

of the expressions for the waves traveling in the line. The physical insight lent by the scattering interpretation has led to a much greater understanding of the result first noted in [1].

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Coaxial Circular Array: Studies Related to Sidelobe Suppressed Beam Patterns for Underwater Transducer Application

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An analysis of sidelobe suppressed beam patterns other than usual shading methods are carried out for uniformly excited elements in a coaxial circular array at different operating wavelengths. Improved beamwidths and suppressed sidelobes are obtained for multiplicative array techniques useful for high resolution bathymetry, whereas, in the case of an array with tilted element pattern, the sidelobes are not suppressed up to a satisfactory level.

In many sonar applications, circular array transducers are useful. The geometry of such arrays allows them to have symmetrical and narrow beams which ultimately help in obtaining well resolved bathymetry. The present analysis is devoted towards the development of a suitable method to obtain sidelobe suppressed beam patterns other than the usual shading techniques (Taylor's approximation, or methods involving nonuniform excitation of elements) at different operating wavelengths, e.g., 0.5λ , 1.0λ , 1.5λ , 2.5λ and 7.5λ (where λ is the design wavelength).

A study related to the beam patterns of a coaxial circular array (Fig. 1) of three turns kept on an xy -plane which consists of a total of 24 circular disc elements of a radius equivalent to 0.5λ was carried out [1]. The radii of innermost, middle, and outermost circles were 4.0λ , 6.0λ , and 8.0λ expressed as a_1 , a_2 , and a_3 respectively, and each circle consisted of eight elements. The necessary expression for array patterns of such arrays of uniformly excited elements were developed. The appearance of the narrow beams (2° , 3.3° , 5.3° , 8.6° , and 26° respectively) at the above mentioned different operating wavelengths (Fig. 2) confirm wide applicability of such array patterns for well resolved underwater transducer applications [1]. But the presence of sidelobes (more than -10 dB) in such array patterns is the major problem which requires attention. Amplitude shading technique was used to suppress the sidelobes of the above mentioned arrays, and coefficients of excitations were calculated at different wavelengths on the basis of Taylor's approximations [2]. Difficulties with the amplitude shading techniques are not unknown due to their degraded element sensitivity and require a complex feed network. In this letter, different methods of sidelobe suppression other than usual shading techniques, which involve uniform excitation or reception of the array elements, are presented for comparison.

Hill [3] has shown a novel method of amplitude tapering, exciting each element uniformly to reduce the sidelobes by controlled element pattern, i.e., by tilting the elements to a suitable angle. The above method is known as the "tilting method," and is attempted for coaxial circular array elements. The elements of different circles are tilted at different angles (7.5° , 10° , and 15° for the innermost,

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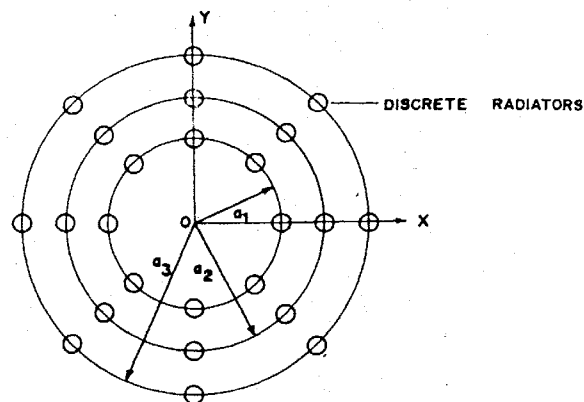


Fig. 1. Geometry of the coaxial circular array.

middle, and outermost circles, respectively) with respect to the xy -plane.

