

Link Node Model for the Simulation of Tides in the Gironde Estuary, France

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Utility of a simplified model such as the link node model in simulating the tides in the Gironde estuary is discussed. In this method, the grid network consists of polygons of variable dimensions whose centres of gravity are called junctions. Any 2 junctions are connected by channel elements. Equations are solved in 1 dimensional form. The 2 dimensional aspects of the flow are represented by virtue of the grid network. Comparison of observed and computed tides and a few other applications are also given.

Since the numerical methods suggested by Leendertse and Gritton¹, vertically integrated 2 dimensional (2D) models have been widely used for the simulation of barotropic tides and tidal currents in estuaries. Application of a 2D numerical model is laborious, particularly in taking into account tidal flats. In the present study, a link node approach has been used for studying tidal propagation in the Gironde estuary. Link node approach, originally designed to describe the flow in open channels, has been utilised by Feigner² for the San Francisco delta systems.

Methods

The Gironde estuary (Fig. 1) is situated at the southwestern part of France, near Bordeaux. The Gironde is formed at the junction of 2 rivers, Dordogne and Garonne. The estuary is characterised by high tidal ranges of the order of 6 m at the mouth of the estuary during spring tide. The mean discharge rate of the Garonne is $444 \text{ m}^3 \text{ sec}^{-1}$, while for the Dordogne, it is $322 \text{ m}^3 \text{ sec}^{-1}$.

The method consists of calculating velocities for the channel elements using the equation of momentum in one dimensional form, while surface elevations being computed at the junctions based on the flux of water entering and leaving a polygon. The momentum equation in one dimensional form may be expressed as:

$$\frac{\partial U}{\partial t} = -U \frac{\partial U}{\partial x} - k|U|U - g \frac{\partial \zeta}{\partial x} \quad \dots (1)$$

where U is velocity along a channel element; k , frictional coefficient; ζ , water surface elevation and g is acceleration due to gravity. The x direc-

tion is taken positive towards the head of the estuary and t is time

The equation of continuity is written as:

$$\frac{\partial \zeta}{\partial t} = -\frac{1}{b} \frac{\partial}{\partial x} (UA) \quad \dots (2)$$

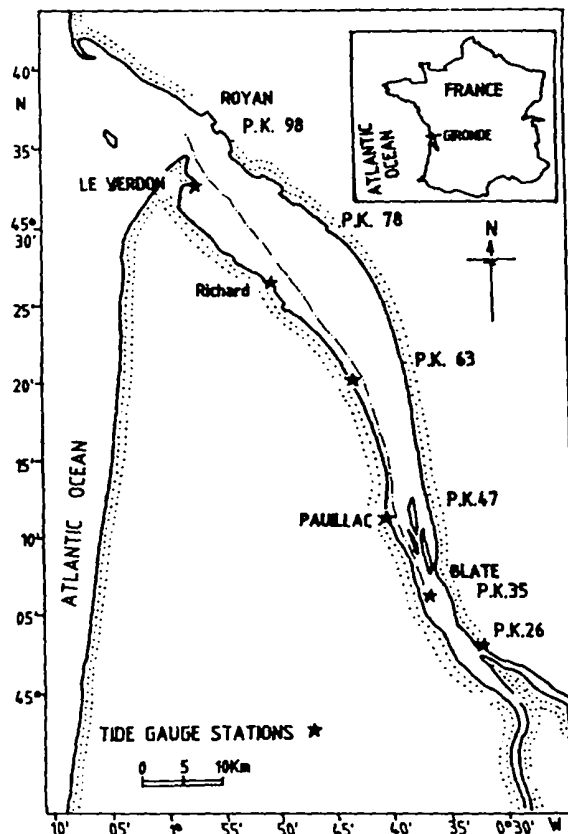


Fig. 1—Gironde estuary

where b is width of a channel element and A is area of cross section of a channel element.

It is to be noted that in applying this model to an estuary, the following assumptions have to be made:

- (a) Wind and Coriolis forces are neglected.
- (b) Parameters like velocity, depth and area of cross section are constant for a channel element.
- (c) Channel element is straight and its cross sectional area is uniform along it.
- (d) Tidal wave is at least twice the channel depth.
- (e) Acceleration normal to the direction of the channel element is neglected.

Evaluation of advection term—Since velocity is constant for a channel element, advection term cannot be evaluated directly. Shubinski *et al.*³ computed the term along the length of the mid point of channel directly ahead of channel to the mid point of the one directly behind it. In the present method, Feigner² computed the advection term indirectly using the continuity equation by a method proposed by Lai⁴.

