

Modelling of wave propagation over a submerged sand bar using SWASH

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A non-hydrostatic numerical model 'SWASH' (Simulating WAVes till SHore) is used to study the wave propagation over a submerged sand bar in a wave flume. The SWASH model is calibrated and further used to validate the wave propagation for two different cases. The wave heights and wave induced velocities obtained from the model and the laboratory experimental results are compared. The model without the morphology feedback provided good correlation with the measurements for case of low wave energy, whereas for the case of a moderately high wave energy, due to significant variations in the bed morphology, the model under-performed towards the later part of the simulation. However, incorporating a modified bathymetry considering the variation in the bed morphology, the model results were reasonable.

[Keywords: SWASH model, Surfzone, Sand bar, Wave model, Non-hydrostatic]

Introduction

The SWASH model (Simulating Waves till SHore) is a time domain numerical model for simulating non-hydrostatic, free surface and rotational flows. The SWASH model is essentially applicable for coastal regions up to the shore. The basic SWASH codes help to run a wide range of time and space scales in both surface waves and shallow water flows. SWASH uses explicit, continuity and momentum equations in second order Leapfrog scheme¹. The governing equations are the Non Linear Shallow Water Equations (NLSWE) which are derived from the incompressible Navier-Stokes equations including non-hydrostatic pressure and provide a general basis for describing complex changes. Description about SWASH model, boundary conditions, numerical schemes and applications are explained elsewhere².

SWASH is used in non-hydrostatic one dimensional mode in this study to estimate and compare the wave characteristics with the measured data collected from a wave flume with a movable bed sand bar. Recent studies on the

SWASH model validation include both wave transformation and wave overtopping based on physical model experiments and sensitivity analysis to bottom friction³; swash zone studies⁴; beach nourishment effects⁵; solitary wave propagation over horizontal and sloped bed⁶ etc. The incompressible Navier-Stokes equations for the depth averaged, non-hydrostatics, free surface flow is given below⁷.

$$\frac{\partial \zeta}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0 \quad \dots (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial \zeta}{\partial x} + \frac{1}{h} \int_{-d}^{\zeta} \frac{\partial q}{\partial x} dz + c_f \frac{u \sqrt{u^2 + v^2}}{h} \quad \dots (2)$$

$$= \frac{1}{h} \left(\frac{\partial h \tau_{xx}}{\partial x} + \frac{\partial h \tau_{xy}}{\partial y} \right)$$

$$\begin{aligned} & \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial \zeta}{\partial y} \\ & + \frac{1}{h} \int_{-d}^{\zeta} \frac{\partial q}{\partial y} dz + c_f \frac{u \sqrt{u^2 + v^2}}{h} \quad \dots (3) \\ & = \frac{1}{h} \left(\frac{\partial h \tau_{yx}}{\partial x} + \frac{\partial h \tau_{yy}}{\partial y} \right) \end{aligned}$$

where, t is time, x and y are the positions in water level and z is depth in upward direction. $\zeta(x, y)$ is the surface elevation and h is the still water depth. $u(x, y, t)$ and $v(x, y, t)$ are the velocities in x and y direction respectively and $q(x, y, z, t)$ is the non-hydrostatic pressure, g is the acceleration due to gravity, τ_{xx} , τ_{yy} , τ_{yx} and τ_{xy} are the horizontal stress terms. The water depth is varying with time in accordance with variation of ζ and h .

The vertical velocities at the free surface W_s and at the boundary level W_b .

$$W_b = -u \frac{\partial d}{\partial x} - v \frac{\partial d}{\partial y} \quad \dots (4)$$

Using the linear wave theory, the velocity at a depth is calculated using linear superposition of N-harmonic waves, the amplitude is determined by sampling a variance density spectrum and the phase is randomly chosen for each step:

$$\begin{aligned} u_b(z, t) &= \sum_{j=1}^N a_j \left(\omega_j \frac{\cosh k_j(z+d)}{\sin k_j h} + \sqrt{\frac{g}{h}} \right) \dots (5) \\ & \cos(\omega_j t - a_j) - \sqrt{\frac{g}{h}} \zeta \end{aligned}$$

Where k_j and a_j are the wave number and random phase of the wave of each frequency ω_j ; and kept frequency range at an interval of $\Delta\omega$, i.e. $\omega_j = j\Delta\omega$. The wave number and frequency are related by the dispersion relationship, $\omega^2 = gk \tanh(kh)$, whereas the random phase at each frequency is uniformly distributed between 0 and 2π . The amplitude of each wave is calculated using equation.

