

WAVE HINDCASTING/FORECASTING: THEORY

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INTRODUCTION

The term wave hindcasting means prediction of waves based on historical wind data; forecasting is the prediction of waves based on real time wind data. But meteorologists generally use the term wave forecasting and wave hindcasting as wave predictions.

Sverdrup and Munk (1947) developed a wave prediction method based on the significant wave height approach. This is the first method developed and widely being used, largely because of its simplicity and efficiency. The application of spectral concept is incorporated in the early 1950's. The widely accepted energy balance equation is introduced in 1960. The source functions of the energy balance equations represent the process of wave generation, wave breaking, dissipation and nonlinear wave-wave interaction. The JONSWAP (Joint North Sea Wave Project) experiment (1973) shows the importance of the nonlinear wave-wave interaction in the energy balance equation. Based on energy balance equation and nonlinear interaction, many wave models are developed suitable to particular region. This paper describes the ocean wave formation in the form of numerical expressions of energy spectrum and wave prediction theory.

BASIC WAVE THEORY

The wave can broadly be classified into two categories, depending on the ratio of the wave length (L) to the mean water depth (h). If the ratio L/h is much greater than one ($L/h \gg 1$), we call it as *long wave*. Tides and flood-waves in the rivers are examples of long waves. If L/h is not much greater than one, we call it as *gravity wave* or *short wave*. Wind generated waves and ship-waves are examples of gravity waves.

There are certain differences in the properties of long waves which have led to different mathematical theories, for which reasons the two categories are usually treated separately. The present study is restricted to gravity waves. The wind generated waves are more complicated in

structure and appearance. Their description requires spectral and statistical methods, which are covered in the section 'Ocean wave prediction theory'. The mathematical theory of sinusoidal progressive waves is the basis for understanding of wave phenomena (available in any hydrodynamics book).

WIND GENERATED OCEAN WAVES

In the description of ocean waves it is useful to distinguish three time scales. In the first time scale, the sea surface can be described as a stationary Gaussian with zero-mean process. In the second time scale, the wave characteristics are considered as slowly varying functions of time and place. Here the waves travel for a period of few days. The third time scale is very large, this is required for the design of structures which protect large communities from the sea. Initially the waves are considered as a stationary Gaussian zero-mean process. The relationship between sea surface elevation and wave spectrum are explained below.

Description of Ocean Waves

The sea surface elevation of a time scale of about one hundred characteristic wave periods can be expressed as a stationary Gaussian process with zero-mean. This implies that the joint probability of the sea surface elevation $\mu(t)$ at t arbitrarily chosen different moments is fully described with the auto-covariance function $c(\tau)$ which is equal to $\langle \eta(t) \eta(t+\tau) \rangle$.

The theoretical basis for equating the sea surface elevation to a Gaussian process is the central limit theorem which states that sum of a large number of statistically independent variables is Gaussian distributed. In the case of ocean waves, the sea surface elevation η at a moment t is considered to be the sum of an infinite number of fourier components, each with its own frequency and direction. Each of these components is generated independently from the other components. They are therefore statistically independent in origin and remain independent during their journey across the ocean because they travel independently according to linear wave theory. If the waves become very steep (eg, in a severe storm or in a very shallow water), the central limit theorem no longer applies due to nonlinear effects. Here the auto-covariance function $c(\tau)$ describes the sea surface elevation completely in a statistical sense. The variance density spectrum can be expressed as

$$E'(f) = \int_{-\infty}^{\infty} c(\tau) e^{-2\pi i f \tau} d\tau \quad (1)$$

This spectrum has an additional advantage over the auto-covariance function for ocean waves and many other physical phenomena; the hydrodynamic processes are well described in time of fourier components. As the relationship (Eq. 1) is reversible,

$$c(\tau) = \int_{-\infty}^{\infty} E'(f) e^{2\pi i f \tau} df \quad (2)$$

and no information is lost in the transformation, indicating that the spectrum contains exactly the same information as the auto-covariance function, only the format of the information is changed. In the field of ocean wave theory it is conventional to define a spectrum $E(f)$ slightly different from $E'(f)$:

$$E(f) = \begin{cases} 2E'(f) & \text{for } f \geq 0 \\ 0 & \text{for } f < 0 \end{cases} \quad (3)$$

