

# Role of “Effective Density” in Improving the Accuracy of Bottom-Pressure based Sea Level Measurements — A Case Study from Gulf of Guinea

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**Abstract** - Seasonal observations of a set of bottom pressures and concurrent tide-staff measurements from Takoradi harbour located in the Gulf of Guinea indicated that the effective depth-mean density value,  $\rho_{\text{eff}}$  of this clear water body is less than ( $\approx 0.25\% - 1.84\%$ ) its bulk density,  $\rho_b$  obtained from precision density measurements made on discrete water samples. Thus, use of  $\rho_b$  to estimate sea level elevation yields an underestimation of this order in sea level measurements. Although, this effect was observed in association with both wind and rainfall, the comparative reduction of  $\rho_{\text{eff}}$  value was more prominent during the period when rainfall was strong. Presence of microbubbles introduced into the upper layers of the sea water, as a result of turbulence induced by wind forcing and impact of rain drops, is suspected to be responsible for the observed effective reduction in the in situ density of this shallow water body. The accuracy of bottom-pressure based sea level measurements can be improved by introducing  $\rho_{\text{eff}}$  value into sea level estimation. It is shown that  $\rho_{\text{eff}}$  value can be estimated by using a statistically derived simple linear model, which is constructed from a set of bottom pressures and concurrent tide-staff measurements.

## I. INTRODUCTION

Conventionally, water density is used to convert bottom pressure measurements to sea level measurements. However, Joseph *et al.*, [1, 2] reported inaccuracies creeping into such sea level records, particularly from shallow coastal turbid water bodies. These inaccuracies arise primarily from a combination of various poorly understood site-specific influences such as turbulence, micro-bubbles, suspended particulate matter, etc., which render the effective density of the water body to be lower than that obtained from conventional measurements using CTD or precision density meter. The objective of the present study is to incorporate the use of ‘effective density’ parameter into bottom pressure based sea level measurements by exploring efficient use of seasonally made tide-staff measurements

Bench-mark leveled tide-staff can be found in every harbor. The Inter-governmental Oceanographic Commission (IOC) of UNESCO has recommended the application of tide-staff measurements for quality control of sea level measurements made from autonomous instrumentation (IOC Manuals and Guides, No. 14, Vol. 3, 2002). However, no studies have been reported on the optimal quantum of tide-

staff measurements to be used, or the most appropriate sequence of such measurements in relation to spring/neap cycles for the purpose of efficient utilization of tide-staff measurements to achieve the best possible accuracy from bottom pressure based sea level measurements. Fig. 1 indicates the location of the GLOSS sea level station at Takoradi harbor on the Ghanaian coast, from where measurements in the present study are reported.

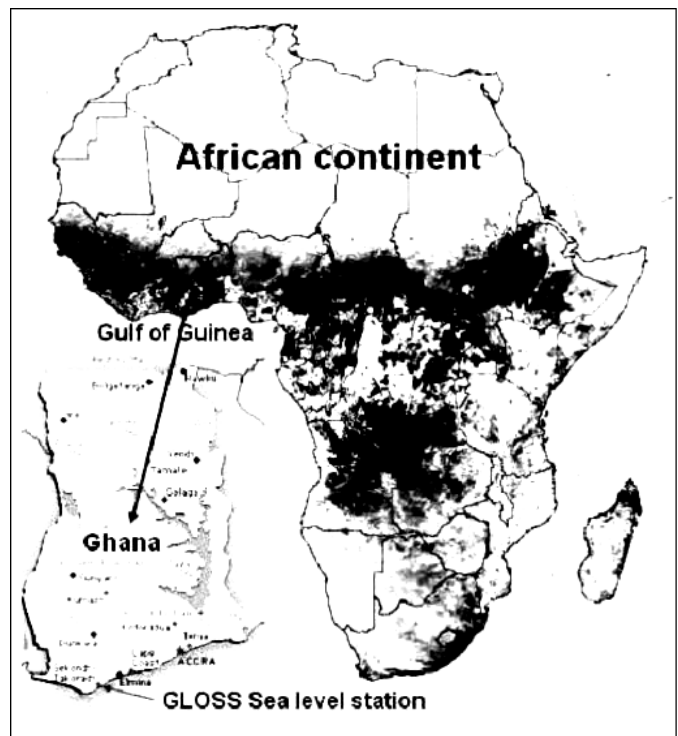


Fig. 1 Map indicating the measurement site at Takoradi harbor, Ghana.