$$a_j = \sqrt{2E(\omega_j)} \Delta\omega \quad \dots (6)$$

Materials and Methods

The SWASH model was used to study the wave

propagation over a sand bar and analysis of the resulting wave characteristics was carried out. Similar studies carried out in the literature are by⁸⁻⁹. In this study the computational grid is made of an equal horizontal spacing of 0.05 m, and 25 layers were considered in the vertical. The surveyed flume bathymetry was provided as input to the model. A Fourier series with wave amplitude of 0.05 m and period of 1 s was used initially to validate the model. A constant friction factor of 0.002 and viscosity factor of 0.005 was provided initially. The simulations were carried out for one hour similar to the laboratory experimental data. The water level and velocity at a water depth of 20 cm on the top of the sand hump was used to test the model results.

Experimental setup

This laboratory experiment setup consists of a sand bar 3 m long and 25 cm high initially placed as a trapezium, the centre of the sand bar being placed at a distance of about 8.5 m from paddle and initially subjected to periodic waves for more than 6 hours so that the sand bar gets consolidated, with the sand hump covered with a geo-fabric arresting the movement of sand from the bar. The flume length was 25 m with water depth kept at 0.5 m. The other end of the flume is provided with a wave absorber, however, the reflection coefficient for higher waves from the absorber was found to be 0.30. The measured data consisted of water levels close to wave paddle as well as over the sand bar and beyond the sand bar. Velocity measurements were made on top of the sand bar. The schematic diagram of the experimental setup is shown in Fig. 1.

Results and Discussion

The SWASH model is calibrated using a part of the wave flume data consisting of both water levels and velocities at a distance of 8.5 m from the wave paddle, corresponding to the top of the submerged sand bar. This calibrated model was further used to simulate the waves for two different experiments viz., (case i) wave height of 0.1 m and wave period of 1.0 s and (case ii) wave height of 0.15 m and wave period 1.5s.

The model was simulated for different vertical layers (5, 10 and 25 layers) to find optimum

number of layers to be used in the model runs. The model results are compared with measured data and it is seen that for 5 and 10 vertical layers, the correlation is good (Fig. 2). Further model calibration and analysis is carried out for 5 vertical layers.

Calibration of Model

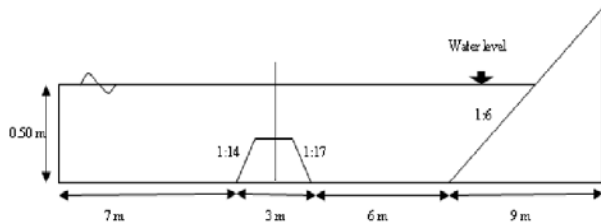


Fig. 1—Schematic representation of wave flume setup

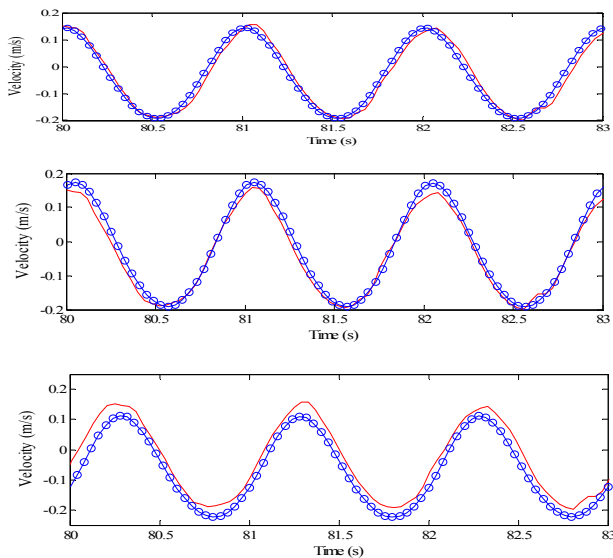


Fig. 2—Comparison between measured and model output for 5, 10 and 25, vertical layers for case 1 (solid line=model; circles=measured).

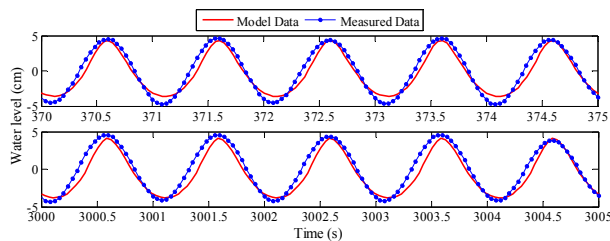


Fig. 3—Comparison plot between model and observed water level in case 1

Table 1—Correlation coefficients for different cases.

