

**Application of generalised inverse for analysis of magnetic anomalies due to a dyke Model –  
- Some numerical experiments**

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## **ABSTRACT**

Forward and Inverse methods of magnetic interpretation have been used to estimate the parameters of dyke like intrusion over the eastern continental shelf of India. In the forward problem, the tentative parameters of the causative source are assumed and anomalies are calculated using the anomaly equation, while inverse problem refers to methods of tracing the boundaries or outlines of anomalous bodies using an iterative approach. While solving the inverse problem, data kernel has been generated through the model (i.e. partial derivatives of magnetic anomaly function with respect to model parameters at each station forming the kernel). In solving the coupled eigenvalue problem of covariance matrices of data kernel, Singular Value Decomposition (SVD) has been performed to build Generalised Inverse Operator (GIO). This GIO is operated on the observed anomaly (with reference to the calculated) to yield improved model parameters. Data and model resolution matrices are computed to check the correctness of the solution and further analysis. The marine magnetic total field anomaly of continental shelf of Visakhapatnam is interpreted using Generalised Inverse (GI) technique, which revealed the causative source of the anomaly as a dyke model at a depth of 120 m below seabed.

The software routine details are given in Annexure-I and sample input/output and FORTRAN coding are given in Annexure-II.

**Key words:** Magnetic anomaly, forward problem , inverse problems, GIO, SVD, data resolution, model resolution, closeness ratio, dyke intrusive, Visakhapatnam.

## Introduction

Among the many geophysical techniques, magnetic method is easy to execute and economically viable and therefore widely used in geophysical exploration. A general procedure is to assume most plausible shape for the magnetic anomaly source and compute its magnetic effect at the surface and to modify the model progressively until a reasonable fit with the observed anomaly is obtained<sup>(1-3)</sup>. The refined model represents a possible solution. The process of making inferences about the causative source from observed data is what is widely known as the inverse problem. Backus & Gilbert<sup>4</sup> made an attempt for exploring the mathematical structure of the inverse problems.

The marine magnetic data that were collected in the Bay of Bengal, are interpreted in terms of the geological structures, causing the magnetic anomalies<sup>5,6</sup>. In the present work, marine magnetic total field anomalies off Visakhapatnam, east coast of India (Fig. 1) inverted by the generalised inverse approach, to yield the intrusive dyke model parameters.

Ultimately the above physical problem gives a mathematical relationship between the theoretical framework ( $M$ : kernel) to observed data ( $dr$ ) and leading to system of equations (i.e.,  $M dy = dr$ ) for solving parameters ( $dy = M_g^{-1} dr$ , where  $M_g^{-1}$  is Generalised Inverse Operator (GIO)). In general the exploration problems are two types: 1) over-determined and 2) under-determined. The trivial solution of even-determined case is of rare occurrence. In all the cases a good resolution of both the model and data are desirable. The least square method solves the over-determined problem minimizing the squared Euclidean length of the error and has a perfect model resolution. Instead, the under-determined problem can be solved with a perfect data resolution by minimizing the model length. In nature, the problems are neither purely over-determined nor purely under-determined, they are mixed type. The entire field or observation over which the determination of the parameters of the model is to be carried out comes up in observation blocks, some of which are over-determined while some are under-determined leading to non-uniqueness of the solution. The generalized inverse technique solves such a mixed determined problem over the whole region of observations and has both

data and model resolutions intermediate between the two extremes<sup>7</sup>. Generalised Inverse (GI) technique through Singular Value Decomposition (SVD) is employed to solve such a mixed determined problem<sup>7</sup>. The advantages of involving SVD in construction of the GIO are viz., (1) SVD is objective and does not impose a pre-determined form to the data; (2) It provides an objective means of ranking uncorrelated modes of variability to determine weak signals or noise from the data; and (3) it provides the modes of variability which are not correlated with one another<sup>7,8</sup>.

Earlier, Malleswara Rao<sup>1</sup> *et,al.*, suggested interpretation of magnetic anomalies due to bodies of polygonal cross section using Marquardt<sup>9</sup> optimisation technique. In the present study, magnetic anomalies due to intrusive dyke model are analysed using the approach of Natural Generalised Inverse (GI) technique<sup>10</sup> as it throws light on data and model resolution and individual model parameter contribution. All these procedures and algorithms attempt to relate physical models to readily observable data and determine the "best-estimates" of model parameters.

## Notation

- $J$  - intensity of effective magnetization of two-dimensional body lying in a plane perpendicular to the strike
- $\phi$  - dip of the effective magnetization
- $\alpha$  - strike of the body measured from the magnetic north due east or west
- $i$  - dip of the earth's magnetic field
- $F$  - Earth's magnetic field expressed in gammas
- $D_m$  - direction of measurement, is 0 for horizontal  $\Delta H$ ,  $\Pi/2$  for vertical  $\Delta V$  and  $i$  for the total field  $\Delta F$  anomalies.
- $\Delta T$  - anomaly in any component : it is equal to  $\Delta H$ ,  $\Delta V$  and  $\Delta F$  when  $D_m$  is 0,  $\Pi/2$  and  $i$  respectively.
- $\Delta T_{cal}$  - calculated anomaly
- $dT$  -  $(\Delta T_{obs} - \Delta T_{cal})$
- $K$  - susceptibility contrast of the body
- $\theta$  - geological dip of the dyke model
- $Q$  - an angle, which is a function of  $\theta$ ,  $\phi$ ,  $D_m$ ,  $i$  and  $\alpha$
- $D$  - distance from an arbitrary reference point to the position of centre of the dyke
- $x$  - distance co-ordinates of anomalies
- $w$  - half width of the dyke
- $C$  - size factor, being a function of magnetisation(in gammas) and dip of dyke
- $z$  - depth to the top of the dyke
- $N_{OBS}$  - Number observations
- $A'$  - regional gradient
- $B'$  - regional at the first station

### Magnetic dyke - forward problem

Dykes and thin sheets are frequently used geophysical models in magnetic studies (Radhakrishna Murthy<sup>11-13</sup>). The dyke is assumed to be a model with a horizontal top bounded by parallel sides (Fig. 2). Equation for its magnetic anomaly can be written by treating it as a model bounded by four first order surfaces. The conventional method is to write down the magnetic anomaly of a line dipole of cross-sectional area  $du \cdot dv$  and integrating it throughout the cross-section of the dyke. When the origin is placed above the center of the dyke the generalized equation for its magnetic anomaly in any component can be written as (Radhakrishna Murthy<sup>12</sup>):

$$\Delta T(x) = 2J' \int_{z_1 - w + (v - z_1) \cot \theta}^{z_2} \int_{-w}^{w + (v - z_1) \cot \theta} \frac{(v^2 - (x - u)^2) \sin \phi' - 2(x - u)v \cos \phi' dv du}{((x - u)^2 + v^2)^2} \text{----- (1)}$$

where  $w$  is the half-width of the dyke,  $\theta$  is its dip,  $z_1$  and  $z_2$  are the depths to its top and bottom and

$$J' = J(1 - \cos^2 \alpha \cos^2 D_m)^{1/2},$$

$$\phi' = \phi - \arctan(\sin \alpha \cot D_m)$$

with  $J$ ,  $\phi$ ,  $\alpha$  and  $D_m$  as defined in notation. The usefulness of the above equation is that one can use the same for calculating vertical, horizontal or total field anomalies by suitably taking the values of  $D_m$ . Equation 1 after integration simplifies to

$$\Delta T(x) = 2 J' \sin \theta [\cos Q \cdot (\theta_A - \theta_B + \theta_C - \theta_D) + \sin Q \ln (r_B r_D / r_A r_C)] \text{-----(2)}$$

where various parameters are defined in Fig.2. For special case when the dyke is extending to a large depth  $\theta_C = \theta_D$  and  $r_D = r_C$ . Thus, Eq.(2) takes the form (Radhakrishna Murthy<sup>3</sup>):

$$\Delta T(x) = C [\cos Q (\theta_A - \theta_B) + \sin Q \ln (r_B / r_A)] \text{----- (3)}$$

The distances and angles involved in Eq. (3) are defined as follows.

$$\theta_A - \theta_B = \arctan((x+w)/z) - \arctan((x-w)/z), \text{ for } z \neq 0,$$

$$r_A^2 = (x+w)^2 + z^2, \quad r_B^2 = (x-w)^2 + z^2$$

with  $Q = \theta - \phi$  and  $C = 2 J' \sin \theta$ .

### The inverse problem

The generalised equation for the magnetic anomaly of a dyke model is of infinite depth extent shown in Eq:(3). The parameters to be determined from its anomalies are  $w$ ,  $z$ ,  $Q$  (from  $\theta - \phi$ ),  $J \sin \theta$  (from  $J' \sin \theta$ ) and  $D$  the position of the center of the dyke. For this, the anomalies  $\Delta T(x_i)$  are digitized at distances measured from a convenient reference on the profile. When the distances are measured relative to the reference point, the anomaly equation is rewritten as

$$\Delta T(x_i) = C [\cos Q (\theta_A - \theta_B) + \sin Q \ln (r_B / r_A)] + A'x_i + B', \quad \text{----- (4)}$$

where  $A'x_i + B'$  is the associated regional anomaly. Initialization of these parameters is achieved by the procedure suggested by Radhakrishna Murthy<sup>14</sup>, which considers the dominant maximum and minimum anomalies and their corresponding distances from the chosen reference point on the magnetic anomaly profile. To calculate the initial values these characteristic distances and/ or anomalies are scaled by the computer from the field profile and their ratios or magnitudes are used to calculate the parameters by properly derived empirical relations. The characteristic distances to be measured are chosen, by trial and error, such that they or their ratios vary significantly with one of the parameters and less significantly with others.

Most often the anomalies  $\Delta T_{cal}$  computed involving these initial parameters differ from the observed anomalies  $\Delta T_{NOBS}$ . The difference between observed and calculated anomalies at the  $i^{th}$  anomaly point is expressed by the Taylor's expansion in terms of errors in the initial values of the model parameters which can be expressed, in terms of errors  $dw$ ,  $dz$ ,  $dD$ ,  $dQ$ ,  $dC$ , regional gradient ( $A'$ ) and constant ( $B'$ ), in the parameters as follows:

$$dT(x_i) = \Delta T_{obs}(x_i) - \Delta T_{cal}(x_i) \quad (i=1, \dots, N_{OBS}) \quad \text{-----(5)}$$

$$dT(x_i) = \sum_{k=1}^7 \frac{\partial \Delta T(x_i)}{\partial a_k} da_k,$$



$$k=1 \quad \partial a_k$$

where  $da_1 = dw$ ,  $da_2 = dz$ ,  $da_3 = dD$ ,  $da_4 = dQ$ ,  $da_5 = dC$ ,  $da_6 = dA'$ , and  $da_7 = dB'$ .