## II. METHOD

The method examined in this study involves the use of a statistically derived simple linear mathematical model, which relates a sufficiently large set of bottom pressure measurements and concurrently made tide-staff

measurements. The model representing the linear regression between these two measurements is of the form:

$$T = (G \times P_w) - O \quad (1.1)$$

where  $T$ ,  $P_w$ ,  $G$ , and  $O$  represent respectively chart-datum-referenced tide-staff measurement, concurrent water pressure measurement, gain, and offset of the model equation. The value of  $O$  is negative in the present case because, conventionally, the pressure port is located below the chart-datum (CD) level (Fig. 2), preferably 1m or more. The premise is that if a ‘sufficiently large’ and ‘representative’ dataset during a given season have been used in constructing the model, then application of this model to a much larger time-series of water-pressure dataset during the same season should provide realistic estimates of sea level measurements.

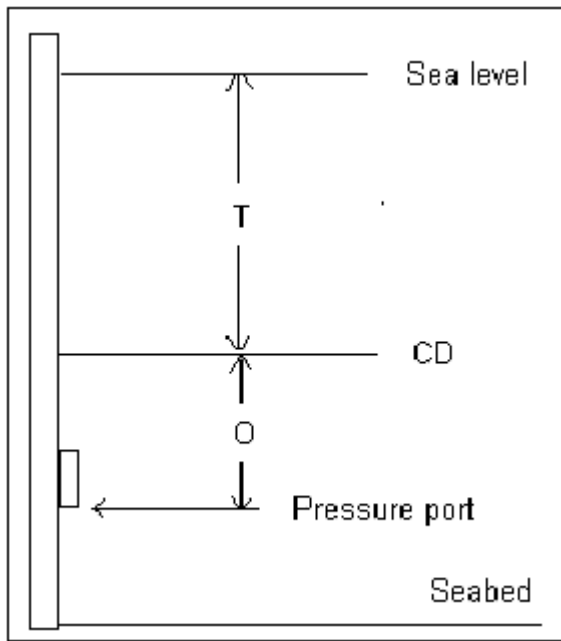


Fig.2 Schematic diagram illustrating the model parameters  $T$  and  $O$ .

### III. OPTIMIZATION OF TIDE-STAFF MEASUREMENTS

While the model considered above provides a simple and broad framework for conversion of water pressure measurements to sea level measurements; it remains to be understood as to what constitutes “sufficiently large” and “representative” dataset to be used for the construction of the model.

In an attempt to find an answer to these questions, we constructed model equations of the type mentioned above, wherein datasets (acquired at 15-minutes sampling interval) corresponding to various tidal cycles during both neap and spring tides were used. We then examined the optimal number

of tidal cycles and their neap/spring relationships in achieving stabilization of the model parameters  $G$  and  $O$  (Fig. 2).

In this study, it was observed that in the case of neap tide measurements (indicated by  $\times$  on the graphs), even after several tidal cycles the values of  $G$  and  $O$  never stabilized. However, spring tide measurements (indicated by  $\bullet$  on the graphs) provided significantly improved results. In this case,  $G$  and  $O$  stabilized with the use of dataset corresponding to 3 cycles of tide (one and half days). However, incorporation of more tidal cycles for construction of the model did not provide any added benefits in terms of stabilization of  $G$  and  $O$ . We then examined if the size of the dataset for construction of the model can be reduced by centering the dataset on spring tide peak. With reference to Fig. 3(b, c); points a, b, c, d, e, f, and g represent, respectively, the results achieved with the use of datasets acquired at 15-minutes sampling interval during spring tide between points 4-5 (1 cycle), 5-6 (1 cycle), 4-6 (2 cycles), 3-7 (4 cycles), 2-8 (6 cycles), 1-9 (8 cycles), and 0-10 (10 cycles) shown in Fig. 3(a). It is seen that, in relation to several of the  $G$  and  $O$  values obtained from the models, use of datasets corresponding to a single cycle that encompasses the largest tidal range (bounded between points 4-5 at spring tide) provided stabilized  $G$  and  $O$  values (indicated by  $a$  on the Gain and Offset graphs). However, datasets corresponding to another adjacent single cycle that represented a much smaller tidal range (bounded between points 5-6 at spring tide) provided  $G$  and  $O$  values (indicated by  $b$  on the Gain and Offset graphs) that differed significantly from the stabilized  $G$  and  $O$  values.

