

Objective Mapping of Temperature Field By Stochastic Inverse Method Using Acoustic Tomography Experimental Data of Eastern Arabian Sea

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Abstract

An acoustic transmission experiment was conducted in the eastern Arabian Sea along 12.5°N latitude for a duration of ten days (2-12 May, 1993), with two transceiver systems deployed on deep sea moorings, separated by a range of 270.92 km. The data are revisited for objective mapping of the temperature fields using Stochastic Inverse Method. Hourly reciprocal transmissions were carried with time lag of 30 minutes between each direction. From the multipath arrival patterns, significant peaks corresponding to the predicted ray arrivals were identified and travel time perturbations of the most stable Eigen rays enabled reconstruction of temperature anomaly from the sound velocity perturbations using stochastic inverse method. For formulation of stochastic inverse, generation of data-data and model-data covariance matrices are needed for computing the modal amplitudes to operate on observed time perturbations to estimate the sound velocity perturbations. For present simulation, the dependence of travel time on the oceanic sound velocity has been calculated using ray paths traced by computer with mean profile and vertical and horizontal velocity structures were modeled as a combination of Empirical Orthogonal Functions (EOFs) from insitu CTD data in the study area. A linear relation was used to transform sound speed perturbation in the vertical plane to temperature perturbations, for the first four days of transmission, following Munk and Wunsch (1979). The 2-D temperature anomaly derived from the six hourly mean travel-time data showed a gradual warming of the top layers, significant of diurnal variability and intrusion of Red sea water.

1. Introduction

Ocean Acoustic Tomography (OAT) is a technique by which real time oceanic fields could be measured synoptically by acoustic means (Munk and Wunsch, 1979). A characteristic feature of the ocean sound speed profile is the existence of an acoustic waveguide which enables propagation of sound over long distances without loss of energy. In OAT, one measures times of arrivals of acoustic pulses and obtain travel time perturbations between the measured and the corresponding predicted ray arrivals (known as *forward problem*). From these, one could obtain the sound speed/temperature anomalies to the first order by means of appropriate generalized

inversion schemes like deterministic and stochastic. Since deterministic inverse procedure doesn't permit a continuous representation of unknown field, Stochastic Inverse technique (Cornuelle, 1983) has been adopted for the objective mapping of temperature field using Acoustic Tomography Experimental Data of Eastern Arabian Sea.

During summer 1993, an acoustic transmission experiment was conducted in the eastern Arabian sea, for a duration of 10 days (2-12 May) with an objective of mapping the time evolution of temperature anomaly from travel time perturbations. This OAT experiment has many components; preparation of available data on ocean parameters relevant to OAT studies, acoustic model simulation in reference ocean, deployment of acoustic system and data collection, preprocessing the data collected during OAT experiments, optimal estimation of sound speed perturbations prevailing at the time of observation and finally the interpretation of results for oceanographic applications. Prasanna Kumar et.al, (1997) detailed the OAT's experimental components and interpretation of results on physical aspects using deterministic generalized inverse technique. In this paper we present the results from the above experiment and examine time evolving temperature anomaly obtained through tomographic inversion using stochastic inversion method.

2. Methodology

2.1 Inverse Problem

(a) Formulation of the inverse problem

Following Munk and Wunsch (1979) the vertical section containing the source and the receiver has been divided into a number of rectangular grid areas. The travel time deviations for sound pulses propagating along N eigenrays in M grid areas give rise to a set of N linear equations in M unknowns. The matrix equation is generally under-determined, and several generalized inverse solutions exist (Ramana Murty et al., 1992). The most widely used procedure is the Gauss-Markov method of minimum-mean-square estimation (Liebelt, 1967), also referred to as the stochastic inverse.