The partial derivatives are calculated through the following equations:

(Radhakrishna Murthy<sup>13</sup>):

$$\frac{\partial \Delta T(x_i)}{\partial a_1} = 2C \left[ \frac{[(x_i - D)^2 + z^2 + w^2]z \cos Q - (x_i - D)[(x_i - D)^2 + z^2 - w^2]^2 \sin Q}{[(x_i - D + w)^2 + z^2][(x_i - D - w)^2 + z^2]} \right],$$

$$\frac{\partial \Delta T(x_i)}{\partial a_2} = 2Cw \left[ \frac{[(x_i - D)^2 + z^2 - w^2] \cos Q + 2z(x_i - D) \sin Q}{[(x_i - D + w)^2 + z^2][(x_i - D - w)^2 + z^2]} \right],$$

$$\frac{\partial \Delta T(x_i)}{\partial a_3} = 2Cw \left[ \frac{2z(x_i - D) \cos Q - [(x_i - D)^2 - z^2 - w^2] \sin Q}{[(x_i - D + w)^2 + z^2][(x_i - D - w)^2 + z^2]} \right],$$

$$\frac{\partial \Delta T(x_i)}{\partial a_4} = C \left[ \frac{[\arctan((x_i - D - w)/z) - \arctan((x_i - D + w)/z)] \sin Q}{+ 0.5 \cos Q \ln [((x_i - D - w)^2 + z^2)/((x_i - D + w)^2 + z^2)]} \right],$$

$$\frac{\partial \Delta T(x_i)}{\partial a_5} = C \left[ \frac{[\arctan((x_i - D + w)/z) - \arctan((x_i - D - w)/z)] \cos Q}{+ 0.5 \sin Q \ln [((x_i - D - w)^2 + z^2)/((x_i - D + w)^2 + z^2)]} \right],$$

$$\frac{\partial \Delta T(x_i)}{\partial a_6} = x_i,$$

and

$$\frac{\partial \Delta T(x_i)}{\partial a_7} = 1.0. \quad \text{----- (6)}$$

The necessary normal equations to be solved for the increments  $da_k$  ( $k=1, \dots, 7$ ) are framed involving Eq.(3), the observed data and partial derivatives with respect to model parameters and are given by

$$\sum_{i=1}^{N_{OBS}} \sum_{k=1}^7 \frac{\partial \Delta T(x_i)}{\partial a_j} da_k \frac{\partial \Delta T(x_i)}{\partial a_k} (1 + \delta \lambda) da_k \quad \text{----- (7)}$$

$$= \sum_{i=1}^{N_{OBS}} \frac{\partial \Delta T(x_i)}{\partial a_j} dT(x_i), \quad (j = 1, \dots, 7)$$

with  $N_{OBS}$  = number of magnetic anomaly points, the constant  $\delta = 1$  for  $i = j$ ,

and 0 otherwise and  $\lambda$  is Marquardt<sup>9</sup> damping factor. The Marquardt damping factor  $\lambda$  is chosen by the relation  $\lambda = 0.5 [ 2^{n-1} - 1 ]$  ( Radhakrishna Murthy <sup>13</sup>). The value of  $\lambda$  is initially set to zero by putting  $n=1$ . Subsequently, it is increased or decreased, each time, by one depending upon the resulting objective function is increased or decreased. The Eq.(7) may be put in the matrix form:

$$M dy = dr \text{ ----- (8)}$$

The Eq. set (8) (for  $i = 1, \dots, N_{OBS}$ ) is to be solved for the increments to the various parameters. To handle this problem there are different approaches that include - a) ridge regression, frequently called Marquardt<sup>9</sup> iteration and b) Generalized Inversion (GI) approach. In the present magnetic data inversion, we follow the second method viz. GI, as it has additional advantages over the Marquardt method (as already mentioned in the beginning).

Equation(8) is written in following discrete form for computational convenience:

$$dr = M dy ; dr = (dr_i) ; dy = (dy_k) ; (i = 1, 2, \dots, N_{OBS} ; k = 1, 2, \dots, 7) \text{ ----- (9)}$$

Equation (9) is solved by generalised inverse<sup>(8,15-17)</sup> (GI) through singular value decomposition (SVD) to obtain the following optimised model parameters<sup>10,18</sup>  $dy_P$  as follows:

$$\begin{matrix} M & = & U & \Gamma & V^T & \text{----- (10)} \\ (N_{OBS} \times m) & & (N_{OBS} \times r) & (r \times r) & (r \times m) & \end{matrix}$$

The columns of U and V are orthonormal i.e  $U^T U = I_{N_{OBS}}$  and  $V^T V = I_m$

with  $I_{N_{OBS}}$  and  $I_m$  are unit matrices of order  $(N_{OBS} \times N_{OBS})$  and  $(m \times m)$  respectively and  $m =$  number of model parameters.

$$\text{In general, } UU^T \neq I_{N_{OBS}} \text{ and } VV^T \neq I_m ; p \leq r = \min(N_{OBS}, m) \text{ ----- (11)}$$

where  $r$  is the rank of the matrix  $M$  (data kernel) ,  $p$  number of factors to be

considered and the matrices  $U$  and  $V$  are the respective coupled Eigen vector matrices for the Eigen value problem defined as:

$$(M M^T) U \equiv \Gamma^2 U \text{ -----(12)}$$

and  $(M^T M) V \equiv \Gamma^2 V \text{ -----(13)}$

In Eq.(10),  $\Gamma$  is diagonal matrix of non-zero singular values ( $\Gamma_i^2 = \lambda_i'$ ) of  $M$  arranged in the decreasing order. Once  $U$ ,  $\Gamma$  and  $V$  are obtained by solving above Eigenvalue problem (Eqs.12 and 13) the generalised inverse solution (Jackson<sup>18</sup>) is given by

$$dy_p = V V^T dy = M_g^{-1} dr \text{ -----(14)}$$

where  $M_g^{-1} = V_p \Gamma_p^{-1} U_p^T$  is the natural generalised inverse operator(GIO). Let  $s(d) = \{u_1, u_2, \dots, u_n\}$  and  $s(m) = \{v_1, v_2, \dots, v_m\}$  be sets of orthogonal vectors ( i.e., column vectors of complete data space  $U$  and model space  $V$ ) as detailed below:

$U = \{U_p, U_{r-p}, U_0\}$  is the complete data space where

$U_p$  : optimised data space - set of  $p$  eigenvectors corresponding to dominant eigenvalues such that  $\lambda'_1 > \lambda'_2 > \lambda'_3 > \dots > \lambda'_p \neq 0$ .

$U_{r-p}$  : set of  $(r-p)$  eigenvectors corresponding to remaining dominant eigenvalues such that  $\lambda'_{p+1} > \lambda'_{p+2} > \dots > \lambda'_r \neq 0$  and

$U_0$ : Null space - having eigenvectors corresponding to  $r - \min(N_{OBS} \times m)$  zero eigenvalues.

Similarly it follows in the case of complete model space (i.e.  $V = \{V_p, V_{r-p}, V_0\}$ ) and can also be explained .

If  $VV^T$  equals to  $I_m$  (i.e. the rank of the matrix  $M = m$ ) then the solution of the Eq. (8) is

$$dy_p = dy = M_g^{-1} dr \text{ -----(15)}$$

If  $VV^T$  not equals to  $I_m$  (for the case of, presence of noise in the model space)

$$dy_P \approx dy = M^{-1}_g dr \quad \text{-----(16)}$$

For better estimates the resolution in the model space  $VV^T$  of Eq.(16) has to be improved. This is done judiciously by selecting the  $p(\leq r)$  eigenvectors (also called factors or modes) in the activated model space  $s(m)$  corresponding to the ranking of the  $p$  singular values ( $\lambda'_i$ ) of the data kernel ( $M$ ) in descending order. The noise in the data kernel prevailing in the form of small values increases the ranking of the matrix apart from amplifying the solution. This however does not provide any additional information on the model parameters. So, it can be treated as though the solution to the present problem is obtained through the optimisation.

The above process is continued to obtain optimised model based on the best fit between the observed and calculated anomalies. Once the optimised solution is obtained, it is necessary to assess how best the data determines the model parameters. This is done through model resolution ( $V_p V_p^T$ ) and data resolution ( $U_p U_p^T$ ) matrices.

The following mathematical presentation shows us how the data and model space are interrelated. Pre-multiplying (Eq. 14) by  $V_P^T$  on both sides of the equation and by defining the general parameters

$$\alpha = V_P^T dy_P \quad \text{----- (17)}$$

$$\Rightarrow \alpha_j = v_j^T \cdot dy_P$$

and

$$\beta = U_P^T dr_P \quad \text{----- (18)}$$

$$\Rightarrow \beta_j = u_j^T \cdot dr_P$$

it reduces to

$$\alpha = \Gamma^{-1} \beta \quad \text{----- (19)}$$

The relation shows how data errors in  $\beta$  component belong to small eigenvalues  $\lambda'_i$  (of diagonal matrix  $\Gamma$ ) are strongly amplified giving rise to large errors of  $\alpha_j$  and hence those components  $dy$ , which contributes to  $\alpha_j$ . For the investigation of parameter errors and information content of the data it has become tradition to use the model resolution matrix  $R = V V^T$  and information density matrix  $S = U U^T$ .

From Eq. (17) we see that an investigation of the vector  $v_j$  will reveal which

parameters of  $dy$  are specifically attached to the generalised parameters  $\alpha_j$ . From Eq. (18) we see that similar investigation of the vector  $u_j$  will reveal which data of  $dr$  are particularly contributing to the content of the generalised datum  $\beta_j$ . The components of  $v_j$  directly indicate which of the parameters of  $dy$  are determined. On other hand the components of  $u_j$  indicate where to find those data points that contribute to the determination of these parameters  $v_j^{10}$ .

**Closeness ratio:** The ratio between the sum of the factors considered and that of the data matrix (kernel) is the measure of closeness of the model data

$$\text{Measure of closeness} = \frac{\sum_{i=1}^p \lambda'_i}{\sum_{i=1}^r \lambda_i} \quad \text{----- (20)}$$

where  $r$  is the rank of the data matrix  $M$  (Kernel). The first eigen function,  $u_1$  associated with the largest eigenvalues  $\lambda'_1$  represents the gross features (i.e.,  $\|u_1^T M\|$ ) in the data in the least square sense, while the second function  $u_2$  associated with second largest eigenvalue  $\lambda'_2$  describes the residual mean square data [ i.e.,  $\|u_2^T (M - (u_1 u_1^T M))\|$  ] in the least square sense and so on. The closeness ratio is expressed in percentage to judge the contribution of different parameters.

## SOFTWARE DETAILS:

The software program details for the dyke model analysis using GI are presented in program *GIDYKE.FOR* (**ANNEXURE – I**) with another synthetic dyke model as input and output details with FORTRAN coding in **ANNEXURE – II**.

### Dyke inversion - Synthetic and field examples:

The performance of the natural Generalised Inverse (GI) explained above is illustrated with synthetic and field data sets by carrying two numerical experiments as detailed below.

#### *Synthetic data:*

A synthetic dyke model with parameters  $w = 4.0 \text{ km}$ ,  $z = 1.0 \text{ km}$ ,  $D = 10.0 \text{ km}$ ,  $Q = 90^\circ$  and  $J = 100$  are taken and computed the magnetic anomaly at 21 points (NOBS) using the forward model Eq.(3). The synthetic anomaly so generated is subjected to GI and the derived model anomaly denoted by stars and corresponding model (dotted circles) shown in Fig. 3A, which is well coincided with actual synthetic model anomaly denoted solid line. The GI optimized model parameters are  $w = 3.99 \text{ km}$ ,  $z = 1.07 \text{ km}$ ,  $D = 10.0 \text{ km}$ ,  $Q = 90^\circ$  and  $J = 101.06$  shows the efficacy of the method. Root Mean square Error (RMS) at initial (zero) iteration is 9.83 and the objective function is 2806. At 5th iteration, RMS error in GI optimized model is 1.68 while in Marquardt Inversion (MI) after 24 iterations the RMS error is 4.72 and objective function is 468 (Table I b). The success of synthetic model inversion with GI prompted to carryout the field data inversion using GI.