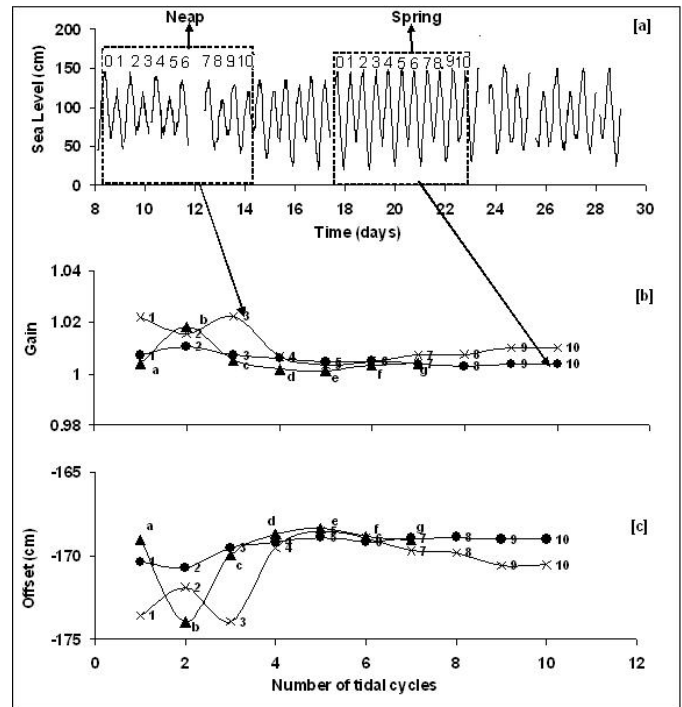


Fig.3 Illustration of optimal number of tidal cycles and their neap/spring relationships in achieving stabilization of model parameters  $G$  and  $O$ .

Our experience shows that in many cases, a single tidal cycle at spring tide may not represent the maximum span of the water column height. This explains as to why sometimes dataset corresponding to three or more tidal cycles during a given spring tide becomes necessary to achieve stabilization of the model parameters  $G$  and  $O$ .

It may, therefore, be concluded that the crucial factor which determines stabilization of the model parameters  $G$  and  $O$  is incorporation of datasets from that (those) tidal cycle(s) which encompasses the largest span of water column height during spring tide.

#### IV. SEA LEVEL MEASUREMENTS FROM TAKORADI HARBOR

*In-situ* sea-level data from coastal and island tide gauge network is a useful method to validate altimetric measurements [3]. However, the Gulf of Guinea (GOG) witnessed a severe lack of *in-situ* sea-level measurements. Thus, it became vital to have quality-controlled sea level data availability from the Guinea Gulf; to contribute to the Global Sea Level Observing System (GLOSS) program in Africa. Because of the presence of seasonally variable Guinea Current flowing along the Ghanaian coast, and its proximity to areas of seasonal coastal upwelling and increased biological productivity [5, 6, 7 & 8], long-term close monitoring of the sea-level variability from GOG assumes special significance in the context of oceanographic and climate studies. Considering the fact that during the 26 December 2004 ‘‘Sumatra Tsunami’’ the only sea level measurements available from the entire West African coast (except those from South Africa) was from the Takoradi harbour [9], the data obtained from Ghana based on the quality control scheme described in this paper could be of great importance to evaluating the global character of this tsunami, as well as for the verification of numerical models.