(b) The stochastic inverse

In the stochastic inverse method both the data (d) and the unknown field $\delta C(x, z)$ are assumed random variables having known mean and covariance functions. The best estimate $\delta \hat{C}(x, z)$ of the model parameters $\delta C(x, z)$ (i.e., $C(x, z) - C_0 = D^T$) can then be expressed in the form

$$\delta \hat{C}(x, z) = \sum a_i^1(x, z) \delta T_i^M = A(x, z) d, \quad (1)$$

such that the ensemble average of the square of the difference between the estimate of the unknown field and the true field at each point in the medium is a minimum, i.e., a_i 's are determined such that the square of the deviation (ensemble) is a minimum at each point (x, z) . Using the Gauss-Markov theorem (Liebelt, 1967), a_i 's are given by

$$A(x, z) = \langle \delta C(x, z) \delta T_i^T \rangle [\langle \delta T_i T_j^T \rangle]^{-1}, \quad (2)$$

where $\langle \delta C(x, z) \delta T_i^T \rangle$ and $\langle \delta T_i T_j^T \rangle$ are the model-data and data-data covariance matrices. These are evaluated using the linearized direct problem,

$$\delta T_i = - \int_{\tau_{i0}} \frac{\delta C(x, z)}{C_0^2(z)} ds + \varepsilon_i \quad (3)$$

where ε_i represents noise due to smaller scale phenomena and measurement errors, assumed to have zero ensemble average and to be uncorrelated with the mesoscale sound speed fluctuation. Using (3) the model-data and data-data covariance matrices become

$$\begin{aligned} \langle \delta C(x, z) d^T \rangle &= \langle \delta C(x, z) \delta T_i^T \rangle \\ &= \int_{\tau_{oi}} \frac{\langle \delta C(x, z) \delta C(x', z') \rangle}{C_0^2(x', z')} ds' \end{aligned} \quad (4)$$

$$= \int_{\Gamma_{oi}} \frac{\left[\left[\sum_{i=1}^{M-1} a_i(x) f_i(z) \right] \left[\sum_{i=1}^{M-1} a_i(x) f_i(z^1) \right] \right]}{C_0^2(x^1, z^1)} \frac{dx^1}{\cos \theta^1}$$

$$\langle d d^T \rangle = \langle \delta T_i \delta T_j^T \rangle$$

$$= \int_{\tau_{oi}} ds'' \int_{\tau_{oj}} \frac{\langle \delta C(x', z') \delta C(x'', z'') \rangle}{C_0^2(x', z') C_0^2(x'', z'')} ds'' + \gamma_{ij} \quad (5)$$

$$= \int_{\tau_{oj}} \frac{dx^{11}}{\cos \theta^{11}} \int_{\tau_{oi}} \frac{\left[\left[\sum_{i=1}^{M-1} a_i(x) f_i(z) \right] \left[\sum_{i=1}^{M-1} a_i(x) f_i(z^1) \right] \right]}{C_0^2(x^1, z^1) C_0^2(x^{11}, z^{11})} \frac{dx^1}{\cos \theta^1}$$

where $\gamma_{ij} = \langle \varepsilon_i \varepsilon_j \rangle$ is the error covariance matrix. $(x', z'); \theta^1$ and $(x'', z''); \theta^{11}$ are the coordinates and angles of the i -th and j -th rays respectively. In eqns.(4) and (5), the integrations are carried along the eigenrays $(\tau_{oi} \ \& \ \tau_{oj})$. In the above equations, $f_i(z)$ are the vertical EOF modes ($M-1$ column vectors of U) of covariance

matrix $D^T D = U \Gamma V^T$ (SVD form) and horizontal modal amplitudes $a_i(x)$ are obtained as follows:

$$a_i(x) = \sum_{n=1}^M \delta C(x, z_n) f_i(z_n) \quad (6)$$

$i = \text{mode index}$, $n = \text{layer index}$

For each point in a given region, the coefficients $a_i^1(x, z)$ are determined from the eigenrays and operated on the travel time perturbations to objectively estimate the model parameter. This procedure permits a continuous representation of the unknown field. The data-data and model-data covariance matrices are determined for the reference sound speed distribution with a suitably selected source-receiver configuration. Using these matrices the sound speed field ($\delta \hat{C}(x, z)$) of the real ocean is obtained for the given travel time perturbations (δT^M).