#### *Field data:*

Marine magnetic total field data was collected, using Baringer Proton precession magnetometer (model M123) off Lawson's Bay (opposite Kailash Hill) of Visakhapatnam(  $17^\circ 42' \text{ N}$  ,  $83^\circ 17' \text{ E}$ ) along a coast parallel profile (HH') (Fig. 1) at 40 m water depth, covering a distance of 2.5 km, at one-minute polarisation

interval. Figure 3B shows the observed magnetic anomaly along this coast parallel profile HH' of Fig.1. The observed total field marine magnetic data were corrected using 1985 IGRF coefficients (DGRF<sup>19</sup>) and the residual data at 26 points on the profile is considered in the present GI analysis. The residual marine magnetic anomaly ranges between -460 to 170 nT with a low of -460nT at 1.25 km from the reference point, flanked by highs on either side, 60nT(left) and 170 nT (right)(Fig.3B). This magnetic anomaly is subjected to GI to yield optimized dyke parameters. Figure 3B also shows the calculated anomalies of initial and final dyke model using GI.

### Field example - Results and discussion

In the present section, computed results of the field data (Fig.3B) are discussed. SVD of kernel  $M_{(26 \times 7)} = U \sqrt{\Lambda}^T$  has been performed on residual magnetic data and shown as observed data in Fig. 3B. The column matrices  $U = (u_1, u_2, \dots, u_7)$  and  $V = (v_1, v_2, \dots, v_7)$  are orthonormal eigenvectors spanning data space  $s(d)$  and model  $s(m)$  corresponding to eigenvalues  $\lambda_i'$  of diagonal matrix  $\sqrt{\Lambda}$  (Table 1) computed by solving coupled eigen value problem of two covariance matrices  $MM^T$  and  $M^T M$  of 7 components contributing 100% of total variance. Based on the point of inflection or minimum point of cumulative eigenvalues, four factors ( $p=4$ ) are retained for interpretation as shown in Figs.4 and 5 (however other figures in respect of other 3 factors are also shown for the sake of completeness). The GIO  $M_g^{-1} = V_p \sqrt{\Lambda_p}^{-1} U_p^T$  has been built after removing null spaces  $\{U_0, V_0\}$  to operate on observed magnetic anomaly to yield dyke intrusive model parameters in the present studies (Fig.3B, field model).

Using vectors in activated space, data resolution matrix  $\{U_p U_p^T\}$  and model resolution matrix  $\{V_p V_p^T\}$  have been computed and the elements of these matrices are presented in 3-D contour form (Figs.6 and 7) respectively. The diagonal elements of these resolution matrices are presented in Tables 2 and 3. The column Vectors  $u_i$  ( $i=1, \dots, 7$ ) of  $s(d)$  of corresponding eigenvalues of  $\lambda_i'$  (i.e. measure of variance  $M$  in descending order), having 26 components and with each component a value that indicates amplitude, which explains sharing contribution of its variance mode  $i$ . The component  $u_i$  represent the trend or

direction of spatial common features contained in kernel  $M$  (partial derivatives of magnetic function with respect to model parameters). Its corresponding eigenvalue  $\lambda_i$  represents energy level. In a similar manner, the components of vectors in model space  $s(m)$  can be explained. The percentage contribution of individual components of  $s(m)$  and  $s(d)$  have been computed through closeness ratio approach (Eq. 20) and are shown in Tables 4 and 5. Eigenvectors derived from the data kernel of present field example have been presented in Figs. 4 and 5 to explore the spatial and model parameter variability. From these Figs. 4 and 5 and Tables 4 and 5 the first four eigenfunctions of  $s(d)$  and  $s(m)$  account for more than 98% of variance relatively dominated by the first function alone. The first function  $u_1$  and  $v_1$  contribute 58% of total distribution representing model parameters  $Q$  followed by  $D$ ,  $w$ , and  $z$  in that order. In general,  $Q$  - an angle, function of  $\theta$ ,  $\phi$ ,  $D_m$ ,  $i$  and  $\alpha$ , contributes to the extent of 45% between stations 18 and 21. Further  $z < w$ , indicating that it is thick dyke intrusive. The contribution of remaining three functions in  $s(d)$  and  $s(m)$  is only 2% of total information, and is too insignificant to consider for interpretation.

Therefore the first four energetic spatial and model functions of  $s(d)$  and  $s(m)$  have been used to bring the predictable rich part of the original signal to obtain inverse solution (Fig.3B) by operating GI on magnetic anomaly and for construction of data and model resolution matrices in 3-D form (Figs. 6 and 7). At the end of 21 iterations the results obtained from GI indicate that the width of the dyke intrusive model is 0.72 km at a depth ( $Z$ ) of 0.12 km from the surface located at ( $D$ ) 1.44 km away from the reference point and at an angle ( $Q$ ) of  $140^\circ$  and with intensity of magnetization ( $C$ ) 225.86 nT (Fig 3B). The optimized value of intensity of magnetization ( $C$ ) is reasonably comparable with local value (250. nT)<sup>12</sup>. Computed model anomaly with refined parameter set almost coincides with observed field anomaly to quantify (Fig. 3B). The initial objective function (=111039) is reduced up to (=4645) after 21 iterations, and the corresponding refined model parameter set is considered for interpretation. The refined intrusive dyke model corresponding to minimum objective function is shown in Fig.3B. The Root Mean Square (RMS) error for the field data at zero iteration is 65.35. After 21



iterations it has reduced to 13.36.

### **Data resolution:**

The data resolution matrix is an indication of the information density of the data kernel i.e. it indicates which data contribute independent information to the solution. The diagonal elements of  $U_p U_p^T$  are shown in Table 3 for factors 1 to 7. A value of unity indicates contribution of information independent of other observations. From Table 3 one can infer which of the data points contribute strong / poor information resolution. The data resolution is given by  $N = MM^1_g = U_p U_p^T$ . The data are completely resolved if  $U_p$  spans the complete space of data. The small eigenvalues in the data kernel increases the rank of the matrix (i.e. the dimension of activated space) besides amplifying the solution due to the presence of noise. Such a filtered data kernel has been utilized in 3-D form (Fig. 8), which almost represents the original data kernel to build GIO. In Fig. 6 Factor 1 describes the contour map of data resolution matrix  $u_1 u_1^T$ , where  $u_1$  is the highest spatial energetic mode of  $s(d)$ . It gives 58.003% variance of the total information i.e. mostly gross features of the kernel (Table 5). High resolution is found at stations 10 and 18 indicating the sudden change at these stations likely indicating the width of the dyke intrusive (Fig. 6 and Tables 3-5)

### **Model resolution:**

The model resolution of the generalised inverse is given by  $R = M^1_g M = V_p V_p^T$ . Here  $p$  indicates number of factors used in SVD, which is less than or equal to the rank of the matrix  $M$ . The model parameters will be perfectly resolved if  $V_p$  spans the complete space of the model parameters i.e.  $V_p V_p^T = I_m$ . The model resolution is perfect for the contribution of magnetic anomaly due to  $Q$  (Fig. 7, factor 1 and shown in Table 4) and one can conclude that  $Q$  is independent of other model parameters and of homogeneous nature which is also reflected in the data resolution Fig. 6, factor 1 at stations 12 and 18 (also see Table 5). Table 4 presents the individual parameter contribution in each of the seven factors. In first factor the contribution of  $Q$  is 45.222% out of 58%, and in second factor the contribution of  $W$  is 9.7% out of 18.5%, and in third factor  $z$  contributes 7.1% out of 13.2% and finally in fourth factor that of  $D$  is 5.5% out of 8.1%. The total

contribution of  $Q$ ,  $w$ ,  $z$  and  $D$  is 62% out of 89.8% from the first four factors.

From the above model and data resolution study, the partial information contained in the data space is adequate enough to reconstruct the approximate model parameters. This reveals the efficacy of natural generalised inverse GI with a least square sense in handling problems of over determinacy.

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## Synthetic Example – Dyke Model:

Table: I (a) **Analysis by generalised inverse**

### Magnetic anomalies – due to dyke model

ITERATION NUMBER 0			
DIS- TANCE	OBSERVED ANOMALY	CALCULATED ANOMALY	ERROR
.00	83.6	82.4	1.2
1.00	93.9	95.2	1.3
2.00	107.2	110.7	3.5
3.00	125.1	129.5	4.4
4.00	150.3	152.1	1.8
5.00	185.7	176.3	9.4
5.50	197.5	185.7	11.8
6.00	208.7	189.2	19.5
6.50	185.0	183.3	1.7
7.00	160.9	167.2	6.3
7.50	130.0	143.6	13.6
8.00	100.1	116.1	16.0
8.50	74.0	87.1	13.1
9.00	47.8	58.0	10.2
10.00	.0	.0	.0
11.00	-47.8	-58.0	10.2
11.50	-74.0	-87.1	13.1
12.00	-100.1	-116.1	16.0
12.50	-130.0	-143.6	13.6
13.00	-160.9	-167.2	6.3
13.50	-185.0	-183.3	1.7
14.00	-208.7	-189.2	19.5
14.50	-197.5	-185.7	11.8
15.00	-185.7	-176.3	9.4
16.00	-150.3	-152.1	1.8
17.00	-125.1	-129.5	4.4
18.00	-107.2	-110.7	3.5
19.00	-93.9	-95.2	1.3
20.00	-83.6	-82.4	1.2

OBJECTIVE FUNCTION IS 2806.

#### MODEL PARAMETERS

DEPTH TO TOP OF DYKE IS	1.83
HALF THE THICKNESS IS	3.66
THE ORIGIN IS AT	10.00
THETA-PHI IS	90.00
CONST IS	136.09
REGIONAL AT 1ST STATION IS	-17.81
REGIONAL GRADIENT IS	1.78

(contd ...)

**Table I(a) ITERATION NUMBER 5**

DIS- TANCE	OBSERVED ANOMALY	CALCULATED ANOMALY	ERROR
.00	83.6	83.4	.2
1.00	93.9	93.9	.0
2.00	107.2	107.4	.2
3.00	125.1	125.4	.3
4.00	150.3	150.4	.1
5.00	185.7	184.3	1.4
5.50	197.5	200.5	3.0
6.00	208.7	204.7	4.0
6.50	185.0	188.5	3.5
7.00	160.9	159.9	1.0
7.50	130.0	129.3	.7
8.00	100.1	100.3	.2
8.50	74.0	73.4	.6
9.00	47.8	48.0	.2
10.00	.0	.0	.0
11.00	-47.8	-48.0	.2
11.50	-74.0	-73.4	.6
12.00	-100.1	-100.3	.2
12.50	-130.0	-129.3	.7
13.00	-160.9	-160.0	.9
13.50	-185.0	-188.5	3.5
14.00	-208.7	-204.7	4.0
14.50	-197.5	-200.5	3.0
15.00	-185.7	-184.3	1.4
16.00	-150.3	-150.4	.1
17.00	-125.1	-125.4	.3
18.00	-107.2	-107.4	.2
19.00	-93.9	-93.9	.0
20.00	-83.6	-83.4	.2

OBJECTIVE FUNCTION IS 83.