Expanding Eq. 2 and rearranging terms,

$$\frac{\partial U}{\partial x} = -\frac{b}{A} \frac{\partial \zeta}{\partial t} - \frac{U}{A} \frac{\partial A}{\partial x} \quad \dots (3)$$

Method of resolution—Eq. 1 can be written in finite difference form as follows:

$$\frac{\Delta U_i}{\Delta t} = -U_i \frac{\Delta U_i}{x_i} - k U_i |U_i| - g \frac{\Delta \zeta}{x_i} \quad \dots (4)$$

where i indicates the index of the channel element.

The equation of continuity (Eq. 2) becomes

$$\frac{\Delta \zeta_n}{\Delta t} = \frac{\Sigma Q_n}{A_n} \quad \dots (5)$$

where n is the index of the junction; ΣQ_n the algebraic sum of the flux of water entering and leaving a polygon and A_n is the surface area of a polygon.

Computation procedure—(a) Velocities are calculated for the channel elements at time $t + \Delta t/2$ using the momentum equation based on the initial conditions at time t . The initial conditions being velocities to be zero, surface elevations equal to the mean tide and the cross sectional areas calculated from the surface elevations. (b) Flux of water through each side of a polygon is calculated at $t + \Delta t/2$ using the velocities obtained above and the cross sectional areas at t . (c) The water elevations are computed at $t + \Delta t/2$ based on the continuity equation using the flux obtained in the sec-

ond stage. (d) The cross sectional areas are calculated at $t + \Delta t/2$ using the water surface elevations computed in the 3rd stage. (e) The velocities are computed at $t + \Delta t/2$ utilising the cross sectional areas and water surface elevations at $t + \Delta t/2$. The procedure is continued for the other i steps.

Stability—An explicit scheme is used for 1 resolution. The constraint on the time step is given by the following relation.

$$x_i > (\alpha_i \pm U_i) \Delta t \quad \dots (6)$$

x_i is the length of a channel element i and α_i , velocity of tidal wave in the channel element. For shallow water, $\alpha_i = \sqrt{gh}$, where h is the depth.

Evaluation of the coefficient of friction—It is evaluated using a quadratic law based on Manning's equation. It is given as follows.

$$k = \frac{gn^2}{2.208 R^{4/3}} \quad \dots (7)$$

where n is Manning's roughness coefficient and R is hydraulic radius $R = h + \zeta$

Application to the Gironde estuary—The grid network used for the present study is shown in Fig. 2. Since the depth is constant for a channel element, the channel elements are oriented in such a way that the depth variations are minimum along them. A junction is situated at the centre of gravity of the polygon. The distance between any 2 junctions corresponds to the length of a channel element. The side of a polygon is the median of the line connecting 2 junctions. The side of a polygon corresponds to the width of a channel element. The whole network consists of 407 channel elements and 241 junctions. The channel length varies from 900 to 7000 m, with typical lengths between 2000 and 3000 m. The channel widths have variations between 120 and 6000 m, lower values occurring in the rivers. The surface areas of polygons are measured through planimetry. The time step used is 60 sec.

To calibrate the model, Manning's roughness coefficient used in Eq. 7 are in between 0.04 and 0.02, with lower values assigned to the channel elements situated near to the head of the estuary.

The seaward boundary conditions are given in terms of water surface elevation. This is based on tide gauge data at Verdon (Fig. 1). The open boundary conditions at the river sides are given in terms of river discharge rates.

Tidal flats—The model takes into account the tidal flats which get exposed during low tide. The