Case	Profile	Correlation coefficient	
		Water level	Velocity
1	1	0.851	0.842
	1	0.965	0.891
2	2	0.885	0.881

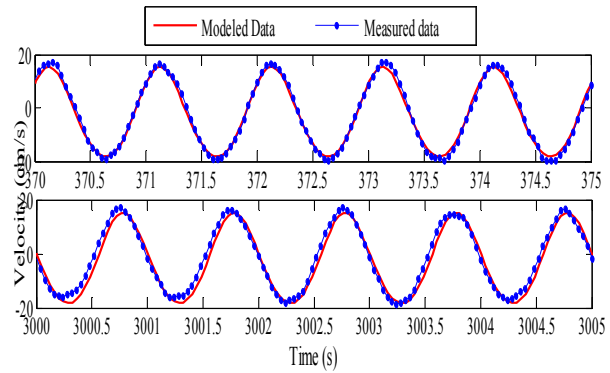


Fig. 4. Comparison plot between model and observed velocity for case 1.

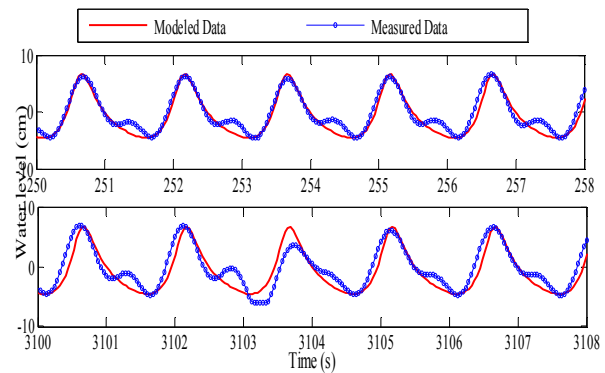


Fig. 5—Comparison plot between model and observed water level for case 2 using first profile.

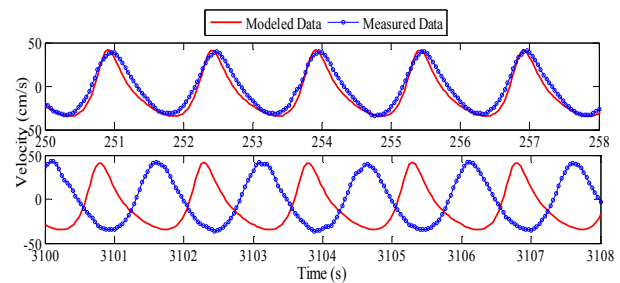


Fig. 6—Comparison between model and observed velocity for case 2 using first profile.

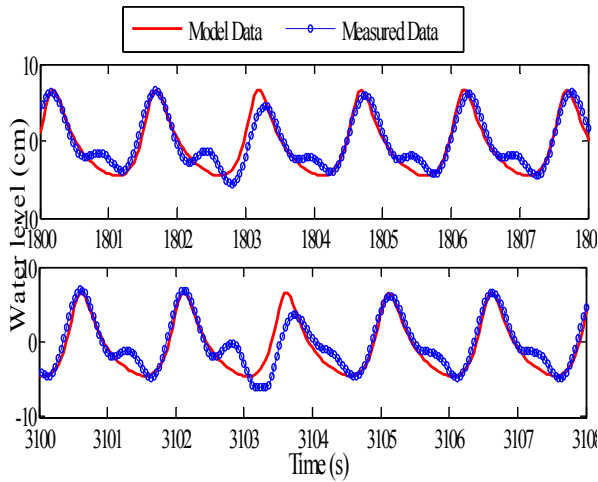


Fig. 7—Comparison between model and observed water level for case 2 using second profile.

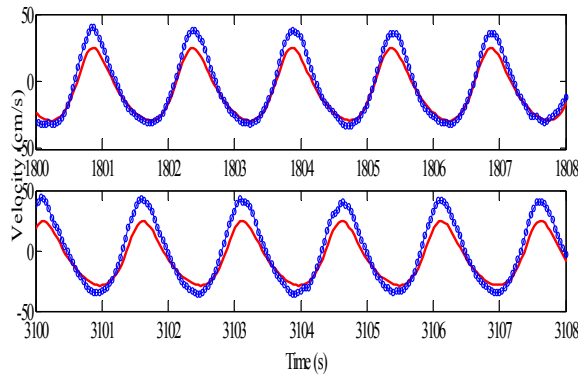


Fig. 8—Comparison between model and observed water level for case 2 using second profile.