MODEL PARAMETERS

DEPTH TO TOP OF DYKE IS 1.07  
 HALF THE THICKNESS IS 3.99  
 THE ORIGIN IS AT 10.00  
 THETA-PHI IS 90.00  
 CONST IS 101.68  
 REGIONAL AT 1ST STATION IS -1.32  
 REGIONAL GRADIENT IS .13

**Table : I (b) GI analysis – RMS error details**

Model	Z	W	D	Q=( $\theta-\phi$ )	CONST.	Reg.	Itr / Obj.fun	RMS err
Exact	1.0	4.0	10.0	90.0	100	0	---	---
Initial	1.83	3.66	10.0	90.0	136.09	-17.8	0/2806	9.83
GI Mdl.	1.07	3.99	10.0	90.0	101.68	-1.32	5/83	1.69
MI Mdl.	1.0	4.01	9.99	89.7	119.2	-23.4	24/468	4.72



**Table 4: Percentage contribution of individual components of model space  $s(m)$**

<b>Parameters</b>	$v_1\%$	$v_2\%$	$v_3\%$	$v_4\%$	$v_5\%$	$v_6\%$	$v_7\%$
W	.837	-9.693	4.972	-.427	.153	.000	.000
Z	-.186	6.917	7.106	-.046	.021	.001	.000
D	10.504	.562	-.340	-5.563	-.185	.000	.000
Q	-45.222	-.108	.009	-1.282	-.078	.000	.000
Const	.000	-.168	-.026	.000	-.050	.046	-.019
A	-1.255	1.041	-.714	-.780	1.361	.004	.002
B	.000	.084	-.061	-.012	.107	-.026	-.033
Total (%)	58.003	18.573	13.228	8.111	1.454	0.0078	0.054



Table 5: Percentage contribution of individual component of data space  $s(d)$ 

Station ID	$u_1\%$	$u_2\%$	$u_3\%$	$u_4\%$	$u_5\%$	$u_6\%$	$u_7\%$
0	1.004	.079	-.138	.233	.042	-.003	-.006
1	1.121	.085	-.0157	.245	.054	-.003	-.005
2	1.223	.095	-.182	.258	.065	-.002	-.004
3	1.325	.106	-.213	.270	.077	-.001	-.003
4	1.465	.116	-.251	.284	.088	-.001	-.002
5	1.618	.137	-.307	.296	.098	.000	-.001
6	1.821	.169	-.383	.299	.106	.000	-.001
7	2.076	.227	-.499	.288	.109	.000	.002
8	2.420	.349	-.681	.227	.098	-.002	.003
9	2.891	.671	-.966	.023	.052	-.003	.004
10	3.401	1.612	-1.217	-.526	-.075	.003	.001
11	3.070	2.732	-.242	-.757	-.104	.003	-.004
12	1.554	1.981	.725	-.117	.068	-.003	.001
13	.153	1.379	.809	.036	.107	-.006	.003
14	-1.006	1.173	.743	-.018	.087	-.006	.003
15	-2.089	1.199	.668	-.074	.053	-.003	.001
16	-3.184	1.437	.546	-.006	.016	.004	-.001
17	-4.330	1.892	.107	.415	-.018	.005	-.002
18	-4.955	1.580	-1.107	.869	-.041	.005	.001
19	-4.114	.132	-1.245	.000	-.028	-.006	.001
20	-3.210	-.275	-.753	-.456	.016	-.006	-.001
21	-2.611	-.301	-.464	-.536	.056	-.002	-.002
22	-2.203	-.264	-.311	-.523	.088	.001	-.001
23	-1.911	-.227	-.220	-.487	.114	.003	-.001
24	-1.694	-.190	-.166	-.450	.136	.005	.000
25	-1.516	-.164	-.129	-.417	.156	.007	.001
Total(%)	58.003	18.573	13.228	8.111	1.954	.078	.054

Table 6: **INTERPRETATION OF MAGNETIC ANOMALIES DUE TO A DYKE BY GENERALISED INVERSE APPROACH**

<b>ITERATION NUMBER 0</b>			<b>ITERATION NUMBER 21</b>		
	↓			↓	
DIS- TANCE	OBSERVED ANOMALY (nT)	CALCULATED ERR.(nT)	CALCULATED ANOMALY(nT)	ERR. (nT)	
0.00	60.	148.	88.	56.	4.
1.00	55.	122.	67.	53.	2.
2.00	52.	95.	43.	49.	3.
3.00	49.	66.	17.	44.	5.
4.00	40.	34.	6.	38.	2.
5.00	20.	-1.	21.	29.	9.
6.00	10.	-41.	51.	17.	7.
7.00	-5.	-87.	82.	-1.	4.
8.00	-30.	-141.	111.	-30.	0.
9.00	-70.	-205.	135.	-81.	11.
10.00	-160.	-281.	121.	-180.	20.
11.00	-350.	-359.	9.	-337.	13.
12.00	-455.	-409.	46.	-436.	20.
13.00	-460.	-410.	50.	-456.	4.
14.00	-450.	-375.	75.	-444.	6.
15.00	-407.	-323.	84.	-412.	5.
16.00	-335.	-261.	74.	-358.	23.
17.00	-245.	-190.	55.	-261.	16.
18.00	-108.	-109.	1.	-95.	13.
19.00	35.	-17.	52.	46.	11.
20.00	115.	77.	38.	102.	13.
21.00	100.	145.	45.	124.	24.
22.00	110.	166.	56.	133.	23.
23.00	130.	153.	23.	137.	7.
24.00	150.	128.	22.	140.	10.

	<b>At Iteration 0</b>	<b>At Iteration 21</b>
OBJECTIVE FUNCTION IS	111039.	4645.

**MODEL PARAMETERS**

DEPTH TO TOP OF DYKE IS	2.25 units	0.12 units
HALF THE THICKNESS IS	4.50 units	0.36 units
THE ORIGIN IS AT	16.10 units	1.44 units
THETA-PHI IS	86.05°	140.78°
CONST IS	67.58 nT	225.86 nT
REGIONAL AT 1ST STN. IS	289.63 nT	95.15 nT
REGIONAL GRADIENT IS	-15.39	1.01 nt

## ANNEXURE - I

### SOFTWARE PROGRAM DETAILS:

#### Main Program *GIDYKE.FOR*

The main program GIDYKE.FOR in FORTRAN – 77 code form consists of seven routines GIO, SVD, MATMLT, TRNSPZ, ARRANGE, OUTPUT and the seventh DYKE.FOR (DYKE forward model). The necessary input to the main program is given in file DYKEIN.DAT that consists of number of anomaly (data) points: NOBS; number of iterations to carry out: NITER followed by corresponding model details. The difference between observed and model anomaly is computed and stored in array ERROR (K) and sum of squares of these errors is stored as single variable FUNCT (objective function). Now data kernel (coefficient matrix) is generated in matrix P of order ( $N_{OBS} \times M$ ) and column matrix PB of order ( $N_{OBS} \times 1$ ), consisting of difference between observed and computed anomaly are also stored. The output file DYKEOUT.DAT consists of model parameter details for each iteration and objective function. The main program opens files like DYMRES.DAT & DYDRES.DAT to write model and data resolution details for each iteration.

At this stage subroutine GIO is called to obtain output argument Q as column matrix of order ( $N_{OBS} \times 1$ ) by sending input arguments P and PB. The model parameters are updated and fed to DYKE.FOR to compute forward anomaly and there by FUNCT1 which is the current objective function. In the main program this process is repeated iteratively such that ensemble average of the squares of the difference between estimated field and true field at each point is minimum, to

compute the best model parameters.

**Subroutines:**

**Subroutine GIO:** This subroutine GIO takes the data kernel A of order  $(N_{OBS} \times M)$  and D is a column matrix of order  $(N \times 1)$  consisting of deviation in the anomaly from the main program and calls the subroutine SVD to perform singular value decomposition to get orthonormal matrices U of order  $(N_{OBS} \times NF)$ , diagonal matrix W of order  $(NF \times NF)$  and V matrix of order  $(N_{OBS} \times NF)$ . It calls subroutine ARRANGE to arrange eigenvalues in descending order and corresponding vectors are arranged in GIO. Then it computes the closeness ratio as described in theory (Eq. 20) after that it generates generalised inverse operator matrix, X. Using U, W and V matrices and it operates on D matrix and computes the inverse solution and stores in column matrix DELC  $(N \times 1)$ . Also it computes the data resolution, model resolution and returns to the calling program.

**Subroutine SVD:** This subroutine SVD takes data kernel A  $(N_{OBS} \times N)$  from GIO and computes covariance matrix to solve the eigen value problem to perform SVD and to yield orthonormal matrices U, V and diagonal matrix W. The routine LINPACK available in public domain is used in SVD routine.

**Subroutine MATMLT:** This subroutine MATMLT is called from the calling program to compute the multiplication of two matrices A and B of order  $(N_{OBS} \times N)$  and  $(N \times P)$  and stores in Matrix C of order  $(N \times P)$ .

**Subroutine TRNSPZ:** This subroutine TRNSPZ is called in the calling program many times to compute the transpose of matrix A of order  $(N \times N_{OBS})$  and stores in matrix B of order  $(N_{OBS} \times N)$ .

**Subroutine ARRANGE:** This subroutine ARRANGE is called in subroutine GIO and arranges eigen values in descending order.

**Subroutine OUTPUT:** This subroutine OUTPUT is simply an output subroutine that writes the SVD results to an access disk file (SVDOUT.DAT).

## ANNEXURE - II

### Synthetic model

GIODYKE PROGRAM INPUT AND OUTPUT FILE DETAILS

**INPUT - FILE:** DYKEIN.DAT

```
17,45.0,90.0,25
0.0,1.0,2.0,3.0,4.0,5.0,6.0,7.0
8.0,9.0,10.0,11.0,12.0,13.0,14.0,15.0
16.0
-220.,-300.,-370.,-460.,-495.,-340.,-210.,-80.
60.,230.,390.,510.,560.,450.,330.,250.
200.
```

OUTPUT - FILE: DYKEOUT.DAT

ANALYSIS BY GENERALISED INVERSE

INTERPRETATION OF MAGNETIC ANOMALIES DUE TO A DYKE  
USING ITERATIVE APPROACH. THE FOLLOWING TABLES PROVIDE  
INFORMATION ON ANOMALIES AND MODEL PARAMETERS AT THE  
END OF EACH ITERATION.

ITERATION NUMBER 0

DIS- TANCE	OBSERVED ANOMALY	CALCULATED ANOMALY	ERROR
.00	-220.	-180.	40.
1.00	-300.	-250.	50.
2.00	-370.	-330.	40.
3.00	-460.	-416.	44.
4.00	-495.	-469.	26.
5.00	-340.	-419.	79.
6.00	-210.	-283.	73.
7.00	-80.	-126.	46.
8.00	60.	31.	29.
9.00	230.	188.	42.
10.00	390.	346.	44.
11.00	510.	481.	29.
12.00	560.	528.	32.
13.00	450.	473.	23.
14.00	330.	387.	57.
15.00	250.	307.	57.
16.00	200.	238.	38.

OBJECTIVE FUNCTION IS 36863.

MODEL PARAMETERS

DEPTH TO TOP OF DYKE IS	1.82
HALF THE THICKNESS IS	3.64
THE ORIGIN IS AT	7.99
THETA-PHI IS	270.00
CONST IS	399.10
REGIONAL AT 1ST STATION IS	184.45
REGIONAL GRADIENT IS	-19.40

\*\*\*\*\* AT THE END OF 9 ITERATIONS \*\*\*\*\*

ITERATION NUMBER 9

DIS- TANCE	OBSERVED ANOMALY	CALCULATED ANOMALY	ERROR
.00	-220.	-217.	3.
1.00	-300.	-287.	13.
2.00	-370.	-375.	5.
3.00	-460.	-472.	12.
4.00	-495.	-494.	1.
5.00	-340.	-366.	26.
6.00	-210.	-204.	6.
7.00	-80.	-57.	23.
8.00	60.	82.	22.
9.00	230.	222.	8.
10.00	390.	372.	18.
11.00	510.	515.	5.
12.00	560.	543.	17.
13.00	450.	445.	5.
14.00	330.	342.	12.
15.00	250.	260.	10.
16.00	200.	195.	5.

OBJECTIVE FUNCTION IS 3072.