3. Results and Discussion

An acoustic transmission experiment was conducted in the eastern Arabian Sea along 12.5°N latitude for a duration of ten days (2-12 May, 1993), with two transceiver systems (TR1 and TR2) having low central frequency 400Hz (100Hz bandwidth) deployed on deep sea moorings, separated by a range of 270.92 km (Fig.1). Hourly reciprocal transmissions were carried out with time lag of 30 minutes between each direction. The received signals were cross-correlated with the replica of the source signal during the post processing, to generate amplitude peaks corresponding to the multipath arrivals (Fig.2). While transmissions were going on, a hydrographic survey was simultaneously carried out using a Sea-Bird CTD system. Vertical profiles of temperature and salinity, at half degree spacing were taken at 16 locations (Fig.1), for the *in situ* information on acoustic profile and the prevailing hydrographic conditions during the experiment. Range-independent and dependent ray tracing was carried out with Levitus Climatological mean sound velocity ($C_0(z)$) and *in situ* sound velocity ($C_0(x, z)$) to compute predicted (simulated) eigen ray (Fig. 3) and arrivals (Fig. 4). A sequence of the predicted ray arrivals in the Arabian sea is as follows: purely refracted rays with emergence angle between $\pm 5-7^\circ$ (with respect to horizontal) reach the receiver first, followed by lower angle near axial rays, and the steep rays with emergence angles $\pm 8-9^\circ$: the refracted- bottom reflected rays with emergence angle $\pm 10^\circ$ are the final arrivals.

A close examination of the ray arrival time and the corresponding intensity of various arrivals revealed that the arrival patterns are stable in time. In view of this, the ray identification is carried out by correlating the axial arrival peaks of the measured travel time (Fig.4) with that of the predicted ones. A typical ray identification is shown in Fig.4. It was found that during the 4 days transmissions, out of the 14 eigen rays, 9 rays were stable in time (Table 1), enabled reconstruction of temperature anomaly from the sound velocity perturbations using stochastic inverse method. For formulation of stochastic inverse, generation of data-data (Eq. 5) and model-data (Eq.4) covariance matrices are needed for computing the modal amplitudes (Eq.6) to operate on observed time perturbations (δT^M) to estimate the sound velocity perturbations ($\delta \hat{C}(x,$

z)). For present simulation, the dependence of travel time on the oceanic sound velocity has been calculated using ray paths traced by computer with mean velocity profile while vertical and horizontal velocity structures were modeled as a combination of Empirical Orthogonal Functions (EOFs) from in situ CTD (surveys) in the study area.

A linear empirical relation ($\delta\theta = \delta C / (\alpha C_{0j})$, $\alpha = \sim 3.2 \cdot 10^{-3} / ^\circ\text{C}$ is the coefficient of thermal expansion) was used to transform sound speed perturbation (δC) in the vertical plane to temperature perturbations ($\delta\theta$) (Fig.5), for the first four days of transmission, following Munk and Wunsch (1979). The 2-D temperature anomaly derived from the six hourly mean travel-time data showed a gradual warming of the top layers, indicating significant diurnal variability and intrusion of Red sea water.

4. Conclusions

During summer 1993, an acoustic transmission experiment was conducted in the eastern Arabian sea for ten days (2-12 May) with an objective to map 2-D time evolving temperature structure from acoustic travel time perturbations using tomography technique. The 2-D temperature anomaly derived from the six hourly mean travel-time data showed a gradual warming of the top layers, indicating significant diurnal variability and intrusion of Red sea water.

Acknowledgements

The authors are grateful to Dr. S.R.Shetye, Director, National Institute of Oceanography, Goa, for his keen interest in this study. The OAT group of N.I.O. during 1989-93 were also acknowledged for their association with the senior author of this paper in the acquisition of the data, processing and interpretation.