The far field pattern can be written as [1]

$$F(\theta, \phi) = \text{Array pattern} \cdot \text{Element pattern} \quad (1)$$

The element pattern of circular disc elements for tilt angle α can be expressed as

$$\text{Element pattern} = \frac{2J_1(\beta a \sin(\theta + \alpha))}{\beta a \sin(\theta + \alpha)} \quad (2)$$

where J_1 is the first order Bessel function of argument of $\beta a \sin(\theta + \alpha)$, a is the radius of the element (0.5λ in the present case) and β is the wave number ($=2\pi/\lambda_0$, where λ_0 is the operating wavelength). The expression for array patterns for coaxial circular array geometry is given by [1]

$$\text{Array Pattern} = \sum_{n=1}^3 \sum_{m=m_1}^{m_2} \exp j\beta [a_n \sin \theta \cos \phi \cos(m-1)\pi/4 + a_n \sin \theta \sin \phi \sin(m-1)\pi/4 \dots] \quad (3)$$

where n is the circle number and m is the radiator number varying from m_1 to m_2 . For circle 1, m_1 is 1 and m_2 is 8; for circle 2, m_1 is 9 and m_2 is 16; and for circle 3, m_1 is 17 and m_2 is 24. Substituting (2) and (3) on (1), the far field pattern for tilted elements are calculated and presented in Fig. 3. From the figure, it is observed that no significant changes in sidelobes are noticed for different wavelengths other than 0.5λ , compared with the uniformly excited pattern of the untilted elements (Fig. 2). It is also noticed from Fig. 3 that the beamwidths are reduced slightly compared to those of Fig. 2. The fall of array gain is also observed, especially in the case of the operating wavelengths of 0.5λ and 1.0λ . It is realized that if the elements are tilted farther from the above configuration, a suppressed sidelobe pattern may be obtained at the cost of the degraded array gain, which is the effect of "aperture blocking," or shadowing [4].

The multiplicative array technique has its own advantage in that it can produce better angular resolutions along with suppressed sidelobes [5]. The expression for a multiplicative array of three turns is given as

$$F(\theta, \phi) = \prod_{n=1}^3 \sum_{m=m_1}^{m_2} \exp j\beta [a_n \sin \theta \cos \phi \cos(m-1)\pi/4 + a_n \sin \theta \sin \phi \sin(m-1)\pi/4] \quad (4)$$

and the usual notations are already mentioned. The added output of eight elements from each circle is multiplied by each other in the present case. Fig. 4 shows the vertical pattern for five different operating wavelengths. It is clear that the sidelobes are suppressed well below -20 dB. The main beamwidths of 0.5λ and 1.0λ are the same (2° , 3.3°) as the additive array of untilted elements (Fig. 2); whereas for 1.5λ , 2.5λ , and 7.5λ , it becomes narrower (3.3° , 5.0° ,

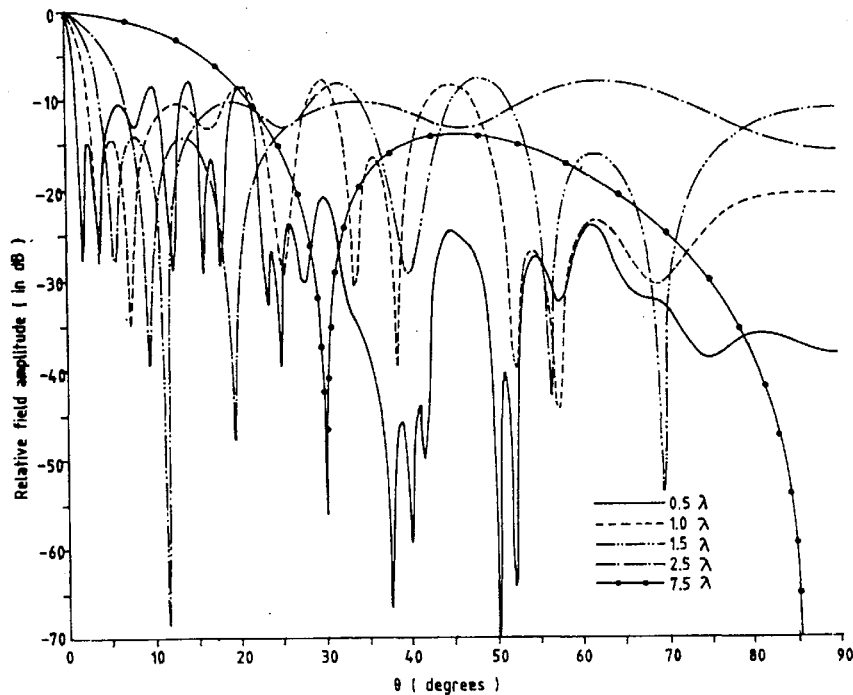


Fig. 2. Vertical beam patterns for a coaxial circular array of untilted elements.