#### V. ESTIMATION EFFECTIVE DENSITY

The seawater pressure data used in the present study has been obtained from a bottom-mounted pressure gauge [10], which incorporated Honeywell’s shallow-water temperature-compensated piezo-resistive semiconductor pressure transducer, whose performance has been reported by Kumar *et al.*, [11]. In conformity with the GLOSS requirements, the gauge logged 15-minutes-averaged time-indexed pressure data at 15-min interval.

In constructing the linear regression model represented by (1.1), tide-staff measurements are represented in *cm* and water pressure in *mb* ( $1\text{ mb} = 10^3\text{ dynes cm}^{-2}$ ). In this,  $(T + O)$  represents the height of the water column over the pressure port of the transducer. Let  $\rho_{\text{eff}}$  represents the depth-mean effective density of the water column that contributes to the pressure detected by the transducer.

Representing all measurements in the same system of units (CGS), and from basic hydrostatics principles,

$$(P_w \times 1000) = (T + O) \times g \times \rho_{\text{eff}} \text{ so that}$$

$$P_w = \frac{(T + O) \times g \times \rho_{\text{eff}}}{1000} \quad (1.2)$$

From Equations (1.1) and (1.2), we get an expression for  $\rho_{\text{eff}}$  ( $\text{g cm}^{-3}$ ) as:

$$\rho_{\text{eff}} = \frac{1000}{(g \times G)} \quad (1.3)$$

With the use of standard gravity ( $g = 980.665\text{ cm s}^{-2}$ ), the expression for  $\rho_{\text{eff}}$  can be simplified as:

$$\rho_{\text{eff}} = \frac{1.020}{G} \quad (1.4)$$

Fig. 4 provides a comparison of the bulk density values,  $\rho_b$ , obtained from precision density measurements made on discrete water samples collected from Takoradi measurements during different periods of a year and the effective density values,  $\rho_{\text{eff}}$ , estimated based on equation (1.4), together with rainfall and wind speed measurements. It is found that the values of  $\rho_{\text{eff}}$  are always less than those of  $\rho_b$ .

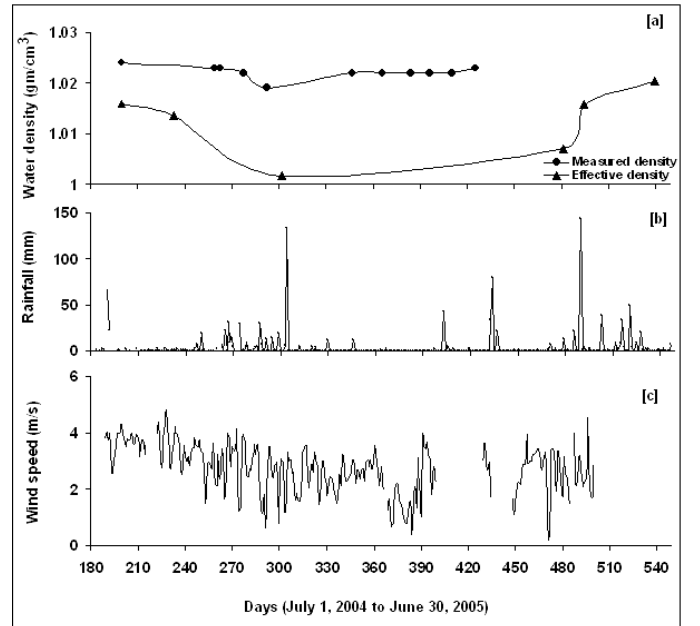


Fig. 4 [a] Comparison of bulk density values,  $\rho_b$ , with effective density values,  $\rho_{\text{eff}}$ , together with [b] rainfall, and [c] wind speed measurements.

Thus, the use of  $\rho_b$  for conversion of water pressure to sea level elevation would lead to under-estimation of sea level measurements. It can be shown that the percentage under-estimation in sea level measurement arising from the use of  $\rho_b$  can be estimated to be  $[(\rho_b - \rho_{eff}) \times 100] / \rho_b$  (See Appendix), which is the percentage difference in the two density parameters. The effective depth-mean density value,  $\rho_{eff}$ , estimated from Takoradi measurements has been found to be less than its bulk density,  $\rho_b$ , by  $\approx 0.25\% - 1.84\%$ .