The variance $\langle \eta^2 \rangle$ of the sea surface elevation is then expressed as

$$\langle \eta^2 \rangle = c(0) = \int_0^{\infty} E(f) df \quad (4)$$

which indicates that the spectrum distributes the variance over frequencies. $E(f)$ should therefore be interpreted as a variance density

$$\int_{f_1}^{f_2} E(f) df$$

equal to the contribution of the frequencies between f_1 and f_2 to the total variance $\langle \eta^2 \rangle$. The dimension of $E(f)$ is m^2/Hz if the elevation is given in m and the frequencies in Hz .

The variance $\langle \eta^2 \rangle$ is equal to the total energy E_{TOT} of the waves per unit surface area if multiplied with a properly chosen coefficient.

$$E_{\text{TOT}} = \frac{1}{2} \rho g \langle \eta^2 \rangle \quad (5)$$

In many wave problems it is not sufficient to define the energy density as a function of frequency alone. It is sometimes required to distribute the wave energy over direction θ as well. Then the wave energy

over frequencies and directions may be denoted with $E(f, \theta)$. It will suffice to state that the integral

$$\int_{f_1}^{f_2} \int_{\theta_1}^{\theta_2} E(f, \theta) df d\theta$$

is equal to the contribution to the total variance $\langle \eta^2 \rangle$ of the components with frequencies between f_1 and f_2 and travelling in directions between θ_1 and θ_2 . This is analogous to the interpretation of $E(f)$ (Fig. 1). As the total energy density at a frequency f is distributed over the directions θ in $E(f, \theta)$ it follows that

$$E(f) = \int_0^{2\pi} E(f, \theta) d\theta \quad (6)$$

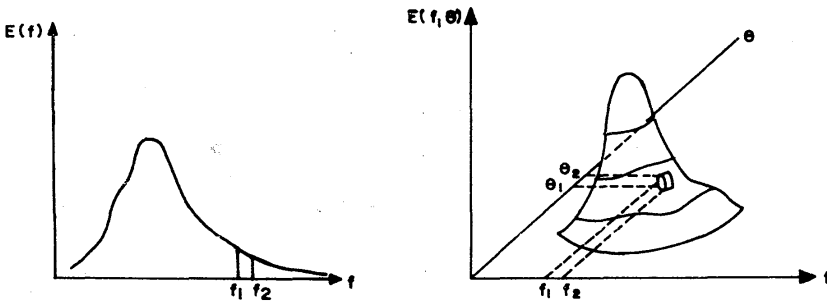


Fig. 1 Contribution of wave components on (f_1, f_2) and (θ_1, θ_2) to the total variance.

Observation of Waves

Visual observations are still being carried out in many situations. Usually only two parameters are obtained from these observations, the significant wave height H_s and significant wave period T_s . If observations of waves are to be used in quantitative manner, it should be carried out with instruments whenever possible. The measurement of sea surface elevation at one horizontal position is usually carried out with a wave gauge or a buoy. The measurement with a wave gauge is commonly based on an electrical resistance measurement. Two wires are run through the sea surface to a certain depth below the water surface. The electrical

current through the wires encounters a resistance, which can be measured and the result is stored on an analog or digital data carrier (eg. paper tape or magnetic tape). The measurements with the waverider buoy are based on accelerations, which are integrated twice thus reproducing the sea surface elevation as a function of time. A radio transmitter on-board the buoy sends the data to a shore-based station where the data are stored on a paper tape or magnetic tape.

The time series thus obtained are analyzed to estimate the energy density spectrum $E(f)$. This procedure is based on the Fast Fourier Transform (FFT), which indicates the conventional values for the spectral resolution, the Nyquist frequency and the statistical reliability of the spectral estimates. These values are determined from desired characteristics on the one hand and practical considerations on the other. High resolution combined with high statistical confidence of the estimates require long periods of observations during which the waves should be stationary. In practice, a wave record is between 15 and 30 min long. This length is primarily based on assumed stationariness of the wave field. Generally a resolution of about 0.01 Hz is chosen (actually 1/128 Hz due to the fact that most FFT procedures require series with a number of samples equal to a power of 2). The Nyquist frequency is usually 0.5 Hz or 1.0 Hz corresponding to a sample interval in time of 1.0 to 0.5 s.