MODEL PARAMETERS

DEPTH TO TOP OF DYKE IS	1.33
HALF THE THICKNESS IS	3.86
THE ORIGIN IS AT	7.73
THETA-PHI IS	272.90
CONST IS	332.51
REGIONAL AT 1ST STATION IS	127.43
REGIONAL GRADIENT IS	-16.16

STOP END OF PROGRAM



## SOFTWARE CODING

```

C      GIODYKE.FOR
C      TO REFINE DYKE MODEL PARAMETERS BY GENERALISED INVERSE
C      TECHNIQUE FROM MAGNETIC DATA
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL LAMDA,LAMDA1
      PARAMETER (NM=17,M=17,N=7,NF=7)
      DIMENSION X(17),T(17),TCAL(17),ERROR(17), 1P(17,7),
      PB(17), DELC(7),PA(1500),DIF(17)
      OPEN(1, FILE='DYUCOMP.DAT', ACCESS='SEQUENTIAL')
      OPEN(2, FILE='DYVCOMP.DAT', ACCESS='SEQUENTIAL')
      OPEN(3, FILE='DYGENSOL.DAT', ACCESS='SEQUENTIAL')
      OPEN(4, FILE='DYMRES.DAT', ACCESS='SEQUENTIAL')
      OPEN(5, FILE='DYDRES.DAT', ACCESS='SEQUENTIAL')
      OPEN(6, FILE='DYBCAL.DAT', ACCESS='SEQUENTIAL')
      OPEN(7, FILE='DYSVDOUT.DAT', ACCESS='SEQUENTIAL')
      OPEN(8, FILE='DYRESYL.DAT', ACCESS='SEQUENTIAL')
      OPEN(9, FILE='DYGIO.DAT', ACCESS='SEQUENTIAL')
      OPEN(10,FILE='DYKEIN.DAT', ACCESS='SEQUENTIAL')
      OPEN(11,FILE='DYKEOUT.DAT', ACCESS='SEQUENTIAL')
      OPEN(12,FILE='DYPER.DAT', ACCESS='SEQUENTIAL')
      OPEN(13,FILE='DYMDIA.DAT', ACCESS='SEQUENTIAL')

C
      READ(10,801)NOBS,STRIKE,DM,NITER
      READ(10,802) (X(K),K=1,NOBS)
      READ(10,803) (T(K),K=1,NOBS)
      WRITE(8,2)
      WRITE(13,2)
      RAD=0.0174532924
      NN=7
      N2=3
      NLAMDA=1
      CON=1.0
      XFAR1=0.0
      XFAR2=0.0
      STR=STRIKE*RAD
      DMR=DM*RAD
      IF(DMR.EQ.0.0)THEN
        DM1=1.57079637
      ELSE
        DM1=DATAN(DSIN(STR)/DTAN(DMR))
      ENDIF

C      LOCATION OF MAXIMUM AND MINIMUM ANOMALIES
      CALL MAXMIN(X,T,NOBS,MAX,MIN,XMAX,XMIN,TMAX,TMIN)
C      LOCATION OF ORIGIN
      IF(TMIN.NE.0.0) THEN
5        WIDTH=DABS(XMAX-XMIN)
        D=ORIGIN(X,T,NOBS,MAX,MIN,XMAX,XMIN,TMAX,TMIN)
      ELSE
        FAR=0.5
      WIDTH=FMAX(X,T,NOBS,TMAX,FAR,XFAR1,XFAR2)

```

```

        D=XMAX
        ENDIF
C      CALCULATION OF INITIAL VALUES OF W,Z AND Q
        R=DABS(TMIN/TMAX)
        IF(R-0.05)10,15,15
10     Q=0.0
        Z=WIDTH/4.472
        GO TO 45
15     IF(R-0.27)20,25,25
20     Q=30.0
        GO TO 40
25     IF(R-0.7)30,30,35
30     Q=60.0
        GO TO 40
35     Q=90.0
40     S=DSIN(Q*RAD)
        Z=WIDTH*S/(2.0*DSQRT(1+4.0*S*S))
45     IF(XMAX-XMIN)50,60,60
50     IF(TMAX)55,75,75
55     Q=Q+180.0
        GO TO 75
60     IF(TMAX)65,70,70
65     Q=180.0-Q
        GO TO 75
70     Q=360.0-Q
75     W=2.0*Z
        Q=Q*RAD
        CONST=1.0
        WRITE(11,911)
        DO 80 I=1,9
80     PA(I)=0.0
        DO 85 I=1,3
85     PB(I)=0.0
C      CALCULATION OF INITIAL VALUES OF THE SIZE FACTOR A AND B
        DO 95 K=1,NOBS
            XX=X(K)-D
            TCAL(K)=DYKE(XX,W,Z,CONST,Q)
            ERROR(1)=X(K)
            ERROR(2)=1.0
            ERROR(3)=TCAL(K)
            ERROR(4)=T(K)
            DO 95 J=1,N2
                DO 90 I=1,N2
                    L=(J-1)*N2+I
90             PA(L)=PA(L)+ERROR(J)*ERROR(I)
                PB(J)=PB(J)+ERROR(J)*ERROR(I)
95     CONTINUE
        CALL SIMQ(PA,PB,N2,KS)
        A=PB(1)
        B=PB(2)
        CONST=PB(3)
        FUNCT=0.0

```

```

C      CALCULATION OF ANOMALIES OF THE INITIAL MODEL
DO 100 K=1,NOBS
      XX=X(K)-D
      TCAL(K)=DYKE(XX,W,Z,CONST,Q)+A*X(K)+B
      DIF(K)=(T(K)-TCAL(K))
      DIF(K)=DABS(DIF(K))
100    FUNCT=FUNCT+(T(K)-TCAL(K))**2
C      INVERSION SCHEME STARTS
DO 175 ITER=1,NITER
      ICOUNT=0
      ITER1=ITER-1
      QQ=(Q-DM1)/RAD
      WRITE(11,908)ITER1
      WRITE(11,902)
      WRITE(11,903)(X(K),T(K),TCAL(K),DIF(K),K=1,NOBS)
      WRITE(11,906)
      WRITE(11,900)FUNCT
      WRITE(11,901)Z,W,D,QQ,CONST,B,A
      IF(FUNCT-NOBS)105,110,110
105    WRITE(11,912)
      STOP
110    IF(ITER-NITER)115,115,180
115    DO 120 L=1,NOBS
      DO 120 MM=1,NN
120    P(L,MM)=0.0
      C=DCOS(Q)
      S=DSIN(Q)
      Z2=Z*Z
C      CALCULATION OF DERIVATIVES
DO 130 K=1,NOBS
      R1=X(K)-D
      R2=R1+W
      R3=R1-W
      R4=Z2+R2*R2
      R5=Z2+R3*R3
      R12=R1*R1
      W2=W*W
      R45=R4*R5
      R6=R12-Z2-W2
      R7=R12+Z2+W2
      R8=R12+Z2-W2
      A1=DATAN(R2/Z)
      A2=DATAN(R3/Z)
      ERROR(1)=2.0*CONST*(R7*Z*C-R1*S*R8)/R45
      ERROR(2)=2.0*CONST*W*(R6*C+2.0*Z*R1*S)/R45
      ERROR(3)=2.0*CONST*W*(-R6*S+2.0*Z*R1*C)/R45
      ERROR(4)=-S*(A1-A2)-0.5*C*DLOG(R4/R5)
      ERROR(4)=ERROR(4)*CONST
      ERROR(5)=(TCAL(K)-A*X(K)-B)/CONST
      ERROR(6)=X(K)
      ERROR(7)=1.0
      ERROR(8)=T(K)-TCAL(K)

```

```

        PB(K)=ERROR(8)
        P(K,1)=ERROR(1)
        P(K,1)=ERROR(1)
        P(K,2)=ERROR(2)
        P(K,3)=ERROR(3)
        P(K,4)=ERROR(4)
        P(K,5)=ERROR(5)
        P(K,6)=ERROR(6)
        P(K,7)=1.0
C      CONSTRUCTION AND SOLUTION OF NORMAL EQUATIONS
130     CONTINUE
135     CONTINUE
        ICOUNT=ICOUNT+1
DO 155 L=1, NOBS
        DO 150 MM=1, NN
        IF( L.EQ.MM) P(L,MM)=P(L,MM)*CON
150     CONTINUE
155     CONTINUE
        CALL GIO( P,PB,DELC,ITER,NITER,ICOUNT)
        IF(KS.EQ.1) THEN
            WRITE(11,904)
            STOP
        ENDIF

C      IMPROVEMENT OF THE INITIAL PARAMETERS OF THE MODEL
        WW=0.25*W
        IF(DABS(DELC(1)).GT.WW) DELC(1)=WW*DELC(1)/DABS(DELC(1))
        WD=W+DELC(1)
        ZZ=0.25*Z
        IF(Z.NE.0.0) THEN
            IF(DABS(DELC(2)).GT.ZZ)
                DELC(2)=ZZ*DELC(2)/DABS(DELC(2))
            ENDIF
            ZD=Z+DELC(2)
            DO 160 ID=1,NOBS
                IF(D.GT.X(ID)) THEN
                    DX=X(ID+1)-X(ID)
                    GO TO 165
                ENDIF
160     CONTINUE
            DDX=0.25*DX
            IF(DABS(DELC(3)).GT.DDX) DELC(3)=DDX*DELC(3)/DABS(DELC(3))
            DD=D+DELC(3)
            IF(DABS(DELC(4)).GT.0.2618)
                DELC(4)=0.2618*DELC(4)/DABS(DELC(4))
            QD=Q+DELC(4)
            CC=0.50*CONST
            IF(DABS(DELC(5)).GT.CC) DELC(5)=CC*DELC(5)/DABS(DELC(5))
            CD=CONST+DELC(5)
            AD=A+DELC(6)
            BD=B+DELC(7)
            IF(ZD.LT.0.0)ZD=0.001

```

```

      FUNCT1=0.0
C     CALCULATION OF THE ANOMALIES OF THE IMPROVED MODEL AND
C     OPTIMIZATION
      DO 170 K=1,NOBS
          XX=X(K)-DD
          TCAL(K)=DYKE(XX,WD,ZD,CD,QD)+AD*X(K)+BD
          FUNCT1=FUNCT1+(T(K)-TCAL(K))**2
          DIF(K)=(T(K)-TCAL(K))
170    DIF(K)=DABS(DIF(K))
          IF(FUNCT1.LE.FUNCT)THEN
              Z=ZD
              W=WD
              Q=QD
              D=DD
              CONST=CD
              A=AD
              B=BD
              FUNCT=FUNCT1
              NLAMDA=NLAMDA-1
              IF(NLAMDA.EQ.0.0)NLAMDA=1
              LAMDA=0.5*(2**(NLAMDA-1)-1)
              CON=1+LAMDA
          ELSE
              IF(LAMDA.GT.15.0)THEN
                  WRITE(11,909)
                  STOP
              ENDIF
              LAMDA1=LAMDA
              NLAMDA=NLAMDA+1
              LAMDA=0.5*(2**(NLAMDA-1)-1)
              CON=1+LAMDA
              WRITE(11,910)ITER,FUNCT1,LAMDA1,LAMDA
              GO TO 135
          ENDIF
175  CONTINUE
180  WRITE(11,905)
      WRITE(8,2)
      WRITE(13,4)
      2  FORMAT(5X,/'DIAGONAL ELEMENTS OF DATA  RESOLUTION ',/ )
      4  FORMAT(5X,/'DIAGONAL ELEMENTS OF MODEL  RESOLUTION ',/ )
801  FORMAT(I5,2F10.2,I5)
802  FORMAT(8F10.2)
802  FORMAT(8F10.0)
900  FORMAT(5X,'OBJECTIVE FUNCTION IS',F10.0,///5X,
      'MODEL PARAMETERS'/5X,16('-'))
901  FORMAT(5X,'DEPTH TO TOP OF DYKE IS',5X,F10.2/
      *5X,'HALF THE THICKNESS IS',7X,F10.2/
      *5X,'THE ORIGIN IS AT',12X,F10.2/
      *5X,'THETA-PHI IS',16X,F10.2/
      *5X,'CONST IS',20X,F10.2/
      *5X,'REGIONAL AT 1ST STATION IS',2X,F10.2/
      *5X,'REGIONAL GRADIENT IS',8X,F10.2///)