## VI. INFLUENCE OF WIND AND RAINFALL ON EFFECTIVE DENSITY

It has been observed that the values of  $\rho_{eff}$  have always been less than those of  $\rho_b$  in the Takoradi measurements. As the visibility of the water body in the measurement site was reasonably good, role of suspended sediments in causing a reduction in the effective in-situ density as reported by [1] may not be significant. Although the observed reduction in the effective in-situ density of Takoradi waters was in association with both wind and rainfall, the effect was more prominent during the period when rainfall was strong. Presence of microbubbles in clear sea water as reported by [12, 13], and probably introduced into the upper layers of the sea water as a result of turbulence induced by wind forcing and impact of rain drops, is suspected to be responsible for the observed effective reduction in the *in-situ* density of this water body. The measurement site being shallow (less than 5 m during high tide), the influence of microbubbles are likely to be significant.

## VII. CONCLUSIONS

It has been shown that the conventional practice of applying the bulk density of sea water for conversion of bottom pressure to sea level would introduce inaccuracy in sea level estimates. This inaccuracy can be minimized by the use of ‘effective density’ parameter,  $\rho_{eff}$ . Presence of microbubbles introduced into the upper layers of the sea water, as a result of turbulence induced by wind forcing and impact of rain drops, is suspected to be responsible for the observed effective reduction in the *in-situ* density of this shallow water body. It is shown that  $\rho_{eff}$  can be introduced, thereby improving the quality of bottom-pressure based sea level measurements, by using a statistically derived simple linear model, which is constructed from a set of bottom pressures and concurrent tide-staff measurements, which encompass the maximum span of water column height over a few tidal cycles during the spring tide. The model provides chart-datum-referenced sea-level record directly from bottom pressure measurements without the use of seawater density measurements, and without having to know precisely the depth of the gauge’s pressure port relative to the chart datum level. This methodology is applicable for all types of water bodies; and particularly useful for turbulent and suspended sediment laden coastal water bodies whose effective density has been reported

to have been less than that obtained from conventional measurements. The quality-control scheme adopted in the present study was found valuable in the detection of a clear tsunami signal from Takoradi harbour in Ghana, which is the only sea level gauge site along the entire West African coast (except those from South Africa) to provide the valuable sea level data during the 26 December 2004 “Sumatra Tsunami” at the Eastern Atlantic Coast.

## ACKNOWLEDGMENT

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## APPENDIX

### Relation between percentage under-estimation in sea level measurement and percentage difference in the two density parameters $\rho_b$ and $\rho_{eff}$ .

Let us assume that flow- and wave-induced errors in the measurement of pressure have been removed by the use of a suitable hydro-mechanical front end. Let  $P$  be the time-averaged water pressure measured by the sensor during a given sampling, so that wind-waves have been smoothed out. Let  $\rho_b$  be the bulk density used for estimation of the water column elevation above the pressure inlet, and let this elevation be  $H_1$ , so that from the standard hydrostatic equation,

$$H_1 = \frac{P}{g\rho_b} \quad (A1)$$

Let  $\rho_{eff}$  be the density of the water column that actually contributed to the pressure  $P$  sensed by the pressure sensor. Let the water column elevation estimate based on  $\rho_{eff}$  be  $H_2$ , so that,

$$H_2 = \frac{P}{g\rho_{eff}} \quad (A2)$$

From (A1) and (A2), the difference  $\Delta H$  in sea level measurement, given by  $(H_2 - H_1)$ , can be estimated to be,

$$\Delta H = \frac{p}{g} \times \frac{(\rho_b - \rho_{eff})}{\rho_b \rho_{eff}} \quad (A3)$$

Percentage error (under-estimation) in sea level measurement with the use of  $\rho_b$  is given by  $(\Delta H / H_2) \times 100$ . Substituting for  $\Delta H$  and  $H_2$  from (A3) and (A2), we get percentage under-estimation,

$$= \frac{(\rho_b - \rho_{eff})}{\rho_b} \times 100 \quad (A4)$$

Thus, the percentage (%) under-estimation in sea level measurement, arising from the use of bulk density is equivalent to the percentage difference in the two density parameters  $\rho_b$  and  $\rho_{eff}$ .

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