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TIDES

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Study on tides in the ocean started since Galeilio. However, it was Newton and Laplace who developed different theories on the generation of tides. Before we discuss the various causes of tide generation in the ocean, it is to be noted that the interest of an oceanographer lies in the study of the magnitudes of tidal oscillations, their role on the circulation, modifications of tidal waves occurring when they approach shallow water regions, etc.

Tide generating potential:

The tidal oscillations in the ocean are caused due to the resultant force between the gravitational force exerted by an astronomical body such as the moon or sun and the centrifugal force due to the rotation of the earth. For the sake of simplicity, let us consider only the moon. Let the mass of the earth be unity and the mass of the moon be m .

At the point A on the surface of the earth, gravitational force is $\frac{m}{(\gamma-p)^2}$. Centrifugal force is same everywhere on the surface of the earth and it is equal to m/γ^2

$$\text{Resultant force is } \frac{m}{(\gamma-p)^2} - \frac{m}{\gamma^2} \sim \frac{2mP}{\gamma^3}$$

Similarly, at the point B, the resultant force is $\frac{m}{(\gamma+p)^2} - \frac{m}{\gamma^2}$ which is equal to $-2mP/\gamma^3$. The resultant force at B is same in magnitude but opposite in direction to that of A. Similarly, the tide generating force can be found at other points on the surface of the earth also. These forces can be schematically shown as in Fig. 2.

It is to be noted that the occurrence of high tide in a given region takes place after some delay of the passage of moon or sun, because tidal waves propagate at speeds less than that of the earth's rotation. (In shallow water, waves propagate at a speed approximately equal to \sqrt{gh} , where h is the depth).

The sun also produces gravitational attraction. The tide generating force of the sun is about 0.45 times that of the moon. Closest planets are either too small and other planets are too far to produce any effect.

Inequalities in tidal ranges

Mainly, there are four important inequalities in tidal ranges. (i)

Semimonthly inequality - About one or two days after new moon or full moon, tidal range reaches maximum. The delay in the occurrence of maximum tidal range with respect to new moon or full moon is referred to as 'age of tide'. (ii) Monthly inequality - The tidal range becomes particularly high after some delay, when the moon passes through perigee (point of the moon's orbit closest to earth) and after some delay of the moon's passing through apogee (point of the moon's orbit most distant from the earth), tidal ranges are small. (iii) Declinational inequality - The axis of the earth is inclined $23^{\circ}30'$ to the vertical. The plane of the moon's orbit is tilted by about $5^{\circ}9'$ with respect to the plane of the ecliptic. After the greatest northern and southern declinations of the moon ($28^{\circ}N$ and $28^{\circ}S$), tidal range is small and after the moon's passage over the equator, a relatively large tidal range occurs. The time of passing between the greatest northern and southern declination of the moon gives the period of the declinational inequality. It is about 13.66 days (half tropical month). (iv) Diurnal inequality - The difference in height of successive high and low water reach their maximum with a certain delay after occurrence of the greatest northern or southern declination of the moon. They disappear with the same delay after the lunar transit of the equator. This inequality depends also on the sign of the declination of the moon. Its period is given by the full tropical month of 27.32 days.

Tidal constituents

From the relative positions of moon, sun and the earth, one can arrive at a complete picture of tides by introducing a series of fictitious heavenly bodies that will each bring about a symmetrical field of tide producing forces and by considering the total field as composed of all of these partial fields. Each of these partial field corresponds to a partial tide or tidal constituent. Some of the most important constituents having semidiurnal period are M2 (principal lunar) and S2 (principal solar). The most prominent diurnal constituents are O1 (lunar diurnal) and K1 (luni solar diurnal).

Tidal measurement and analysis

Nowadays, self recording electronic tide gauges are used for this purpose. A graduated pole, often referred to as tide pole can also be used. During calm conditions, an accuracy of less than 5 cm can be obtained in surface elevations with the help of a tide pole. The reference level for the tidal heights is the lowest low water mark in that particular location.

A tidal record of sufficiently long duration (for instance, 14.77 days duration are required to separate M2 and S2) can be analysed for the individual constituents by a procedure called harmonic analysis. In harmonic analysis, a tidal function can be fitted as

$$T(t) = Z_0 + \sum H_n f_n \cos[\sigma_n t - g_n + (v_n + u_n)]$$

If $O(t)$ is the observed tide, $R(t) = O(t) - T(t)$. The procedure is to minimise $\sum R^2(t)$. Z_0 is the mean sea level H_n and g_n are the amplitude and phase lag of astronomical forcing of the n th constituent. v_n adjusts the phases to allow for astronomical conditions at the time of origin of the data.