```

```

902  FORMAT(5X,48('-')/5X,'DIS-
      *',13X, 'OBSERVED',10X,'CALCULATED'/5X,
      *'TANCE',12X,'ANOMALY',11X,'ANOMALY',/5X,48('-'))
903    FORMAT(F10.2,8X,F10.0,8X,F10.0,2X,F6.0)
904    FORMAT(5X,'ILL-CONDITIONED MATRIX, THE',
      *'PROGRAM IS TERMINATED')
905    FORMAT(5X,'THE PARAMETERS MAY NOT IMPROVE WITH FURTHER'
      *1X,'ITERATIONS.'/5X,'THE LAST PRINTED TABLE
      *2 PROVIDES THE BEST',1X,'FITTING SOLUTION.'/5X,
      *3'STOP. END OF PROGRAM')
906    FORMAT(4X,48('-'))
907    FORMAT(5X,'THE OBJECTIVE FUNCTION INCREASES WITH CHANGES IN'
      *1X,'PARAMETERS.',/5X,'THE INCREMENTS ARE IGNORED. THE LAST'
      *1X,'PRINTED TABLE PROVIDES THE SOLUTION')
908    FORMAT(5X,'ITERATION NUMBER',I4)
909    FORMAT(5X,'LAMDA IS INCREASED TO UNUSUALLY',
      *'LARGE VALUE.',/5X,
      *'THE LAST PRINTED TABLE MAY PROVIDE THE BEST'
      *'FITTING SOLUTION.'
      */5X,'IF NOT,CHECK DATA OR MODEL. THE ITERATION',
      *'IS TERMINATED.')
910    FORMAT(5X,'ITERATION NUMBER IS',I4,/5X,23('-')/
      *5X,'THE OBJECTIVE FUNCTION IS ',F10.0,/
      *5X,'THE CURRENT VALUE OF LAMDA IS ',F5.1/5X,
      *'AS THERE IS AN INCREASE IN THE OBJECTIVE FUNCTION
      *'/5X,'LAMDA IS INCREASED TO',F5.1,
      *' AND THE CALCULATION IS REPEATED.'//)
911    FORMAT(/5X,'ANALYSIS BY GENERALISED INVERSE APPROACH ',//
      *5X,'INTERPRETATION OF MAGNETIC ANOMALIES OF DYKES'
      *1X,'USING',/5X,'AN ITERATIVE APPROACH. THE FOLLOWING
      *TABLES PROVIDE',/5X,'INFORMATION ON ANOMALIES AND'
      *'MODEL PARAMETERS AT',/5X,'THE END OF EACH
      *ITERATION.',/5X,51('-')//)
912    FORMAT(5X,'OBJECTIVE FUNCTION IS LESS THAN THE ALLOWABLE'
      *'ERROR.',/5X,'SOLUTION IN THE LAST PRINTED TABLE MAY BE'
      *'ACCEPTED.')
      STOP
      END

```

```

C SUBROUTINE SIMQ SOLVES A SET OF N-LINEAR EQUATIONS.
C THIS IS ADAPTED FROM THE IBM SUBROUTINE PACKAGE.

```

```
C
```

```
C INPUT
```

```
C -----
```

```
C   A   : MATRIX OF COEFFICIENTS STORED COLUMN-WISE
```

```
C   B   : VECTOR OF CONSTANTS
```

```
C   N   : NUMBER OF EQUATIONS AND VARIABLES
```

```
C
```

```
C OUTPUT
```

```
C -----
```

```
C   B   : COLUMN VECTOR OF FINAL SOLUTION VALUES
```

```

C   KS   : OUTPUT DIGIT = 0 FOR NORMAL SOLUTUION
C           = 1 FOR SINGULAR MATRIX
      SUBROUTINE SIMQ(A,B,N,KS)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(*),B(*)
      TOL=0.0
      KS=0
      JJ=-N
      DO 40 J=1,N
          JY=J+1
          JJ=JJ+N+1
          BIGA=0
          IT=JJ-J
          DO 10 I=J,N
              IJ=IT+I
              IF(ABS(BIGA)-ABS(A(IJ))) 5,10,10
5              BIGA=A(IJ)
              IMAX=I
10             CONTINUE
              IF(ABS(BIGA)-TOL) 15,15,20
15             KS=1
              RETURN
20             I1=J+N*(J-2)
              IT=IMAX-J
              DO 25 K=J,N
                  I1=I1+N
                  I2=I1+IT
                  SAVE=A(I1)
                  A(I1)=A(I2)
                  A(I2)=SAVE
25             A(I1)=A(I1)/BIGA
              SAVE=B(IMAX)
              B(IMAX)=B(J)
              B(J)=SAVE/BIGA
              IF(J-N) 30,45,30
30             IQS=N*(J-1)
              DO 40 IX=JY,N
                  IXJ=IQS+IX
                  IT=J-IX
                  DO 35 JX=JY,N
                      IXJX=N*(JX-1)+IX
                      JJX=IXJX+IT
35             A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
40             B(IX)=B(IX)-(B(J)*A(IXJ))
45             NY=N-1
              IT=N*N
              DO 50 J=1,NY
                  IA=IT-J
                  IB=N-J
                  IC=N
                  DO 50 K=1,J
                      B(IB)=B(IB)-A(IA)*B(IC)

```

```

        IA=IA-N
50      IC=IC-1
        RETURN
        END

```

```

C FUNCTION SUBPROGRAM FMAX CALCULATES THE DISTANCE BETWEEN
C TWO POINTS OF EQUAL ANOMALY, REDUCED FROM THEIR MAXIMUM
C VALUE BY A FACTOR

```

```

C
C INPUT
C -----
C N : NUMBER OF ANOMALIES IN THE PROFILE
C X : DISTANCE OF ANOMALY FROM AN ARBITRARY REFERENCE
C G : ANOMALY VALUES
C GMAX : AMPLITUDE OF THE MAXIMUM ANOMALY
C FAR : FACTOR BY WHICH THE ANOMALY SHOULD BE REDUCED
C

```

```

        FUNCTION FMAX(X,G,N,GMAX,FAR,XFAR1,XFAR2)
        IMPLICIT REAL*8 (A-H,O-Z)
        DIMENSION X(*),G(*)
        GH=FAR*GMAX
        K=0
        IF(GMAX)35,5,5
5       K=K+1
        IF(GH-G(K))15,10,5
10      XFAR1=X(K)
        GO TO 20
15      XFAR1=X(K-1)+(X(K)-X(K-1))*(GH-G(K-1))/(G(K)-G(K-1))
20      K=K+1
        IF(GH-G(K))20,25,30
25      XFAR2=X(K)
        GO TO 65
30      XFAR2=X(K-1)+(X(K)-X(K-1))*(GH-G(K-1))/(G(K)-G(K-1))
        GO TO 65
35      K=K+1
        IF(GH-G(K))35,40,45
40      XFAR1=X(K)
        GO TO 50
45      XFAR1=X(K-1)+(X(K)-X(K-1))*(GH-G(K-1))/(G(K)-G(K-1))
50      K=K+1
        IF(GH-G(K))60,55,50
55      XFAR2=X(K)
        GO TO 65
60      XFAR2=X(K-1)+(X(K)-X(K-1))*(GH-G(K-1))/(G(K)-G(K-1))
65      FMAX=DABS(XFAR2-XFAR1)
        RETURN
        END

```



```

C  SUBROUTINE MAXMIN CALCULATES THE POSITION AND AMPLITUDES
C  OF MAXIMUM AND MINIMUM ANOMALIES ON A MAGNETIC PROFILE
C  INPUT
C    N : NUMBER OF ANOMALIES IN THE PROFILE
C    X : DISTANCE TO ANOMALY POINTS MEASURED
C        FROM AN ARBITRARY REFERENCE (ANY UNITS)
C    T : GEOPHYSICAL ANOMALY
C  OUTPUT
C    MAX      : STATION NUMBER CLOSEST TO XMAX
C    MIN      : STATION NUMBER CLOSEST TO XMIN
C    XMAX     : POSITION OF THE MAXIMUM ANOMALY
C    XMIN     : POSITION OF MINIMUM ANOMALY
C    TMAX     : AMPLITUDE OF MAXIMUM ANOMALY
C    TMIN     : AMPLITUDE OF MINIMUM ANOMALY
C  SUPPORTING SUBPROGRAM: RINT
      SUBROUTINE MAXMIN(X,T,N,MAX,MIN,XMAX,XMIN,TMAX,TMIN)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(*),T(*)
C  TO FIND THE MAXIMUM ANOMALY
      TMAX=0.0
      DO 10 K=1,N
        IF(DABS(TMAX)-DABS(T(K)))5,5,10
5         TMAX=T(K)
          MAX=K
10        CONTINUE
      GR1=T(MAX)-T(MAX-1)
      GR2=T(MAX+1)-T(MAX)
      IF((GR1+GR2).NE.0.0)THEN
        XMAX=0.5*(X(MAX)+X(MAX-1)-GR1*(X(MAX+1)-X(MAX-1))
*        /(GR2-GR1))
        TMAX=RINT(X,N,T,XMAX)
      ELSE
        TMAX=T(MAX)
        XMAX=X(MAX)
      ENDIF
C  TO FIND THE MINIMUM ANOMALY
      TMIN=0.0
      XMIN=X(N)
      IF(TMAX)15,15,30
15     DO 25 K=1,N
        IF(TMIN-T(K))20,20,25
20         TMIN=T(K)
          XMIN=X(K)
          MIN=K
25        CONTINUE
      GO TO 45
30     DO 40 K=1,N
        IF(TMIN-T(K))40,35,35
35         TMIN=T(K)
          XMIN=X(K)
          MIN=K
40        CONTINUE

```

```

45   IF(TMIN)50,100,50
50   GR1=T(MIN)-T(MIN-1)
      GR2=T(MIN+1)-T(MIN)
      IF((GR1+GR2).NE.0.0)THEN
          XMIN=0.5*(X(MIN)+X(MIN-1)-GR1*(X(MIN+1)-X(MIN-1)))
*      /(GR2-GR1))
      TMIN=RINT(X,N,T,XMIN)
      ELSE
          TMIN=T(MIN)
          XMIN=X(MIN)
      ENDIF
100  RETURN
      END

```

```

C FUNCTION SUBPROGRAM DYKE CALCULATES THE MAGNETIC ANOMALY
C (IN GAMMAS) OF AN ARBITRARILY MAGNETIZED DYKE

```

```

C   INPUT
C       X   : DISTANCE OF POINT OF CALCULATION
C             FROM THE CENTRE OF THE DYKE (ANY UNITS)
C       W   : HALF WIDTH OF THE DYKE (SAME UNITS AS X)
C
C       Z   : DEPTH TO THE DYKE (SAME UNITS AS X )
C   SIZE : SIZE FACTOR, BEING A FUNCTION OF
C             INTENSITY OF MAGNETIZATION,(IN GAMMAS)
C             AND DIP OF THE DYKE
C       Q   : ANGLE, BEING A FUNCTION OF DIRECTION
C             OF MAGNETIZATION, DIP OF THE DYKE AND
C             THE DIRECTION OF MEASUREMENT (IN RADIAN)S)
C