The methods to determine the two dimensional spectrum $E(f, \theta)$ are mostly based on the observation of a limited number of spatial characteristics of the sea surface. A method which is used is the measurement of the slope of the surface in two orthogonal directions with pitch-and-roll buoy. It is possible with such a buoy to determine at each frequency a mean direction and a directional spreading of the wave energy.

With the marketing of a pitch-and-roll version of the waverider (ie, WAVEC) these observations may well become routine. More advanced methods which provide details of the two-dimensional spectrum, require sophisticated equipments (eg. stereophotography and radar). These advanced methods are not yet suited for routine observations.

OCEAN WAVE PREDICTION THEORY

Many techniques are available to forecast or hindcast waves, varying from simple rules to highly complex computer models. All models consider the generation of waves by wind and the propagation of waves over the ocean. Based on the meteorological information, the prediction or hindcasting of wave conditions can be made by two methods. The first is one where the wind field and the wave field can be completely described with only a few parameters in a generally accepted standard situation. The relationships between the wind parameters and the wave parameters in this situation are relatively well known from observations. In the

second method, the wave characteristics can be determined in a wind field varying arbitrarily in time and place. This gives more reliable results.

Wave Prediction in the Standard Wind Field

The wave growth is best described in an ideal situation where a constant wind (U) blows perpendicularly off a long and straight coast over deep water. The most obvious development in the wave field is the increase of significant wave height (H_s) and the significant wave period (T_s) with increasing distance from the coast (fetch, F). Initially the wave growth is quite rapid but the rate of growth decreases as the waves grow higher and after some duration the waves do not grow anymore, then the waves are said to be fully developed. The empirical relationship between the wind parameters and the wave parameters are expressed in terms of a dimensionless fetch $\tilde{F} (=gF/U^2)$, a dimensionless wave height $\tilde{H} (=gH_s/U^2)$ and a dimensionless wave period $\tilde{T} (=gT_s/U)$. The observed dependencies can be approximated analytically with

$$\tilde{H} = \tilde{H}_{\infty} \tanh(k_1 \tilde{F}^{m_1})$$

$$\tilde{T} = \tilde{T}_{\infty} \tanh(k_2 \tilde{F}^{m_2})$$

Expressions which fit well to most of the available data are given in the Shore Protection Manual (CERC, 1973):

$$\tilde{H} = 0.283 \tanh(0.0125 F^{0.42}) \quad (7)$$

$$\tilde{T} = 7.54 \tanh(0.077 F^{0.25})$$

A more advanced description of the wave field is based on the notion that the undulating sea surface is composed of a large number of individual wave components, each characterised by a frequency f and a direction θ . The two-dimensional spectrum $E(f, \theta)$ gives the distribution of wave energy over the frequencies and directions. In the above ideal situation the frequency spectrum, obtained by integrating the two-dimensional spectrum over all directions, develops from high frequencies to the low frequencies (Fig. 2). During an extensive and detailed study of the wave growth (JONSWAP: Hasselmann et al., 1973), it was found that the shape of the spectrum was constant during most of the growth of the waves, and only in fully developed stage does the peak flatten somewhat. To formulate this shape in the analytical expression, the investigators in JONSWAP adapted an expression which was formulated in an independent study of the fully developed sea state:

