

Mutual interaction effects between array elements at different wavelengths in a coaxial circular array

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An analysis of the input impedances as well as excitation velocities of the 24 radiators in a coaxial circular array was carried out at different operating wavelengths suitable for underwater transducer application. Values of mutual impedances and excitation velocities were plotted against radiator number. From the present studies, it is established that, except at the operating wavelength of 7.5λ , the values of mutual impedance and radiation resistance are higher. The highest values of excitation velocities are obtained at the operating wavelength of 7.5λ and compared with others.

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INTRODUCTION

The coaxial circular array is well known for the simplicity of its structure, and some of its properties, including high gain and nearly broadband field patterns at different operating wavelengths, are useful to underwater applications like the echo sounder and subbottom profiler.¹ A suitable combination of radii for a three-turn coaxial circular array was chosen and field patterns were calculated at different operating wavelengths, considering uniform pressure for each radiator. Sherman² emphasized the importance of the acoustic interaction among radiators and stressed the need for working out such effects with suitable mathematical analysis. Wang³ developed an approximate method for calculating the mutual coupling effect between the radiators, which is less accurate but saves computational time. Pritchard⁴ suggested a method to compute the mutual impedances between the radiators that involves a number of iterations and computer time to achieve better accuracy. The present Letter deals with the calculation of the mutual impedances between circular disk-shaped radiators of radius equal to 0.5λ , employing the method of Pritchard along with the present analytical method, which saves computational time while maintaining accuracy. In addition, a study is carried out to observe the change in the excitation velocities with different radiator numbers (Fig. 1) at different operating wavelengths.

I. THEORETICAL ANALYSIS

The 24 radiators are distributed within the array (Fig. 1), which is in the x - y plane. The combinations of radii are $a_1 = 4.0\lambda$, $a_2 = 6.0\lambda$, and $a_3 = 8.0\lambda$, where a_1 , a_2 , and a_3 are innermost, middle, and outermost radii. The complex excitation pressure across the s th radiator is

$$[p_s] = [v_s][Z_s]. \tag{1}$$

Let the reference pressure at the s th radiator be equal to unity. Equation (1) can then be written as

$$[v_s] = [Z_s]^{-1}. \tag{2}$$

Assume Z_{sl} are the complex mutual impedances between the s th and l th radiators of a coaxial circular array, keeping the phase difference between the radiators constant.

The mutual impedance of the s th radiator is

$$Z_s = \sum_{s=1}^N Z_{sl}. \tag{3}$$

Here, N is the total number of radiators and summation is done for radiator No. 1 to N . Hence, Eq. (3) presents the resultant impedance across the s th element, which is the sum of the self-impedance of the s th and the mutual impedances for the s th with respect to the other circular disk elements.

The self-impedances of each circular disk radiator of radius 0.5λ is given,⁴ when $s = l$,

$$Z_{ss} = \rho c \pi a^2 \{ [1 - J_1(2ka)/ka] + jH_1(2ka)/ka \} \Omega \text{ mks}, \tag{4}$$

where a is the radius of each radiator. Here, $J_1(2ka)$ is the first-order Bessel function of arguments $2ka$ and $k = 2\pi/\lambda_0$.

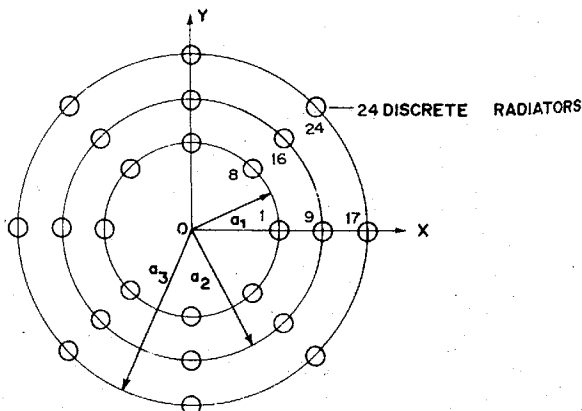


FIG. 1. Geometry of the coaxial circular array.

TABLE I. Self-impedances calculated at different operating wavelengths.

Sr. No.	Operating wavelengths λ	$R_{ss} (\Omega)$	$X_{ss} (\Omega)$	$ Z_{ss} (\Omega)$
1	0.5	3095.36	224.34	3103.48
2	1.0	3224.04	397.24	3248.42
3	1.5	3223.94	1500.04	3555.82
4	2.5	1827.65	2123.69	2801.85
5	7.5	361.31	1029.81	1087.55

and λ_0 is the operating wavelength. Here, $H_1(2ka)$ is the first-order Struve function and ρc is the characteristic impedance of the medium. The values of the self-impedances at different operating wavelengths are presented in Table I.