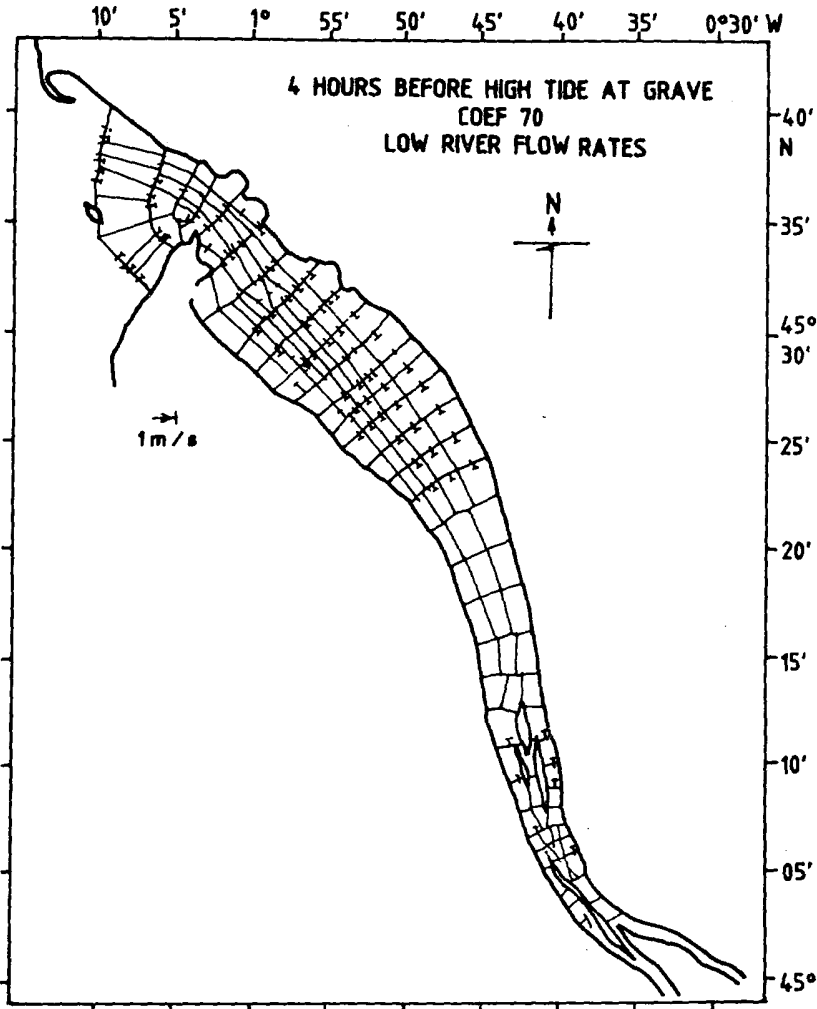


Fig. 2—Grid network used

situation in which the surface elevation at a junction becomes inferior to the depth of an adjacent junction may be considered. In that case, the polygon containing the former junction will be removed from the calculations until the surface elevation in it attains sufficient height. It means that the flux of water through all sides of the removed polygon is zero. Hence, the temporary disappearance of a few polygons makes the calculations to be done on a redefined grid.

Results and Discussion

With reference to the Gironde estuary, the calculation of the advection term by the method suggested by Lai gives numerical instabilities. Accordingly, the advection term has been omitted for the present simulations. In the case of branching

networks, the method suggested by Shubinski³ is also difficult in defining a channel ahead and a channel behind in order to define the velocity gradients. After calibrating the model, the computed water surface elevations are presented in Fig. 3. The accuracy obtained in the computations of amplitudes is ± 5 cm. The phase difference between the observed and the computed tide increase gradually towards upstream, practically absent at Verdon and is maximum at Pauillac. Comparison of tidal elevations between 2 bank sides of the estuary during low river discharge rates ($189 \text{ m}^3 \cdot \text{sec}^{-1}$), shows that small lateral variations in tidal propagation are absent (Fig. 4a). The model cannot fully describe the 2 dimensional aspects of flow. Fig. 4(b) represents the surface elevations as a function of time along the abscissa of the Gi-