$$E(f) = E'(f) \gamma(f) \quad (8)$$

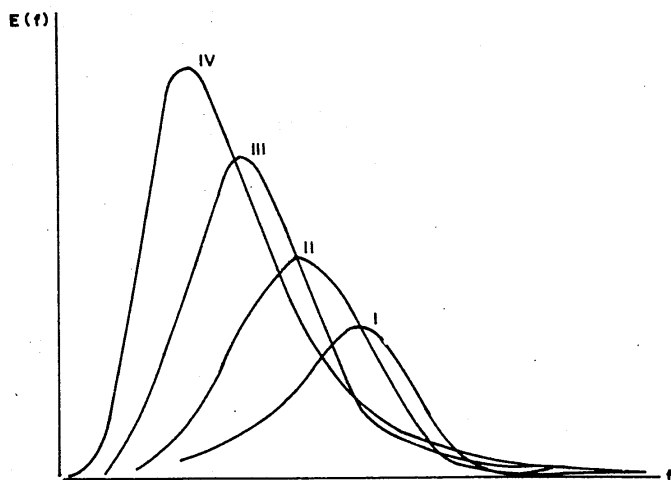


Fig. 2 The development of the spectrum in the ideal wind field.

where $E'(f)$ is a spectrum with the shape of the fully developed spectrum proposed by Pierson and Moskowitz (1964) given in Fig.3, but with arbitrary values for the energy scale coefficient α and the frequency f_m :

$$E'(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp \left[-\frac{5}{4} \left(\frac{f}{f_m} \right)^{-4} \right] \quad (9)$$

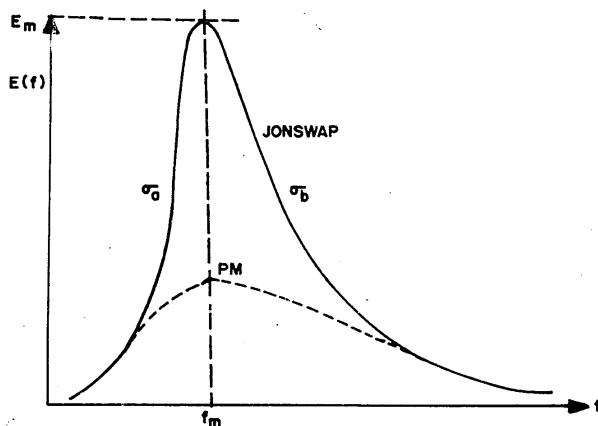


Fig. 3 Shapes of the P-M spectrum and the mean JONSWAP spectrum.

As the spectra of the growing waves were much more peaked than the Pierson-Moskowitz (P-M) spectrum for fully developed waves, it was necessary to use a peak enhancement function $\gamma(f)$ given by:

$$\gamma(f) = \gamma \exp \left\{ \frac{-(f-f_m)^2}{2 \sigma^2 f_m^2} \right\} \quad (10)$$

where γ is the maximum enhancement at the peak frequency and σ is the width of this function. The shape of the P-M spectrum is equal to that of the JONSWAP spectrum for $\gamma=1$. From the observation in JONSWAP (1973)

$$\begin{aligned} \gamma &= 3.3 \\ \sigma &= \begin{cases} \sigma_a = 0.07 & \text{for } f < f_m \\ \sigma_b = 0.09 & \text{for } f \geq f_m \end{cases} \end{aligned} \quad (11)$$

The scale parameters were observed in JONSWAP and the best-fit expressions through observations are:

$$\begin{aligned} \alpha &= 0.076 \tilde{F}^{-0.22}; \quad \tilde{f}_m = 3.5 \tilde{F}^{-0.33} \\ &\text{for } F < 10^4 \text{ (young sea state)} \end{aligned} \quad (12)$$

where the dimensionless peak frequency \tilde{f}_m is defined as

$$\tilde{f}_m = U f_m / g$$

The values of α and \tilde{f}_m for the P-M spectrum are

$$\begin{aligned} \alpha &= 0.0081; \quad \tilde{f}_m = 0.13 \text{ and } \gamma = 1.0 \\ &\text{for } F > 10^5 \text{ (fully developed sea state)} \end{aligned} \quad (13)$$

For many engineering problems, the frequency spectrum gives sufficient wave information but many problems require additional directional information as provided by the two-dimensional spectrum $E(f, \theta)$. The two-dimensional energy spectrum $E(f, \theta)$ was constructed from Eq. 9 by assuming a fixed cosine square spreading factor defined relative to the local wind direction:

$$E(f, \theta) = \begin{cases} E(f) \frac{2}{\pi} \cos^2(\theta - \theta_w) & \text{for } (\theta - \theta_w) \leq \frac{\pi}{2} \\ 0 & \text{for } (\theta - \theta_w) > \frac{\pi}{2} \end{cases} \quad (14)$$