The results of harmonic analysis are used to prepare tidal charts. They

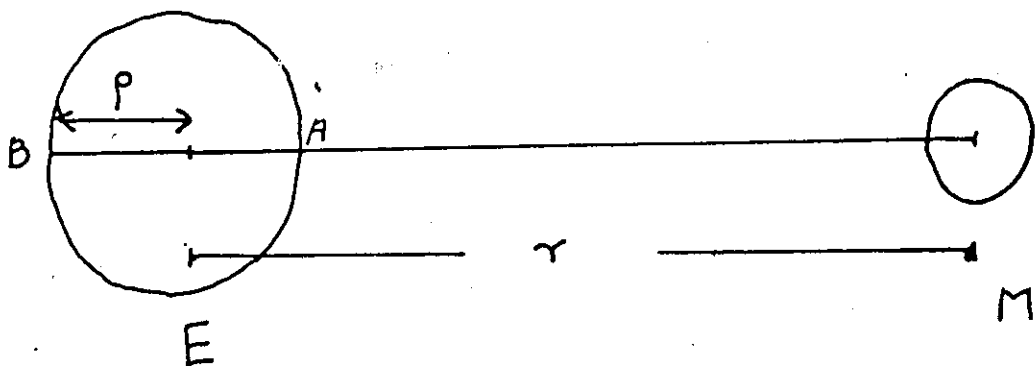


Fig. 1

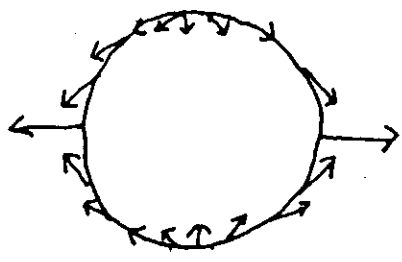


Fig. 2

are represented in the form of co-tidal lines (equal phase) and co-range lines (equal tidal range).

Tides in estuaries

A tidal wave entering into shallow water regions, undergoes modifications. In an estuary, when the tidal wave propagates upstream, due to the shallowing of the bottom, the duration of ebb increases, while the duration of flood decreases, with the period remaining constant. Accordingly, modifications in the pattern of tidal currents also occur. These effects can be studied by means of a numerical model. Usually, a two dimensional vertically integrated model is used for this type of study. The equations used are given below. The equations (1) and (2) are the equations of momentum in x and y directions and (3) is the equation of continuity.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv + g \frac{\partial \zeta}{\partial x} + \frac{g u (u^2 + v^2)^{1/2}}{C^2 H} + \frac{\tau_{sx}}{PH} + \epsilon \Delta u = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu + g \frac{\partial \zeta}{\partial y} + \frac{g v (u^2 + v^2)^{1/2}}{C^2 H} + \frac{\tau_{sy}}{PH} + \epsilon \Delta v = 0 \quad (2)$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} (Hu) + \frac{\partial}{\partial y} (Hv) = 0 \quad (3)$$

where u and v are the components of velocity, ζ is the surface elevation, C is the coefficient of Chezy, h is the water depth, f is the Coriolis parameter, ϵ is the coefficient of pseudo viscosity and τ_{sx} and τ_{sy} are the wind stress components.
 $H = \zeta + h$

It is to be noted that $g \frac{\partial \zeta}{\partial x}$ and $g \frac{\partial \zeta}{\partial y}$ terms are at least one order of magnitude higher than $u \frac{\partial u}{\partial x}$ and $v \frac{\partial v}{\partial y}$ terms. These terms decide the magnitudes of u and v. These equations cannot be solved analytically, since there are non-linear terms in the momentum equations and non-linearity in the continuity equation. However, they can be solved numerically, mostly done by a finite difference technique. In the present discussion, only a brief outline of the procedure will be made. A finite difference mesh of about, say, 1 Km grid size is constructed with the coastline represented by a stair step boundary. A simple open boundary condition will be of the form $\zeta = M + A \sin(\omega t + \phi)$, where M is the mean tide, A the amplitude and ϕ is the phase. At the upstream side, river discharges are used to prescribe the boundary conditions. The model can be used for calculating tides, tidal currents, tidally averaged residual currents, etc.