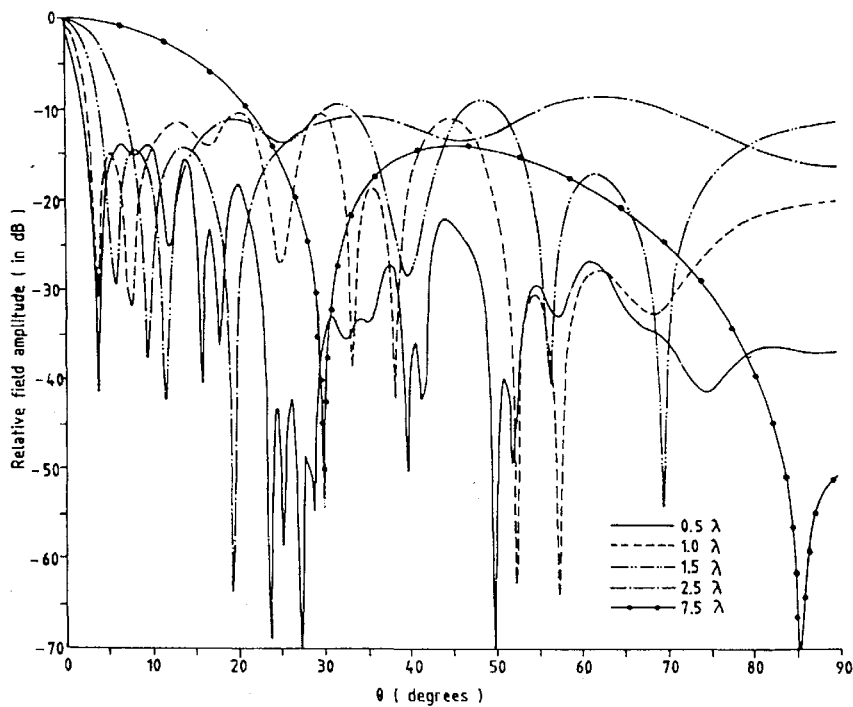


Fig. 3. Vertical beam patterns for a coaxial circular array of tilted elements.

and 14°), thus making it more useful for high resolution underwater bathymetric applications.

CONCLUSIONS

Beam patterns for tilted elements do not seem to suppress the sidelobes up to useful levels. Multiplicative arrays hold well compared to others having patterns for decreased narrow beams, as well as suppressed sidelobes, well below the -20 dB at all chosen operating wavelengths.

