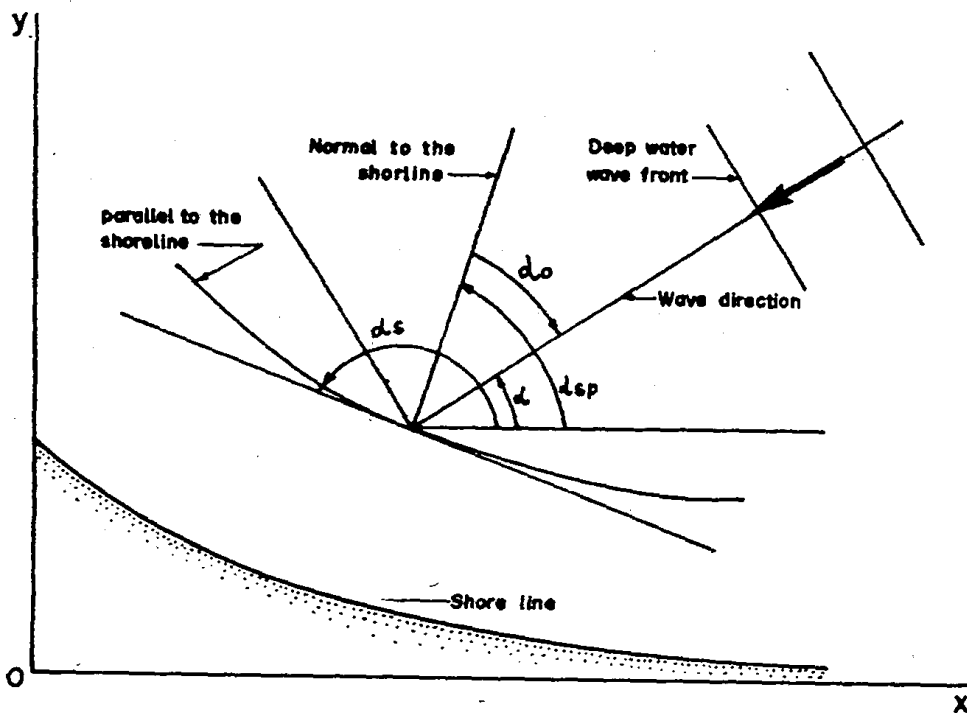


Fig. 2 Deepwater schematic diagram.

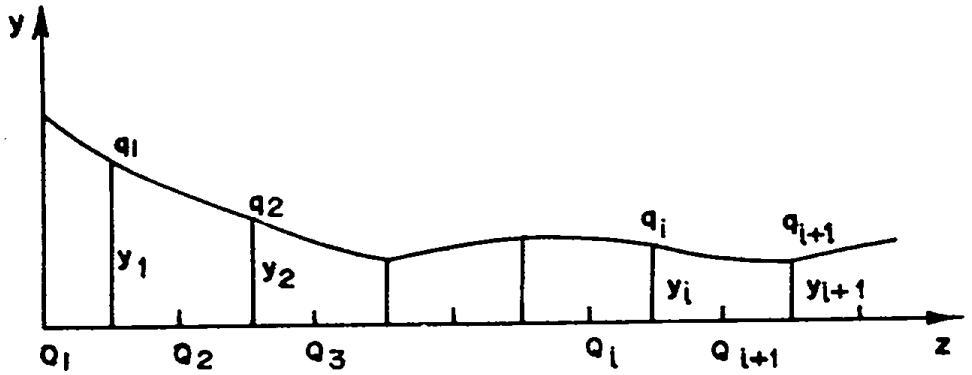


Fig.3. Grid specification for finite difference scheme.