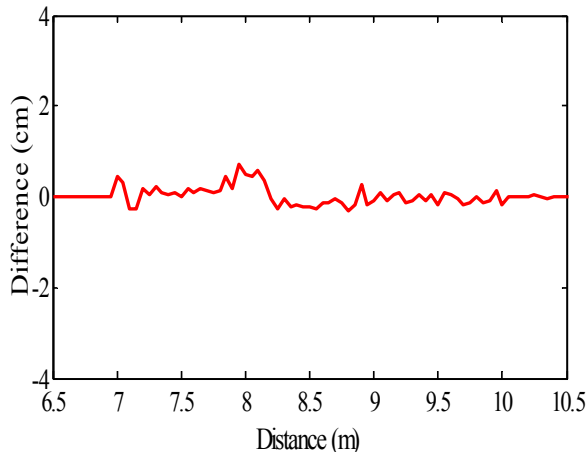


Fig. 9—The difference between two profiles for case 1.

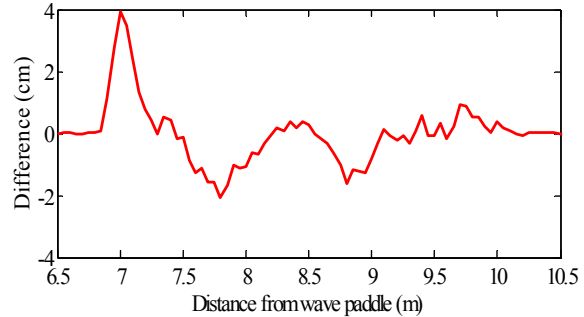


Fig. 10—The difference between two profiles for case 2.

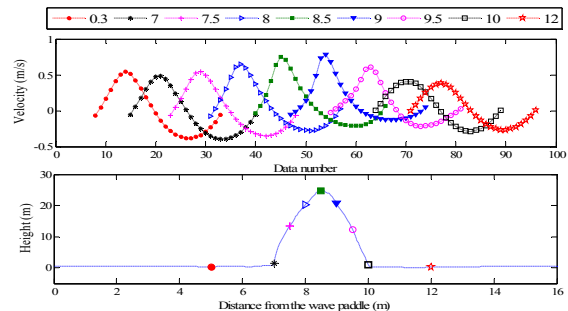


Fig. 11—The velocity variation in x direction

The SWASH model is calibrated using the 1 hour experimental data for wave transformation over a submerged sand bar. The data with wave height of 10 cm and wave period of 1s was used for calibration. The model output obtained for water level and velocity for a depth of 20 cm i.e. 5 cm above the bottom at a distance of 8.5 m from paddle was used to compare with measurements. The model results of velocity and water level showed good comparison with the measurements. The plots of water level comparison are shown in Fig. 3 and correlation is presented in Table 1. The velocity comparison also gave good correlation for the same set of data and the comparison plots are shown in Fig. 4.

Validation of Model

The model was validated for second case of experiment ($T=1.5s$; $H=0.15$ m) observed for 1hr data consist of water level and velocity collected at a distance of 8.5 m from the wave paddle, on top of the sand bar. The model resulted in good correlation with observed water levels (Fig. 5) up to 1800s of simulation. However, the model output after 1800s showed an increased phase difference, especially in velocity comparisons, as the