```

```

FUNCTION DYKE(X,W,Z,SIZE,Q)
IMPLICIT REAL*8 (A-H,O-Z)
PIBY2=1.57079632
R=1.0E-03
C=DCOS(Q)
S=DSIN(Q)
Z2=Z*Z
R1=X+W
R2=X-W
R3=Z2+R1*R1
R4=Z2+R2*R2
ABSX=DABS(X)
IF(Z.NE.0.0)THEN
    ANG=DATAN(R1/Z)-DATAN(R2/Z)
ELSEIF(ABSX.EQ.W)THEN
    ANG=PIBY2
ELSE
    ANG=PIBY2*(1-(ABSX-W)/DABS(ABSX-W))
ENDIF
IF(R3.LT.R)R3=R
IF(R4.LT.R)R4=R

```

```

RTERM=DLOG(R3/R4)
DYKE=SIZE*(C*ANG-0.5*S*RTERM)
RETURN
END

```

```

C FUNCTION SUBPROGRAM ORIGIN LOCATES THE POSITION OF THE
C DYKE'S CENTRE

```

```

C INPUT:

```

```

C N : NUMBER OF ANOMALIES IN THE PROFILE
C X : DISTANCE OF ANOMALY FROM AN ARBITRARY REFERENCE
C T : MAGNETIC ANOMALY (GAMMAS)
C XMAX : POSITION OF MAXIMUM ANOMALY
C XMIN : POSITION OF MINIMUM ANOMALY
C TMAX : AMPLITUDE OF MAXIMUM ANOMALY
C TMIN : AMPLITUDE OF MINIMUM ANOMALY
C MAX : STATION NUMBER CLOSEST TO XMAX
C MIN : STATION NUMBER CLOSEST TO XMIN

```

```

FUNCTION ORIGIN(X,T,N,MAX,MIN,XMAX,XMIN,TMAX,TMIN)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION X(*),T(*)
T0=TMAX+TMIN
IF(XMAX-XMIN)50,5,5
5 IF(TMAX)10,30,30
10 K=MIN+1
15 K=K+1
IF(T0-T(K))15,20,25
20 ORIGIN=X(K)
GO TO 95
25 ORIGIN=X(K-1)+(X(K)-X(K-1))*(T0-T(K-1))/(T(K)-T(K-1))
GO TO 95
30 K=MIN+1
35 K=K+1
IF(T0-T(K))45,40,35
40 ORIGIN=X(K)
GO TO 95
45 ORIGIN=X(K-1)+(X(K)-X(K-1))*(T0-T(K-1))/(T(K)-T(K-1))
GO TO 95
50 IF(TMAX)55,75,75
55 K=MAX+1
60 K=K+1
IF(T0-T(K))70,65,60
65 ORIGIN=X(K)
GO TO 95
70 ORIGIN=X(K-1)+(X(K)-X(K-1))*(T0-T(K-1))/(T(K)-T(K-1))
GO TO 95
75 K=MAX+1
80 K=K+1
IF(T0-T(K))80,85,90
85 ORIGIN=X(K)
GO TO 95
90 ORIGIN=X(K-1)+(X(K)-X(K-1))*(T0-T(K-1))/(T(K)-T(K-1))

```

```

95    CONTINUE
      RETURN
      END

```

```

C FUNCTION SUBPROGRAM RINT CALCULATES BY INTERPOLATION
C THE VALUE OF A FUNCTION F(X) FROM TABULATED FUNCTION
C VALUES AT AN INTERMEDIATE VALUE OF X

```

```

C
C INPUT :
C -----
C       N : NUMBER OF DIGITIZED FUNCTION VALUES
C       X : ARRAY OF VALUES OF X AGAINST WHICH X

```

```

C
C       IS NOTED
C       FX : ARRAY OF VALUES OF F(X)
C       XX : VALUE OF X AT WHICH F(X) NEEDS TO BE
C           CALCULATED

```

```

FUNCTION RINT(X,N,FX,XX)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION X(*),FX(*)
DO 15 K=1,N
  IF(XX-X(K))5,10,15
5    M=K-1
  GO TO 20
10   RINT=FX(K)
  RETURN
15   CONTINUE
20   IF((XX-X(M)).GT.(X(M+1)-XX))M=M+1
  IF(M.LE.2.OR.M.GT.(N-2))THEN
    RINT=FX(M)
    RETURN
  ENDIF
  X1=X(M-2)
  X2=X(M-1)
  X0=X(M)
  X3=X(M+1)
  X4=X(M+2)
  T1=FX(M-2)
  T2=FX(M-1)
  T0=FX(M)
  T3=FX(M+1)
  T4=FX(M+2)
  A=(XX-X1)*(XX-X2)*(XX-X0)*(XX-X4)*(XX-X3)
  B=T1/((XX-X1)*(X1-X2)*(X1-X3)*(X1-X4)*(X1-X0))
  C=T2/((XX-X2)*(X2-X1)*(X2-X3)*(X2-X4)*(X2-X0))
  D=T0/((XX-X0)*(X0-X1)*(X0-X2)*(X0-X3)*(X0-X4))
  E=T3/((XX-X3)*(X3-X1)*(X3-X2)*(X3-X4)*(X3-X0))
  F=T4/((XX-X4)*(X4-X1)*(X4-X2)*(X4-X3)*(X4-X0))
  RINT=A*(B+C+D+E+F)
  RETURN
END

```

```

SUBROUTINE GIO(A,D,DELC,ITR,NITR,ICOOUNT)
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
   IMPLICIT REAL*8(A-H,O-Z)
PARAMETER (NM=26,M=26,N=7,NF=7)
DIMENSION A(NM,NM),W(NM),U(NM,NM),V(NM,NM),RV1(NM)
DIMENSION WW(NM),UU(NM,NM),VV(NM,NM),CT(NM,NM),DELC(NM)
DIMENSION (NM,NM),C(NM,NM),UT(NM,NM),VT(NM,NM),ILT(NM),D(NM)
LOGICAL MATU,MATV
VAR=1.0
MATU=.TRUE.
MATV=.TRUE.
IERR=0
   CALL SVD(NM,M,N,A,WW,MATU,UU,MATV,VV,IERR,RV1)
C   WRITE(2,*)'Unordered - decomposition '
C   CALL OUTPUT(A,UU,VV,WW,NM,M,N,NF)
DO 5 I=1,N
W(I)=-WW(I)
5   ILT(I)=I
   CALL ARRANGE(W,ILT,NM,N)
DO 6 I=1,N
6   W(I)=-W(I)
DO 7 J=1,N
K=ILT(J)
DO 7 I=1,M
7   U(I,J)=UU(I,K)
DO 8 J=1,N
K=ILT(J)
DO 8 I=1,N
8   V(I,J)=VV(I,K)
c*** WRITE(7,*)'ordered - decomposition '
CALL OUTPUT(A,U,V,W,NM,M,N,NF,ITR,NITR)
C   Computing the closeness ratio
SUM=0.0
DO 14 I=1,N
14  SUM=SUM+W(I)
SUM1=0.0
DO 100 II=1,NF
SUM1=SUM1+W(II)
CR=SUM1/SUM*100.0
IF(ITR.NE.NITR) GO TO 9999
WRITE(3,2)II,CR,ITR,ICOOUNT
WRITE(4,2)II,CR,ITR,ICOOUNT
WRITE(5,2)II,CR,ITR,ICOOUNT
WRITE(6,2)II,CR,ITR,ICOOUNT
WRITE(7,2)II,CR,ITR,ICOOUNT
WRITE(9,2)II,CR,ITR,ICOOUNT
WRITE(12,22)II,CR,ITR,ICOOUNT
22  FORMAT(1X,I3,F8.4,1X,I3,1X,I3)
2   FORMAT(/1X,'No.factrs cnsidered = ',I3/1X,
1'Closeness ratio = ',F10.3,3X,'ITR:', I3,3X,'ICOOUNT:',I3/)

```

```

9999 CONTINUE
      DO 1 I=1,M
      DO 1 J=1,II
1     X(I,J)=U(I,J)*W(J)
      CALL TRNSPZ(V,VT,NM,N,II)
      CALL MATMLT(X,VT,C,NM,M,N,II)
      IF(ITR.NE.NITR) GO TO 8888
      WRITE(6,*)'Back calculatin matrix '
      DO 332 I=1, M
      DO 332 j=1, N
332  WRITE(6,333)I,J,C(I,J)
333  FORMAT(1X,I3,1X,I3,1X,E12.5)
8888 CONTINUE
      CALL TRNSPZ(U,UT,NM,M,II)
      CALL MATMLT(U,UT,C,NM,M,M,II)
      IF(ITR.NE.NITR) GO TO 7777
      WRITE(5,*)'Data - Resolution matrix '
      DO 331 I=1, M
      DO 331 J=1, M
331  WRITE(5,333)I,J,C(I,J)
      WRITE(8,222)( C(L,L), L=1, M )
222  FORMAT(1X,20F6.3)
7777 CONTINUE
      CALL MATMLT(V,VT,C,NM,N,N,II)
      IF(ITR.NE.NITR) GO TO 6666
      WRITE(4,*)'Model - Resolution matrix '
      DO 330 I=1, N
      DO 330 J=1, N
330  WRITE(4,333)I,J, C(I,J)
6666 CONTINUE
C     Constructing of the generalized inverse operator
C     WRITE(7,*)'Generalized inverse operator '
      DO 9 I=1,N
      DO 9 J=1,II
9     X(I,J)=V(I,J)/W(J)
      CALL MATMLT(X,UT,C,NM,N,M,II)
      IF(ITR.NE.NITR) GO TO 5555
      WRITE(9,*) ' GENERALISED INVERSE OPERATOR '
      DO 335 I=1, N
      DO 335 J=1, M
335  WRITE(9,333) I,J,C(I,J)
5555 CONTINUE
C     Computing the generalized inverse solution
C     WRITE(7,*)'Generalized inverse solution '
      WRITE(7,*)'Generalized inverse solution '
      DO 11 I=1,N
      DELC(I)=0.0
      DO 11 J=1,M
11   DELC(I)=DELC(I)+C(I,J)*D(J)
      WRITE(7,12)(DELC(I),I=1,N)
12   FORMAT(7(E10.4,1X))
C     Computing the covariance of the solution