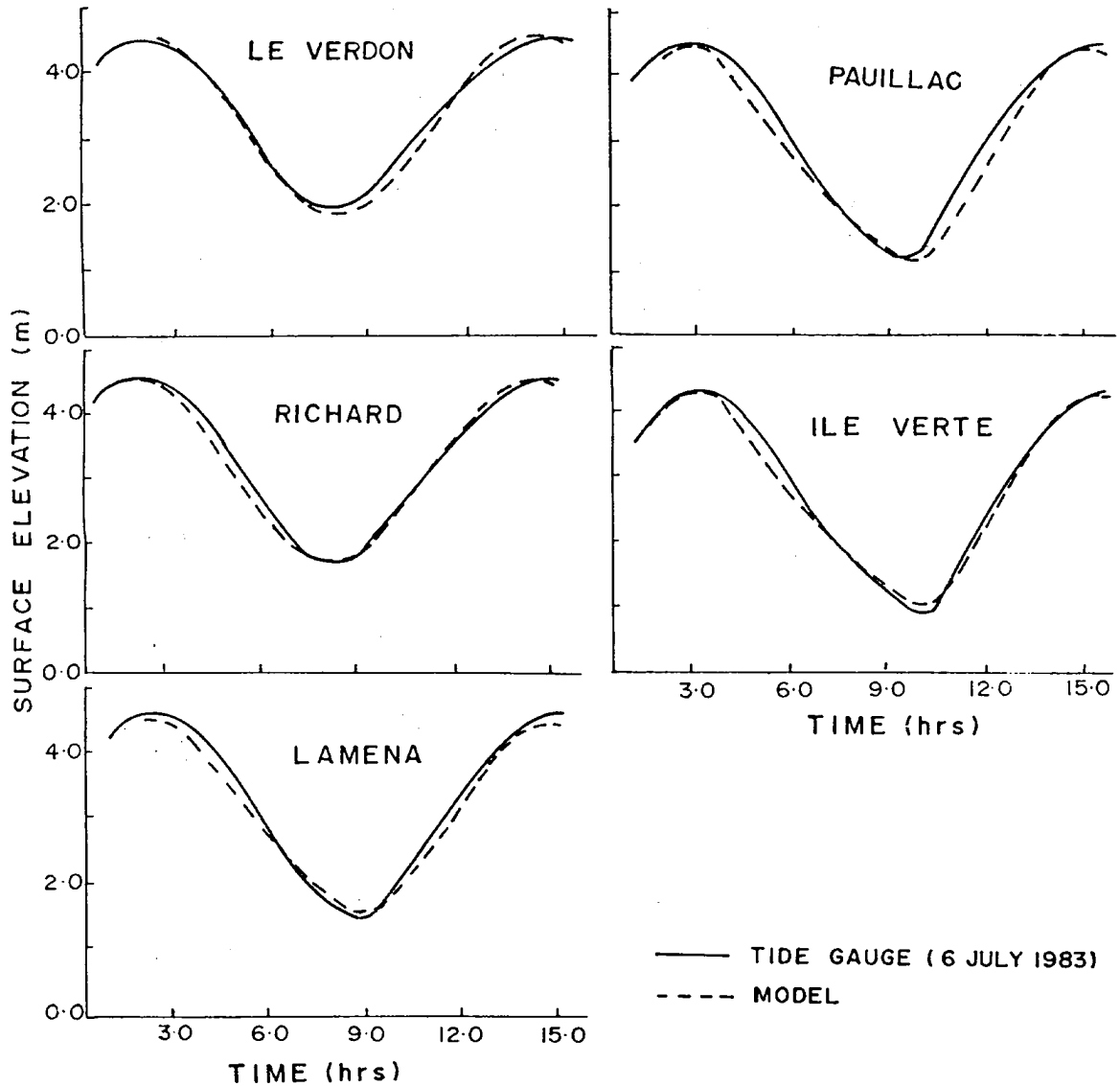


Fig. 3-Comparison between observed and computed water surface elevations at different places

ronde during high river flow rates ($1300 \text{ m}^3 \cdot \text{sec}^{-1}$). For those points situated in the upper estuary, these curves get shifted upwards compared to Fig. 4(a), though there is not much change in tidal ranges. That is, during high river flow rates, both high tide and low tide are higher than during low river flow rates.

An example of current vector diagrams is shown in figure 2. The results are shown at four hours before high tide at the mouth (Grave). The slack occurs downstream of Pauillac. This result is coherent with the results of a 2 dimensional mod-

el⁵ of the Gironde. A comparison of computed currents with *in situ* measurements is not possible. It is not possible to determine a resultant current vector representing a polygon, since the flux of water entering through any side of a polygon depends on the velocity as well as the area of cross section.

The above described model is a relatively simple one for computing tides in estuaries. The method is flexible for taking into account effect of tidal flats. The computed water surface elevations are accurate when compared with those observed.

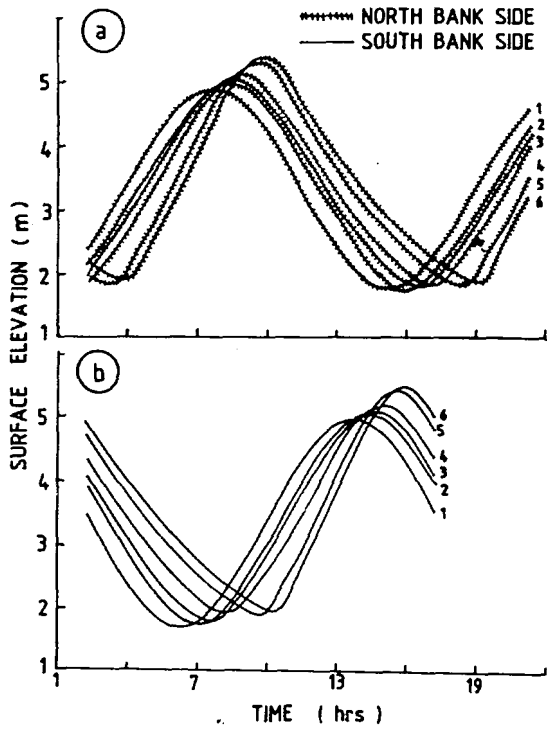


Fig. 4—(a) Comparison of surface elevations between 2 bank sides of the estuary and (b) surface elevations along the axis of the estuary during high river flow rates (1, Grave; 2, Richard; 3, Lamena; 4, St. Christoly; 5, Pauillac; and 6, Ile Verte)

The model is also useful in predicting the hours of slack and the places at which they occur.

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