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Table 1: Ray Identifier

Launch angle (ϵ)	Total turning(N)	Upper-lower turnings (p, -p)	Total travel time T(s)
8.10	9	(5, -4)	181.4543
6.87	8	(4, -4)	181.2587
5.39	8	(4, -4)	181.2166
4.30	9	(5, -4)	181.2865
3.32	10	(5, -5)	181.3380
2.13	11	(6, -5)	181.3595
-1.56	12	(6, -6)	181.3763
-3.14	11	(6, -5)	181.3494
-5.30	9	(5, -4)	181.2357

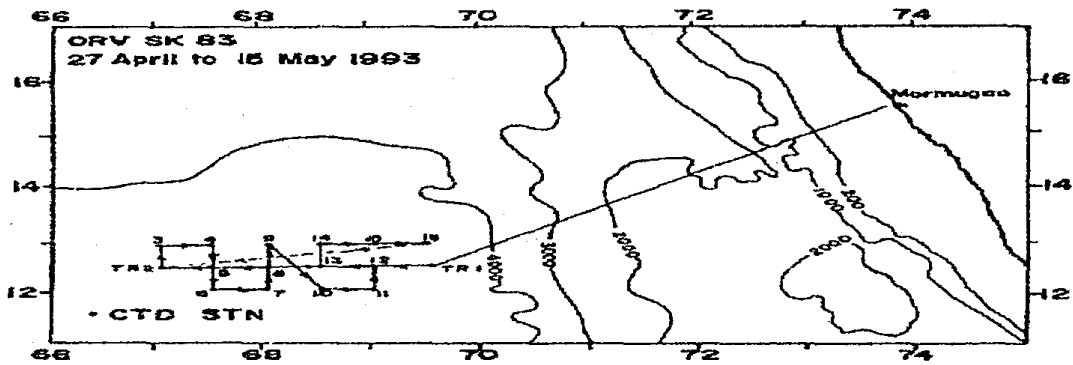


Figure 1. Location map showing transceiver (TR1 & TR2) moorings and CTD stations