Wave Prediction in a Varying Wind Field

In this method, the sea surface is considered to be the sum of a large number of individual wave components, each of which propagates over the ocean surface with a constant frequency in accordance with linear wave theory. To determine the energy density of one such component at one time and one place, conventional computations are used to determine the path of wave component from its origin (usually a coastline) to the point of interest. After leaving the coast, the wave component encounters the influence of wind, bottom, wave breaking and also wave-wave interactions. By considering at regular intervals the energy gained or released by wave component, one can determine its evolution till it reaches the point of forecast. This procedure can be repeated for a larger number of wave components of different frequencies and directions which finally arrive at the point of forecast at a given time. This process can be carried out for a large number of different times and locations, thus providing the final two-dimensional wave spectrum as a function of time over the entire ocean.

Spectral approach

The above process, the evolution of the surface wave field in space and time, is based on the transport or energy balance equation for

$$\frac{dE(f, \theta, \vec{x}, t)}{dt} = S(f, \theta, \vec{x}, t) \quad (15 a)$$

or

$$\frac{\partial E(f, \theta, \vec{x}, t)}{\partial t} + c_g(f, \theta) \cdot \nabla E(f, \theta, \vec{x}, t) = S(f, \theta, \vec{x}, t) \quad (15 b)$$

where E is the energy density of the wave field described as a function of frequency f , direction θ , position \vec{x} and time t . c_g is the deep water group velocity and S is the source function. The source function represents three types of energy transfer (Fig. 4) :

$$S = S_{in} + S_{nl} + S_{ds} \quad (16)$$

where S_{in} represents the energy input from the atmosphere, S_{nl} the transfer from one wave component to another (nonlinear interaction) and S_{ds} represents processes that serve to dissipate wave energy.

The growth of the energy density is described in the existing models for wave prediction as the sum of a linear growth and an exponential growth. For linear growth, Phillips (1957) explained the initial generation of gravity waves on an undisturbed sea surface through a resonant exci-

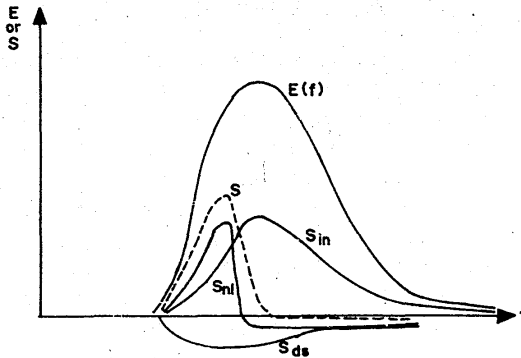


Fig. 4 Source functions responsible for energy transfer.

tation by incoherent atmospheric turbulent pressure fluctuations convected by the mean wind. This theory is commonly referred to as the resonance theory of Phillips. The corresponding source term is (as part of S_{in}) a constant $dE/dt = \alpha'$ for a given combination of frequency, direction and wind speed.

The exponential growth may be contributed by the following processes:

- (a) A feed-back mechanism proposed by Miles (1957) — the wave induced pressure differences across a non-breaking wave crest (no flow separation) transfers energy from the wind to the wave component at a rate proportional to the energy density of the wave component;
- (b) A sheltering mechanism first proposed by Jeffreys (1925) — wind energy is transferred to the waves by wave induced pressure differences across a breaking wave. In this situation, flow separation occurs and the energy transfer is more effective than in the mechanism proposed by Miles (1957). But this mechanism has not been considered seriously for a long time due to the negative results of old experiments. Later on experiments by Banner and Melville (1976) have revived the interest in this mechanism;
- (c) Nonlinear energy transfer between wave components — this is a complex mechanism where groups of wave components interact with single wave components. During the exponential growth the effect for a wave component gives a net gain of energy. The nonlinear wave-wave interaction has been studied theoretically by Hasselmann (1962 and 1963).