Mutual impedances Z_{sl} can be presented, when $s \neq l$,

$$Z_{sl} = \rho c (\pi a^2) 2 \sum_{m=0}^{\alpha} \sum_{n=0}^{\alpha} \frac{m+n+0.5}{\pi^{0.5} m! n!} \times (a/d_{sl})^{m+n} J_{m+1}(ka) J_{n+1}(ka) \times h_{m+n}^{(2)}(kd_{sl}) \Omega \text{ mks.} \quad (5)$$

Here, m, n are the number of iterations made, $h_{m+n}^{(2)}$ is the spherical Hankel function of the second kind, and d_{sl} is the distance between the s th and l th radiators. The values of d_{sl} (in terms of λ) can be written as

$$d_{sl} = [(X_s - X_l)^2 + (Y_s - Y_l)^2]^{0.5}, \quad (6)$$

where $X_s = a_c \cos(s-1)\pi/4$ and $Y_s = a_c \sin(s-1)\pi/4$. Here, a_c is the radius of the coaxial circular array structures. Figure 2 shows the plot between $(a/d_{sl})^{m+n}$ [a term in Eq. (5)] to d_{sl} for different $m+n$. This is clear from

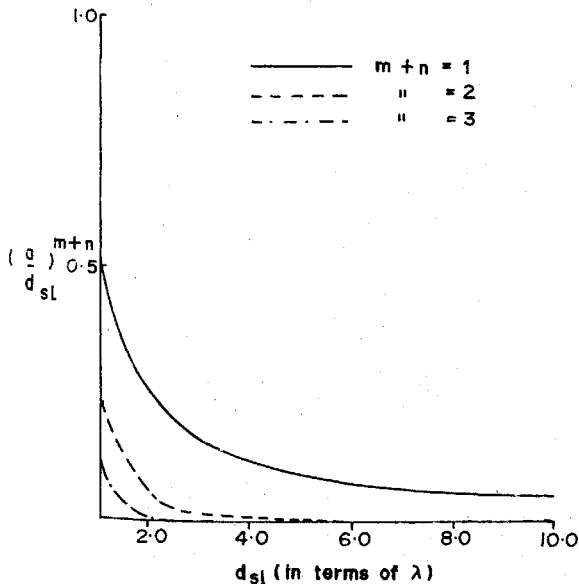


FIG. 2. The $(a/d_{sl})^{m+n}$ for different interelement distances at various $m+n$.

the plot that at $m+n=1$ (i.e., $m=0$ and $n=1$), the values of the $(a/d_{sl})^{m+n}$ are equivalent to 0.167 for $d_{sl} = 3.0\lambda$; beyond that, the values of $(a/d_{sl})^{m+n}$ decrease significantly. Similarly, for higher integer values of $m+n$, $(a/d_{sl})^{m+n}$ becomes negligible, especially after $d_{sl} = 3.0\lambda$. Hence, Eq. (5) is used up to the interelement distance, $d_{sl} = 3.0\lambda$, and the useful number of iterations is reduced to $m+n=1$. According to Skudrzyk,⁴ if the disks are separated by a distance much larger than the disk radius, then Eq. (5) can be rewritten as

$$Z_{sl} = R_{ss} [\sin(kd_{sl})/(kd_{sl}) + j \cos(kd_{sl})/(kd_{sl})] \Omega \text{ mks.} \quad (7)$$

The values of R_{ss} for different wavelengths are given in Table I.