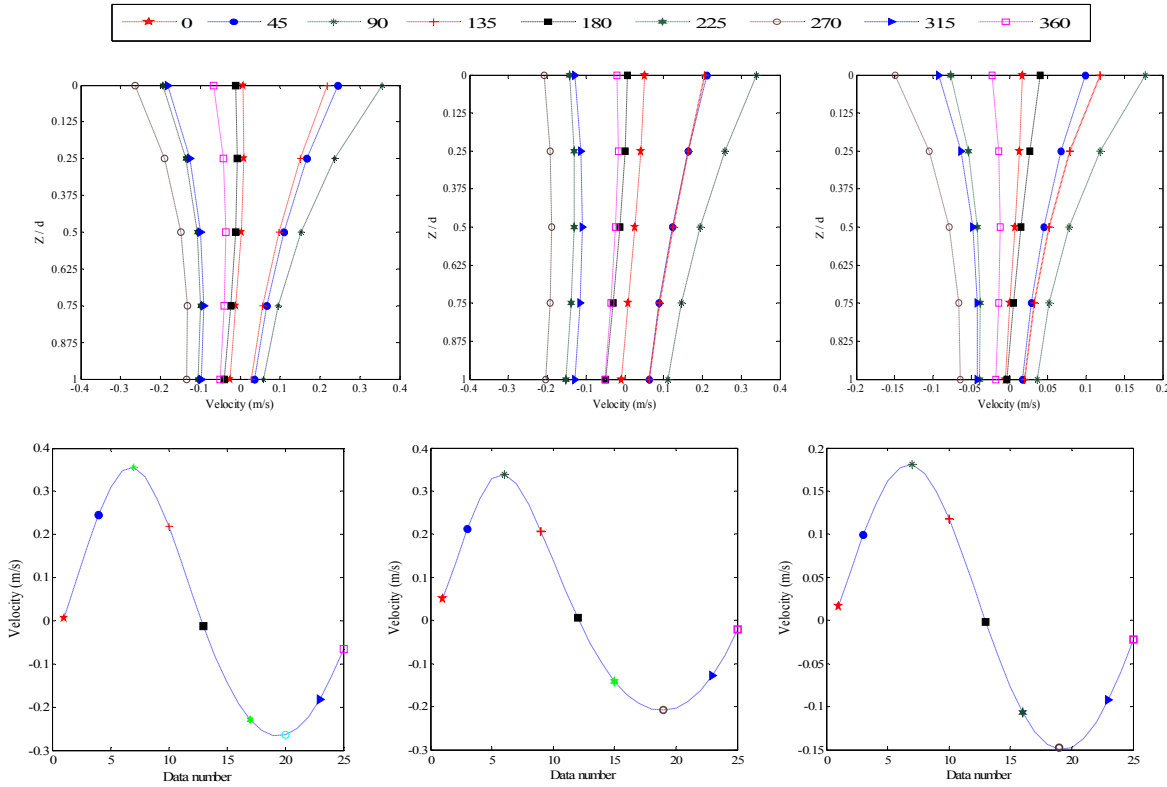


Fig. 12—Velocity profile for 5 layers model simulation (case-1) at x=5, 8.5 and 12 m from the wave paddle

simulation progressed resulting in decreased correlation for this case (Fig. 6). The reason for this increased phase difference is found to be the change in the sand bar morphology during the experiment, which is not captured by the SWASH model. Also the secondary waves could not be reliably predicted by the model.

In order to resolve the discrepancy between model and measured data for case 2, the model was run in two separate parts, the first part consisting of initial bathymetry for the first 1800s and an averaged bathymetry between the initial and final bathymetry is considered for the remaining 1800s. The comparison plot of water levels with the modified bathymetry is shown in Fig. 7.

The comparison plot for velocities with the modified bathymetry (Fig. 8) shows that the model predicts the phase well whereas it under predicts the peak velocity. In the case 1 experiment, the resultant change in bathymetry was smaller (Fig. 9) compared to that of case 2 (Fig. 10), which

resulted in modification of wave and velocity patterns with time.

Velocity profiles

The variation in velocity profile along the flume as well as in the vertical are obtained for an average of 10 waves for case 1. The velocity data at eight different locations viz., 5.0, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0 and 12.0 m from the wave paddle are extracted from the model to understand the variation of velocity in the horizontal as well as in the vertical direction.

The velocity variation in horizontal direction is shown in Fig. 11, which shows a gradual increase in velocity magnitude as the waves approach the sand bar and the velocity decreases after crossing the bar, when the wave encounters relatively deeper waters. The velocity profile is observed to steepen as the wave propagates over the sand bar and after passing over the bar, the velocity profile is observed to flatten out much more than the initial velocity profile due to frequency dispersion.

The vertical velocity profile for different wave phases, viz., 0, 45, 90, 135, 180, 225, 270, 315 and 360, for the waves before the sand bar, over the bar and after the bar is shown in Fig. 12. The vertical velocity profiles clearly show that the near bottom velocity over the bar is almost twice that of the velocity before and after the sand bar. However, the velocity gradient over the sand bar is observed to be less than that for the profiles before and after the sand bar.

Conclusions

The SWASH model with non-hydrostatic pressure formulation was used to study its applicability to wave propagation over movable sand bar. The model results compared well with the experiments for cases where bed morphology is small whereas accounting for the morphology changes in the model bathymetry resulted in better comparisons. For large variations in the morphology the model under-predicted the peak velocity and also the secondary waves were not predicted by the model. The averaged velocity profile variation both in the horizontal and vertical directions showed that the velocity amplitude increases both in the horizontal as well as vertical directions when the wave propagates over the sand bar, albeit with a less vertical velocity gradient, compared to velocity gradient before and after the sand bar.

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