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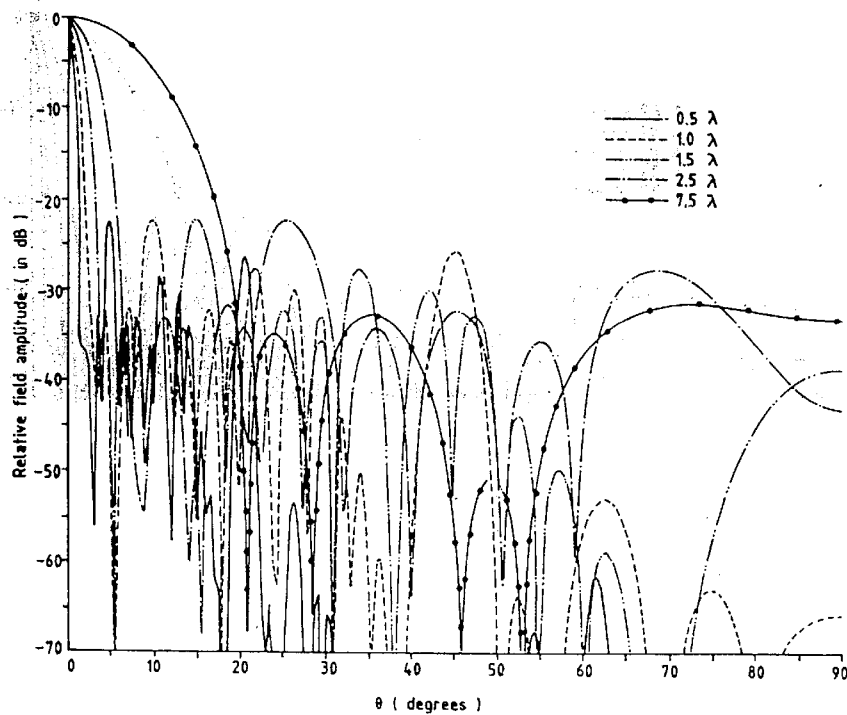


Fig. 4. Vertical beam patterns for a multiplicative coaxial circular array.

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On Three-Dimensional Rotations, Coordinate Frames, and Canonical Forms for It All

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Some properties of the eigenproblem for a three-dimensional rotation matrix are shown, and related to the geometrical rotation parameters. The problem of assigning a unique canonical coordinate frame to a set of three mutually orthogonal axes is considered. The assignment is such that it corresponds to a minimal overall rotation with respect to the reference system. This problem is of interest for the unique and consistent labeling of the principal axes of various tensors related to physical properties of materials, and symmetric matrices that appear in various disciplines of engineering.

INTRODUCTION

In the areas of celestial and applied mechanics, robotics, the theory of elasticity, radar and sonar, and in nuclear, molecular and solid-state physics, one frequently needs to express preferential spatial orientations (attached to a "rigid" body) in terms of some

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fixed reference system (the "laboratory" system). Coordinate transformations are also of interest in expressing material parameters such as dielectric tensors, electrooptic tensors, stress tensors, and so on. Any three fixed mutually orthogonal lines intersecting in 0 (e.g., obtained by solving the eigenproblem for a real symmetric matrix), define 48 possible coordinate frames (of which 24 are right-handed). As the labeling of the preferential axes is usually arbitrary; this paper addresses the problem of providing a "nice" way to uniquely describe or represent a preferential coordinate frame.

As our solution relies on some elementary properties of 3-D rotations, some basic properties of such transformations are first recalled.

BACKGROUND AND NOTATION

A half-line originating in 0 (the origin) will be called an axis. If u is an axis, then the axis parallel to u but extending in the opposite direction will be denoted by $-u$. By a (right-handed) coordinate frame F , we understand an ordered triple of mutually orthogonal axes, following the right-hand rule. A frame consisting of the axes u, v , and w in that particular order will be denoted by (u, v, w) . F is the set of all possible right-handed coordinate frames.

There are many ways to specify the orientation of a coordinate frame relative to another orthogonal coordinate frame with the same origin. Denoting the axes of the fixed reference frame by (x, y, z) , and of the preferential coordinate frame by (x', y', z') , it is standard to represent the rotation by the direction cosines of the primed axes relative to the unprimed ones. One can think of the new (primed) coordinate system as the one resulting by operating on the original (unprimed) system by some transformation, and it is well known that the set of matrices Θ representing these transformations form the rotation group $SO(3)$.

Since any rotation can be represented as a global rotation over $\theta \in [0, \pi]$, measured counterclockwise about some axis u , a representation of the set of three-dimensional rotations can be given in spherical coordinates: Longitude ϕ and latitude ψ suffice to identify the global rotation axis u , and the radius $r = \theta$ describes the angle of rotation. However, $SO(3)$ is not topologically equivalent to the open (or closed) ball, since antipodal points on the surface of the sphere represent the same rotation. A standard homotopy argument shows that the fundamental group contains two ele-