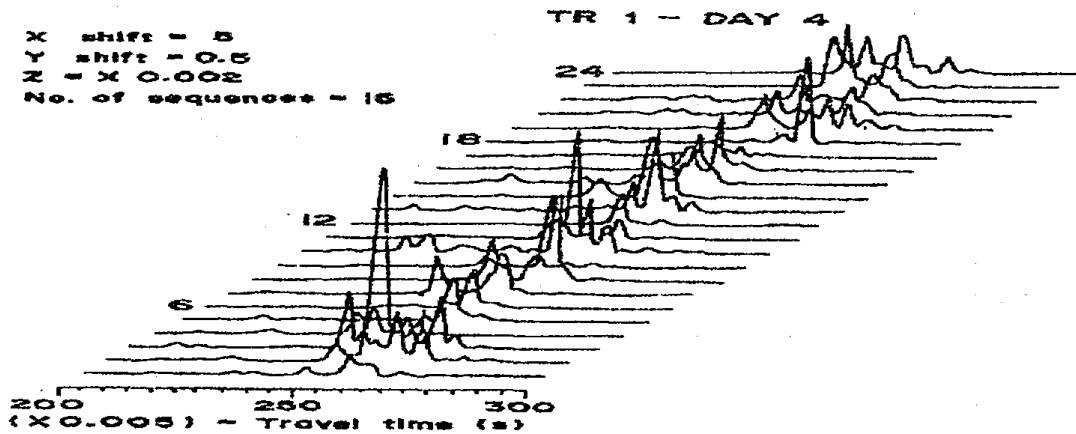


Figure 2. Waterfall plot showing measured signal arrivals at the receiver

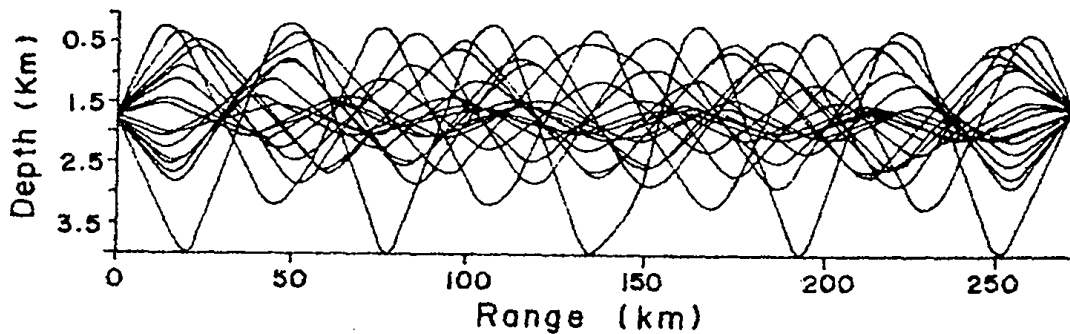


Figure 3. Eigen rays from TR2 to TR1

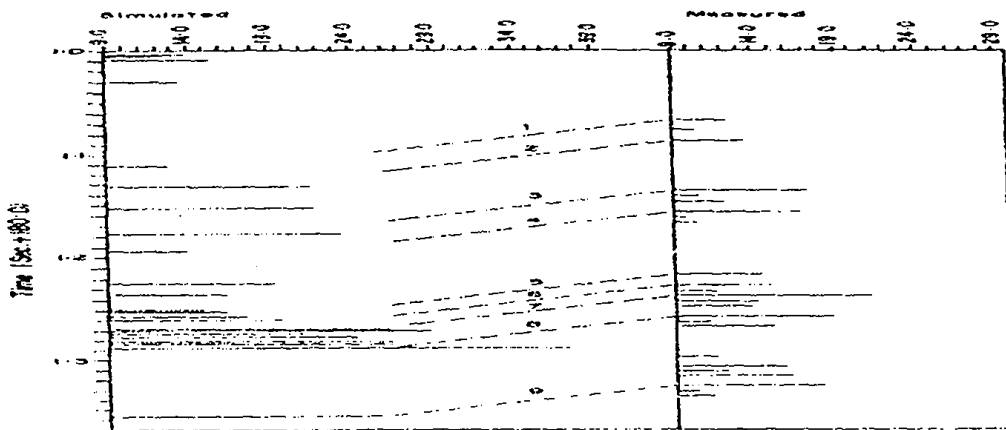


Figure 4. Identification of eigen rays predicted with measured arrivals

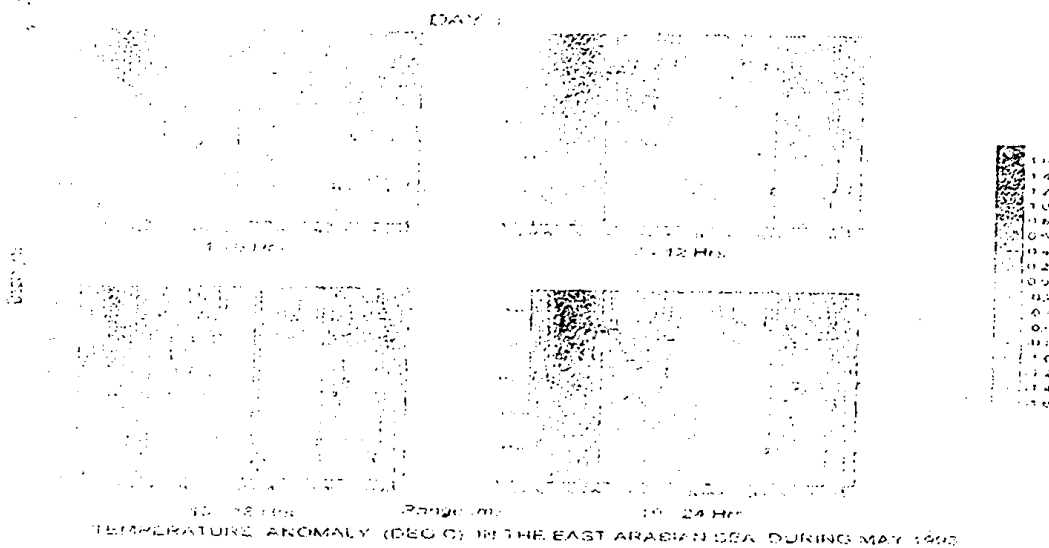


Figure 5. Objective mapping of temperature anomaly using OAT's principle applied to acoustic field transmission data



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