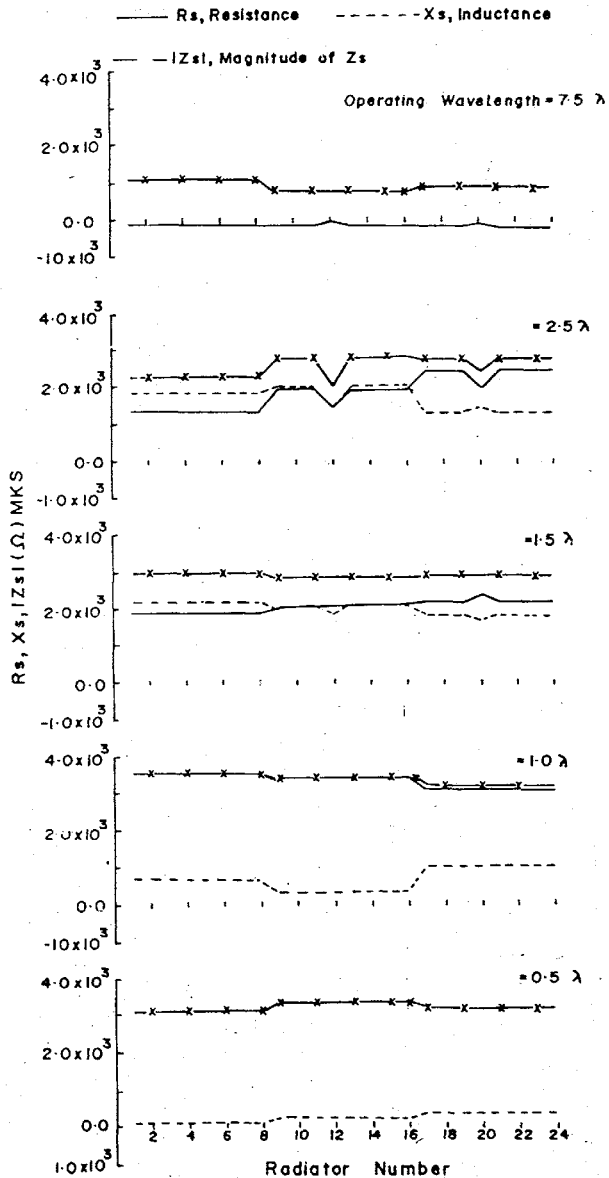


FIG. 3. Radiation resistance, inductance and magnitude of impedances for different radiators at different operating wavelengths.

II. DISCUSSION

The characteristic impedance of seawater ρc is considered to be equivalent to 1.54×10^6 rayls. The design wavelength λ is assumed to be equal to 0.05 m. So the radius of the disks is 0.025 m for our calculation.

It is clear from Table I that the values of self-radiation resistance (R_{ss}) decrease with operating wavelength and the changes in R_{ss} are less, specifically up to 1.5λ , though a low value of R_{ss} is obtained at 7.5λ . Similarly, reactance (X_{ss}) is lowest at 0.5λ , but maximum at 2.5λ and then decreasing at 7.5λ .

The computations for mutual impedances are obtained by Eqs. (3), (5), and (7) and subsequently presented in Fig. 3. It is seen from this figure that the values of impedances are nearly constant for the transducer elements of the same radius, which, in turn, depends on operating wavelength.

Figure 3 also gives an idea of having higher values of radiation resistances (R_s) up to 2.5λ with little undulations, but the highest values of R_s are obtained for an operating wavelength of 1.0λ ; again, the lowest values are obtained at 7.5λ . Similarly, low values of reactances (X_s) at 0.5 and 1.0λ , as well higher values of X_s at 1.5 and 2.5λ , are also observed. It is to be pointed out that at an operating wavelength of 7.5λ , the values of X_s show slightly higher values compared to wavelengths of 0.5 and 1.0λ . Therefore, it is concluded that X_s increases with operating wavelengths up to 2.5λ and again decreases at 7.5λ .

Figure 4 shows the magnitude of complex excitation velocity (IvI) for radiators at different operating wavelengths. The change in IvI up to 2.5λ is not very significant but reaches a maximum at the lowest frequency (7.5λ). The high variation of IvI causes interaction effects at 7.5λ , which can be controlled by means of a suitable network. These effects are not very significant at other operating wavelengths.

III. CONCLUSIONS

An observation is made to determine the frequency dependence of the self-impedance of circular disk radiators of radius 0.5λ . Suitable analytical method has been employed to calculate the mutual impedances between the array elements. The result shows that the higher values of radiation resistance and mutual impedance between the radiators within an operating wavelength of 2.5λ are not significant at an operating wavelength of 7.5λ . Except at the operating wavelength of 7.5λ , no remarkable variations are observed in the values of excitation velocities with radiator number. This suggests that an additional appropriate electrical network

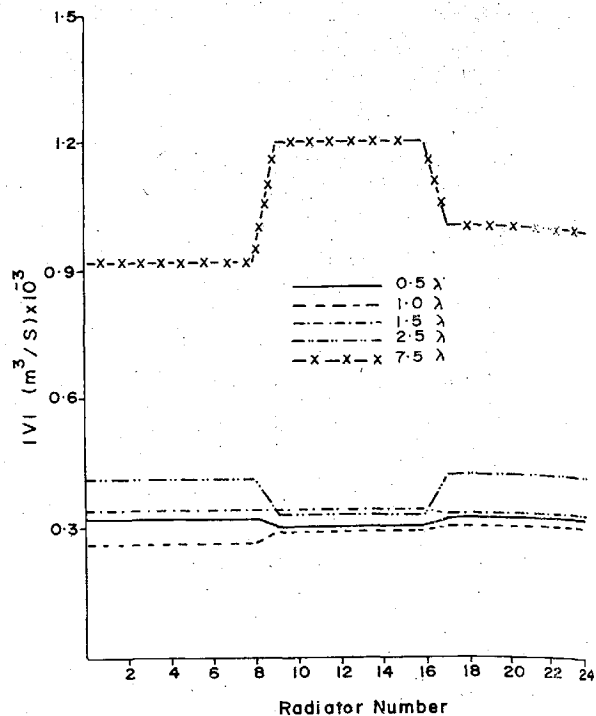


FIG. 4. Magnitude of excitation velocities with respect to radiator number at different operating wavelengths.

for suitable excitations and reception of each element of an array can be used to overcome the effect of interaction at 7.5λ .

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