The above first two mechanisms are not considered explicitly in existing wave prediction. The third mechanism is included only in a few

advanced models in a parameterized form. The exponential growth is therefore usually expressed as

$$\frac{dE}{dt} = B.E \quad (17)$$

where B is determined from observations. To stop the energy density from growing beyond a certain limit E (P-M spectrum limit), the growth is truncated by multiplying the source function by γ :

$$\frac{dE}{dt} = (\alpha + BE) \gamma \quad (18)$$

with $\gamma = 1$ for $E = 0$ and $\gamma \rightarrow 0$ when $E \rightarrow E_{\infty}$. The factor γ represents the dissipative mechanism (S_{ds}) of wave breaking. In the final stage, the energy density is a constant and is determined by a balance between energy input (from wind and nonlinear interaction) and energy dissipation due to wave breaking.

To obtain a review of the energy transfers, consider first the growth of a number of frequencies from one direction (Fig. 5). At lower frequencies (f_5), the wave energy is still increasing. At high frequencies the energy density has already obtained the saturation level where the energy input is balanced by energy dissipation. The overall picture for the spectrum at one moment in time is shown in Fig. 6. In shallow areas of the ocean, bottom effects are included in the models by adding a term for

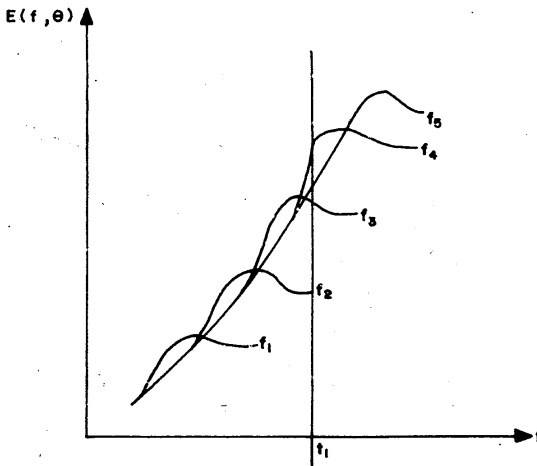


Fig. 5 The wave growth in constant wind field.

bottom dissipation to the source function and by using conventional wave refraction and shoaling theory to account for bottom effects on the energy propagation.

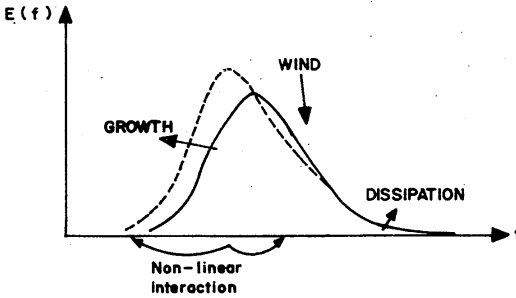


Fig. 6 The energy balance of a wave spectrum in a constant wind field.

Parametric approach

The parametric models, introduced by Hasselmann et al. (1973 and 1976), assume that the spectrum can be given an analytical shape defined by five parameters ($\alpha, f_m, \gamma, \sigma_a, \sigma_b$). These models highlight the role played by the nonlinear wave-wave interaction in the distribution of energy across the spectrum. One important effect of the nonlinear wave-wave interaction is that they tend to force the shape of the spectrum into a universal shape. From numerical experiments and observations it appears that this shape is close to that of the mean JONSWAP spectrum ($\gamma = 3.3, \sigma_a = 0.07, \sigma_b = 0.09$); ie, if the spectrum has a shape different from the universal shape, the nonlinear wave-wave interactions will tend to remove these differences. Hasselmann et al. (1980) explained that the nonlinear interactions also impose a more or less universal relationship between α and f_m in a varying wind field. The wave forecast is thus reduced from predicting a large number of individual wave components to the prediction of one spectral scale parameter. It should be emphasized that this approach is only applicable during wave growth. If the wind speed decreases below the phase speed of the wave components or if the wind direction differs markedly from the main wave direction, the waves do not grow and nonlinear interaction disappears. The wave model should then only transport the swell energy over the ocean surface. Therefore a parametric wave model should always be combined with a pure swell model. The combination is called a coupled hybrid model.

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