```

```

C      WRITE(7,*)'covariance of the solution '
      CALL TRNSPZ(C,CT,NM,N,M)
      DO 13 I=1,M
      DO 13 J=1,N
13     X(I,J)=C(I,J)*VAR
      CALL MATMLT(X,CT,C,NM,N,N,M)
100    CONTINUE
      RETURN
      END

SUBROUTINE SVD(NM,M,N,A,W,MATU,U,MATV,V,IERR,RV1)
      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION A(NM,NM),W(NM),U(NM,NM),V(NM,NM),RV1(NM)
      LOGICAL MATU,MATV
C      THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROCEDURE SVD,
C      NUM. MATH. 14, 403-420(1970) BY GOLUB AND REINSCH.
C      HANDBOOK FOR AUTO. COMP., VOL II-LINEAR ALGEBRA, 134-
      151(1971).
C      THIS SUBROUTINE DETERMINES THE SINGULAR VALUE DECOMPOSITION
C      T
C      A=USV OF A REAL M BY N RECTANGULAR MATRIX. HOUSEHOLDER
C      BIDIAGONALIZATION AND A VARIANT OF THE QR ALGORITHM ARE
      USED.
C
C      ON INPUT.
C
C      NM MUST BE SET TO THE ROW DIMENSION OF TWO-DIMENSIONAL
C      ARRAY PARAMETERS AS DECLARED IN THE CALLING PROGRAM
C      DIMENSION STATEMENT. NOTE THAT NM MUST BE AT LEAST
C      AS LARGE AS THE MAXIMUM OF M AND N.
C      M IS THE NUMBER OF ROWS OF A (AND U).
C      N IS THE NUMBER OF COLUMNS OF A (AND U) AND THE ORDER OF V.
C      A CONTAINS THE RECTANGULAR INPUT MATRIX TO BE DECOMPOSED.
C
C      MATU SHOULD BE SET TO .TRUE. IF THE U MATRIX IN THE
C      DECOMPOSITION IS DESIRED, AND TO .FALSE. OTHERWISE.
C      MATV SHOULD BE SET TO .TRUE. IF THE V MATRIX IN THE
C      DECOMPOSITION IS DESIRED, AND TO .FALSE. OTHERWISE.
C      ON OUTPUT.
C
C      A IS UNALTERED (UNLESS OVERWRITTEN BY U OR V).
C      W CONTAINS THE N (NON-NEGATIVE) SINGULAR VALUES OF A (THE
C      DIAGONAL ELEMENTS OF S). THEY ARE UNORDERED. IF AN
C      ERROR EXIT IS MADE, THE SINGULAR VALUES SHOULD BE
      CORRECT
C      FOR INDICES IERR+1,IERR+2,...,N.
C      U CONTAINS THE MATRIX U (ORTHOGONAL COLUMN VECTORS) OF THE
C      DECOMPOSITION IF MATU HAS BEEN SET TO .TRUE. OTHERWISE
C      U IS USED AS A TEMPORARY ARRAY. U MAY COINCIDE WITH A.
C      IF AN ERROR EXIT IS MADE, THE COLUMNS OF U CORRESPONDING

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C      TO INDICES OF CORRECT SINGULAR VALUES SHOULD BE CORRECT.
C
C      V CONTAINS THE MATRIX V (ORTHOGONAL) OF THE DECOMPOSITION IF
C      MATV HAS BEEN SET TO .TRUE.  OTHERWISE V IS NOT REFERENCED.
C      V MAY ALSO COINCIDE WITH A IF U IS NOT NEEDED.  IF AN ERROR
C      EXIT IS MADE, THE COLUMNS OF V CORRESPONDING TO INDICES OF
C      CORRECT SINGULAR VALUES SHOULD BE CORRECT.
C      IERR IS SET TO
C      ZERO          FOR NORMAL RETURN,
C      K             IF THE K-TH SINGULAR VALUE HAS NOT BEEN
C                   DETERMINED AFTER 30 ITERATIONS.
C
C      RV1 IS A TEMPORARY STORAGE ARRAY.
C
C      THIS IS A MODIFIED VERSION OF A ROUTINE FROM THE EISPACK
C      COLLECTION BY THE NATS PROJECT
C
C      MODIFIED TO ELIMINATE MACHEP
C      IERR = 0
C      DO 100 I = 1, M
C
C      DO 100 J = 1, N
C          U(I,J) = A(I,J)
100    CONTINUE
C      ..... HOUSEHOLDER REDUCTION TO BIDIAGONAL FORM
C      .....
C      G = 0.0D0
C      SCALE = 0.0D0
C      ANORM = 0.0D0
C
C      DO 300 I = 1, N
C          L = I + 1
C          RV1(I) = SCALE * G
C          G = 0.0D0
C          S = 0.0D0
C          SCALE = 0.0D0
C          IF (I .GT. M) GO TO 210
C
C          DO 120 K = I, M
120      SCALE = SCALE + DABS(U(K,I))
C
C          IF (SCALE .EQ. 0.0D0) GO TO 210
C
C          DO 130 K = I, M
C              U(K,I) = U(K,I) / SCALE
C              S = S + U(K,I)**2
130      CONTINUE
C
C          F = U(I,I)
C          G = -DSIGN(DSQRT(S),F)
C          H = F * G - S
C          U(I,I) = F - G

```



```

      IF (I .EQ. N) GO TO 190
C
      DO 150 J = L, N
        S = 0.0D0
C
        DO 140 K = I, M
140      S = S + U(K,I) * U(K,J)
C
        F = S / H
C
        DO 150 K = I, M
          U(K,J) = U(K,J) + F * U(K,I)
150      CONTINUE
190      DO 200 K = I, M
200      U(K,I) = SCALE * U(K,I)
C
210      W(I) = SCALE * G
        G = 0.0D0
        S = 0.0D0
        SCALE = 0.0D0
        IF (I .GT. M .OR. I .EQ. N) GO TO 290
        DO 220 K = L, N
220      SCALE = SCALE + DABS(U(I,K))
C
        IF (SCALE .EQ. 0.0D0) GO TO 290
        DO 230 K = L, N
          U(I,K) = U(I,K) / SCALE
          S = S + U(I,K)**2
230      CONTINUE
C
        F = U(I,L)
        G = -DSIGN(DSQRT(S),F)
        H = F * G - S
        U(I,L) = F - G
C
        DO 240 K = L, N
240      RV1(K) = U(I,K) / H
C
        IF (I .EQ. M) GO TO 270
C
        DO 260 J = L, M
          S = 0.0D0
C
          DO 250 K = L, N
250          S = S + U(J,K) * U(I,K)
C
          DO 260 K = L, N
            U(J,K) = U(J,K) + S * RV1(K)
260          CONTINUE
C
270          DO 280 K = L, N
280          U(I,K) = SCALE * U(I,K)

```

```

C
290     ANORM = DMAX1(ANORM,DABS(W(I))+DABS(RV1(I)))
300     CONTINUE
C
..... ACCUMULATION OF RIGHT-HAND TRANSFORMATIONS
.....
IF (.NOT. MATV) GO TO 410
C
..... FOR I=N STEP -1 UNTIL 1 DO -- .....
DO 400 II = 1, N
    I = N + 1 - II
    IF (I .EQ. N) GO TO 390
    IF (G .EQ. 0.0D0) GO TO 360
C
    DO 320 J = L, N
C
..... DOUBLE DIVISION AVOIDS POSSIBLE UNDERFLOW
.....
320     V(J,I) = (U(I,J) / U(I,L)) / G
C
    DO 350 J = L, N
        S = 0.0D0
        DO 340 K = L, N
340         S = S + U(I,K) * V(K,J)
C
        DO 350 K = L, N
            V(K,J) = V(K,J) + S * V(K,I)
350     CONTINUE
C
360     DO 380 J = L, N
        V(I,J) = 0.0D0
        V(J,I) = 0.0D0
380     CONTINUE
C
390     V(I,I) = 1.0D0
        G = RV1(I)
        L = I
400 CONTINUE
C
..... ACCUMULATION OF LEFT-HAND TRANSFORMATIONS
.....
410 IF (.NOT. MATU) GO TO 510
C
.....FOR I=MIN(M,N) STEP -1 UNTIL 1 DO -- .....
MN = N
IF (M .LT. N) MN = M
C
DO 500 II = 1, MN
    I = MN + 1 - II
    L = I + 1
    G = W(I)
    IF (I .EQ. N) GO TO 430
C
    DO 420 J = L, N
420     U(I,J) = 0.0D0
C
430     IF (G .EQ. 0.0D0) GO TO 475

```

```

      IF (I .EQ. MN) GO TO 460
C
      DO 450 J = L, N
        S = 0.0D0
C
        DO 440 K = L, M
440      S = S + U(K,I) * U(K,J)
C      ..... DOUBLE DIVISION AVOIDS POSSIBLE UNDERFLOW
      .....
        F = (S / U(I,I)) / G
C
        DO 450 K = I, M
          U(K,J) = U(K,J) + F * U(K,I)
450      CONTINUE
C
460      DO 470 J = I, M
470      U(J,I) = U(J,I) / G
C
        GO TO 490
475      DO 480 J = I, M
480      U(J,I) = 0.0D0
C
490      U(I,I) = U(I,I) + 1.0D0
500 CONTINUE
C      ..... DIAGONALIZATION OF THE BIDIAGONAL FORM .....
C      ..... FOR K=N STEP -1 UNTIL 1 DO -- .....
510 DO 700 KK = 1, N
      K1 = N - KK
      K = K1 + 1
      ITS = 0
C      ..... TEST FOR SPLITTING.
C      ..... FOR L=K STEP -1 UNTIL 1 DO -- .....
520 DO 530 LL = 1, K
      L1 = K - LL
      L = L1 + 1
      IF (DABS(RV1(L)) + ANORM .EQ. ANORM) GO TO 565
C      ..... RV1(1) IS ALWAYS ZERO, SO THERE IS NO EXIT
C      ..... THROUGH THE BOTTOM OF THE LOOP .....
      IF (DABS(W(L1)) + ANORM .EQ. ANORM) GO TO 540
530 CONTINUE
C      ..... CANCELLATION OF RV1(L) IF L GREATER THAN 1
      .....
540 C = 0.0D0
      S = 1.0D0
C
      DO 560 I = L, K
        F = S * RV1(I)
        RV1(I) = C * RV1(I)
        IF (DABS(F) + ANORM .EQ. ANORM) GO TO 565
        G = W(I)
        H = DSQRT(F*F+G*G)
        W(I) = H

```

```

      C = G / H
      S = -F / H
      IF (.NOT. MATU) GO TO 560
C
      DO 550 J = 1, M
          Y = U(J,L1)
          Z = U(J,I)
          U(J,L1) = Y * C + Z * S
          U(J,I) = -Y * S + Z * C
550      CONTINUE
C
560      CONTINUE
C
      ..... TEST FOR CONVERGENCE .....
565      Z = W(K)
          IF (L .EQ. K) GO TO 650
C
      ..... SHIFT FROM BOTTOM 2 BY 2 MINOR .....
          IF (ITS .EQ. 30) GO TO 1000
          ITS = ITS + 1
          X = W(L)
          Y = W(K1)
          G = RV1(K1)
          H = RV1(K)
          F = ((Y - Z) * (Y + Z) + (G - H) * (G + H)) / (2.0D0 * H
* Y)
          G = DSQRT(F*F+1.0D0)
          F=((X - Z) * (X + Z) + H * (Y / (F + DSIGN(G,F)) - H))/X
C
      ..... NEXT QR TRANSFORMATION .....
          C = 1.0D0
          S = 1.0D0
C
      DO 600 I1 = L, K1
          I = I1 + 1
          G = RV1(I)
          Y = W(I)
          H = S * G
          G = C * G
          Z = DSQRT(F*F+H*H)
          RV1(I1) = Z
          C = F / Z
          S = H / Z
          F = X * C + G * S
          G = -X * S + G * C
          H = Y * S
          Y = Y * C
          IF (.NOT. MATV) GO TO 575
          DO 570 J = 1, N
              X = V(J,I1)
              Z = V(J,I)
              V(J,I1) = X * C + Z * S
              V(J,I) = -X * S + Z * C
570      CONTINUE
C

```

```

575      Z = DSQRT(F*F+H*H)
        W(I1) = Z
C      ..... ROTATION CAN BE ARBITRARY IF Z IS ZERO .....
        IF (Z .EQ. 0.0D0) GO TO 580
        C = F / Z
        S = H / Z
580      F = C * G + S * Y
        X = -S * G + C * Y
        IF (.NOT. MATU) GO TO 600
        DO 590 J = 1, M
            Y = U(J,I1)
            Z = U(J,I)
            U(J,I1) = Y * C + Z * S
            U(J,I) = -Y * S + Z * C
590      CONTINUE
C
600      CONTINUE
        RV1(L) = 0.0D0
        RV1(K) = F
        W(K) = X
        GO TO 520
C      ..... CONVERGENCE .....
650      IF (Z .GE. 0.0D0) GO TO 700
C      ..... W(K) IS MADE NON-NEGATIVE .....
        W(K) = -Z
        IF (.NOT. MATV) GO TO 700
        DO 690 J = 1, N
690      V(J,K) = -V(J,K)
700      CONTINUE
        GO TO 1001
C      ..... SET ERROR -- NO CONVERGENCE TO A
C      SINGULAR VALUE AFTER 30 ITERATIONS ...
1000     IERR = K
1001     RETURN
        END

```